An application of neighbourhoods in digraphs to the classification of binary dynamics

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Goal

To classify binary dynamics on a network. Our main application is to classification of activity on the Blue Brain Project reconstruction of a small section of a rat's connectome.

Our methodology can also be applied to classification of binary dynamics on other directed graphs.

- How big? 31,346 vertices and 7,803,528 edges.
- The representing graph is directed with no self-loops, no multiple edges in the same direction.
- The vertices represent neurons and the edges synaptic connections.

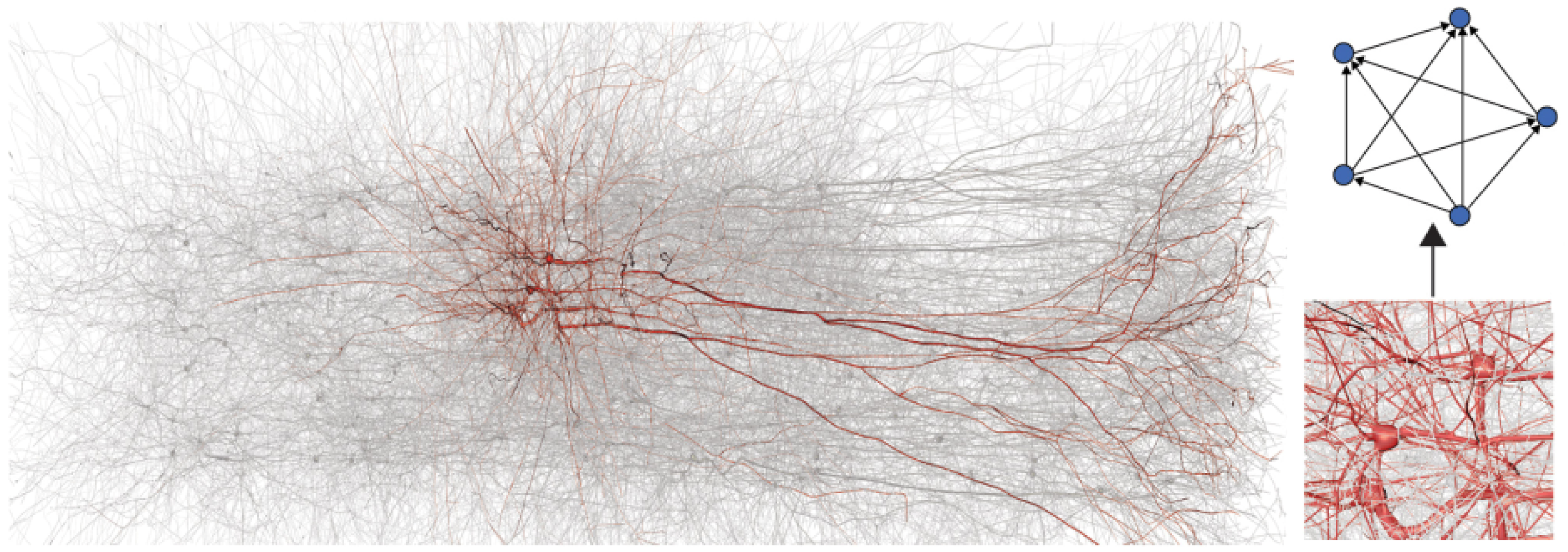
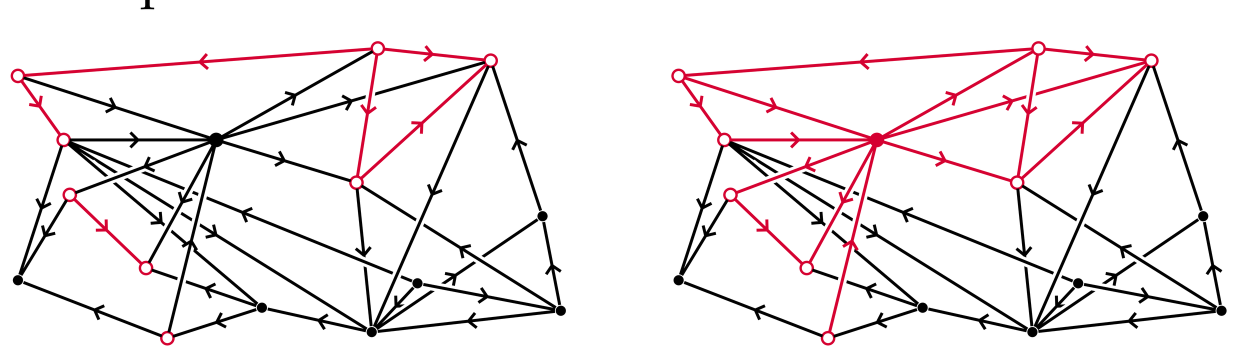


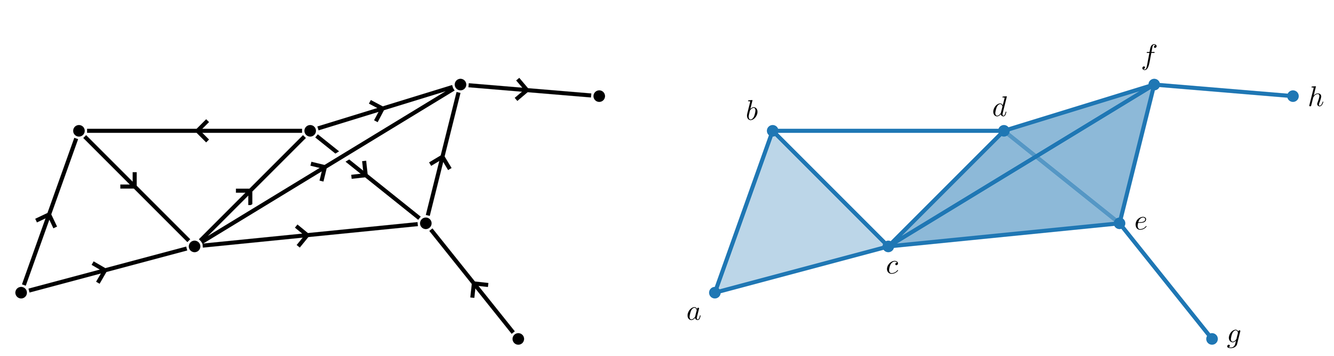
Figure: from https://doi.org/10.3389/fncom.2017.00048

Topology and directed graphs

We consider the closed neighbourhood (**tribe**) of a vertex v_0 (its **chief**) in a digraph $\mathcal{G}$ as computational units.



We realise it topologically by the **directed flag complex**: ordered simplicial complex where a k-simplex is a $(k+1)$-directed clique in $\mathcal{G}$.



$\{b,d\}, \{e,g\}, \{f,h\},$
$\{a,b,c\}, \{c,d,e,f\}$

A $(k+1)$-directed clique is an ordered set of vertices $(v_0,\ldots,v_k)$ such that there is an edge from v_i to v_j in $\mathcal{G}$ whenever $0 \le i < j \le k$

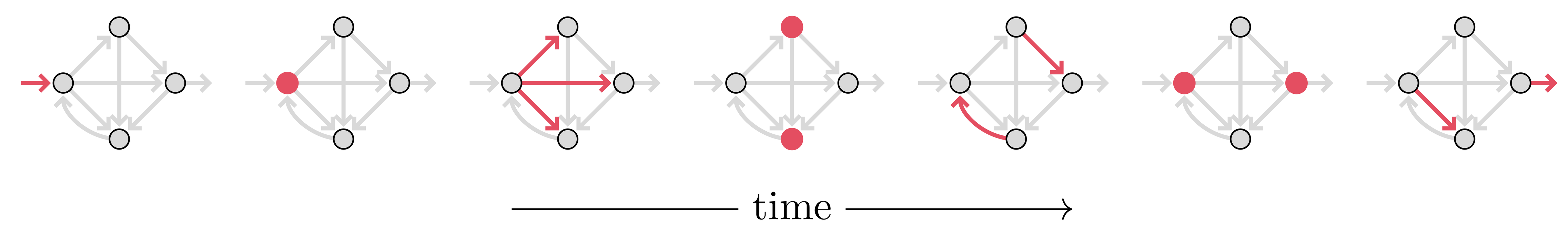
Local parameters

Our approach is *Stay Local* (to keep with the times):

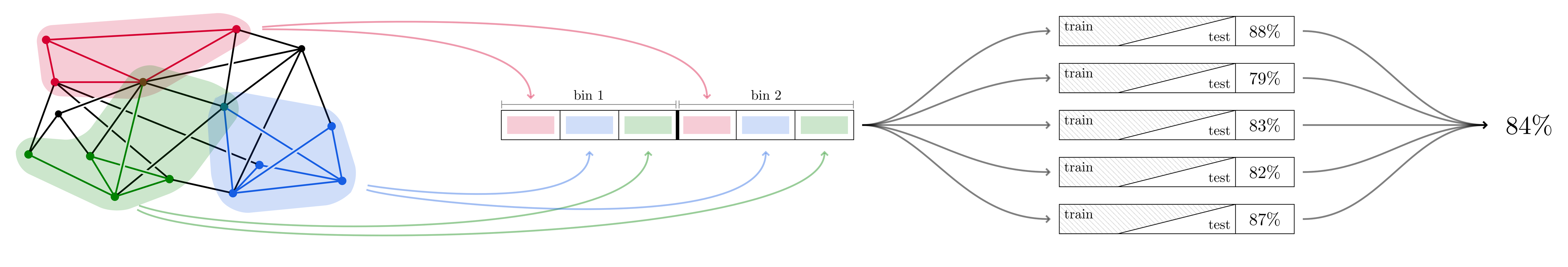
- select a small number of tribes that are **champions** with respect to a **sorting** parameter;
- restrict to specific subcomplexes of each of the tribes;
- compute the value of a given **feature** parameter for those subcomplexes.

Activity and methods

The 8 stimuli activity data: each experiment has a time period of 200 milliseconds and is repeated 557 times for each stimulus in a random sequence $\Longrightarrow$ a big matrix of the recorded activity.



Our pipeline extracts combinatorial/topological information from the active subtribes of the selected tribes and creates a feature vector for a support vector machine .

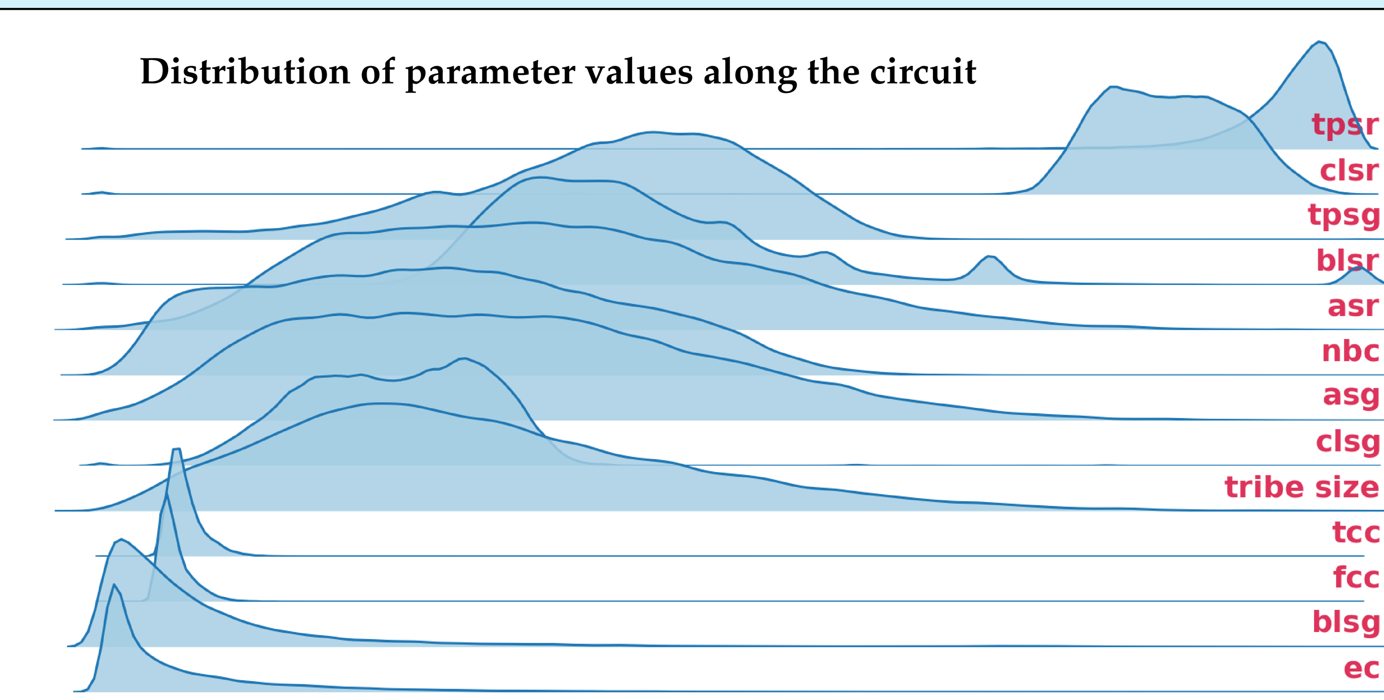


Results and analysis

Classification results

	fcc	tcc	ec	nbc	tribe size	asg	asr	blsg	blsr	clsg	clsr	tpsg	tpsr
fcc	33	33	41	38	44	48	43	51	54	14	43	45	45
tcc	21	21	35	34	36	50	42	50	53	12	25	30	38
ec	61	67	63	60	65	60	58	74	82	85	84	72	71
nbc	48	54	54	42	57	54	52	67	72	82	72	58	55
tribe size	70	75	73	75	75	75	72	83	87	84	88	82	82
asg	43	44	63	63	65	68	64	70	68	19	56	71	65
asr	43	44	63	63	65	68	64	70	68	19	56	71	65
blsg	31	33	30	30	33	31	29	45	57	19	54	39	48
blsr	58	65	42	48	45	41	38	67	73	84	76	67	66
clsg	36	37	37	40	39	41	38	52	65	20	57	50	58
clsr	43	45	44	52	47	44	40	66	73	20	60	67	69
tpsg	34	35	47	43	47	50	46	49	56	20	52	54	51
tpsr	42	44	52	54	56	51	46	66	71	20	58	69	68

Distribution of parameter values along the circuit



*sr denotes the spectral radius - the largest eigenvalue - of a Laplacian matrix

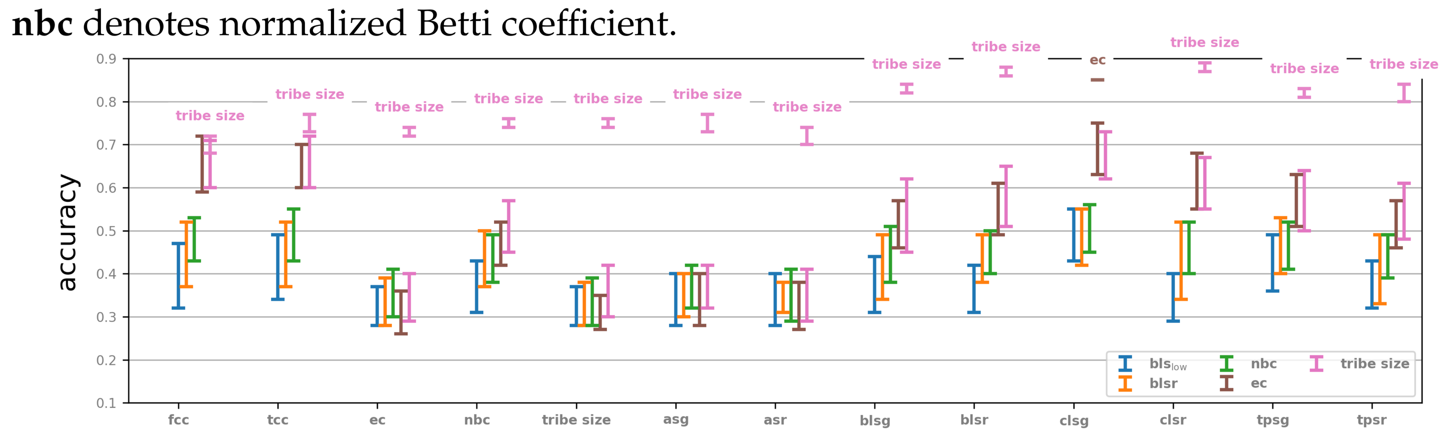
*sg denotes spectral gap - difference between the moduli of the two largest eigenvalues or the minimal non-zero eigenvalue of a Laplacian matrix.

*cc denotes a clustering coefficient for directed graph.

tribe size denotes number of active neurons of the tribe in a time bin.

ec denotes the Euler Characteristic.

nbc denotes normalized Betti coefficient.



One of our validation tests - "tribes" with same chief but different members
The labelled bars are the results from the normal classification.

Further reading

- P. Conceição, D. Govc, J. Lazovskis, R. Levi, H. Riihimäki, J. Smith (2021) - An application of neighbourhoods in digraphs to the classification of binary dynamics https://arxiv.org/abs/2104.06519
- Associated data and visuals: https://homepages.abdn.ac.uk/neurotopology/neighbourhoods