Matrix cracking parallel to the fibres is the initial failure mechanism in continuous fibre-reinforced composite laminates under static or fatigue in-plane tensile loading. It causes degradation of the overall stiffness properties of the laminate and triggers development of other damage modes such as delaminations.

**Introduction**

Crack density (crack/cm) $C^m = (2s)^{-3}$

Relative crack density $D^m = \frac{h_2}{s}$

Relative delamination area $D^\ell = \frac{\ell}{s}$

**Theoretical Modelling**

**Equivalent Constraint Model**

<table>
<thead>
<tr>
<th>Damaged laminate</th>
<th>Equivalent constraint laminate</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Equivalent Constraint Model Diagram" /></td>
<td><img src="image" alt="Equivalent Constraint Model Diagram" /></td>
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</table>

Constitutive equations:

$$\sigma^{(2)} = \left( \begin{array}{c} \sigma_x^{(2)} \\ \sigma_y^{(2)} \\ \tau_{xy}^{(2)} \end{array} \right) = \left[ \begin{array}{c} \sum_{i=1}^{3} A_i \left( \frac{D^m}{h_2} + C_j \right) \right] \left( \begin{array}{c} \epsilon_x^{(1)} \\ \epsilon_y^{(1)} \\ \gamma_{xy}^{(1)} \end{array} \right)$$

**Residual in-plane stiffness matrix of the ‘equivalent’ layer**

Local co-ordinates

$$[\mathbf{Q}^{(2)}] = \left[ \begin{array}{c} Q_{11}^{(2)} \\ Q_{22}^{(2)} \\ Q_{12}^{(2)} \end{array} \right]$$

Local delamination

$$G^m = -\frac{\partial U}{\partial \sigma^m}$$

Total strain energy stored in the laminate

$$U = \frac{1}{2} \frac{\partial U}{\partial \sigma^m}$$

**Strain Energy Release Rates (SERRs)**

Matrix cracking

$$G^m = -\frac{\partial U}{\partial \sigma^m}$$

Local delamination

$$G^\ell = \frac{h_2}{2} [\tau^{(1)} - \frac{\partial \mathbf{Q}}{\partial \sigma^m} | \sin \theta] \frac{\partial \mathbf{Q}}{\partial \sigma^m} \left| \sin \theta \right]$$

**Numerical Results**

**Matrix Cracking: Stiffness Reduction**

Material: glass/epoxy
Lay-ups: [30/-30]s and [55/-55]s

**Delamination: Stiffness Reduction**

Material: T800/3631 carbon/epoxy
Lay-ups: [0/90]s, [0/90]s

Relative delamination area: $D^\ell = 0.1$

**Delamination: Strain Energy Release Rate**

Material: AS4/3506-1 graphite/epoxy
Lay-up: [0/theta, 0/theta]s

**Conclusions**

- An approach based on the Equivalent Constraint Model and the 2-D shear lag method has been developed and applied to analyse damage mechanisms typically exhibited in angle ply laminates subjected to in-plane tensile loading.

- The approach enabled us to derive closed form expressions for strain energy release rates associated with matrix cracking and uniform local delaminations.

- As opposed to O’Brien’s expression, the present approach gives strain energy release rate for delamination that depends both on matrix crack density and delamination area and takes into account the cumulative effect of damage.

**REFERENCES**

