

### Some odd solar system questions answered

The questions here were inspired by reading some of the ‘letters from afar’ submitted by the class. The answers illustrate the application of basic physical principles and in this spirit approximations are made so that simple physics can be applied. For example, asteroids are taken as spherical, though real ones aren’t. Using a realistic shape would complicate the answer a lot and not change the gist of the result. Although most of the questions involve specific numbers, general conclusions can usually be drawn from the answers. The answers also show that ‘common sense’ isn’t much of a guide when it comes to expectations away from Earth.

I should add that the answers to these questions aren’t sitting ready-made in textbooks. You may need to research, on the web for instance, the appropriate physics formulae once you’ve thought through the principles that must be involved. You’ll find the key data needed on the web too. I’ve spelt out the logic of going from the question to the answer. The numbers in the answers have not been checked so it’s worth following the calculations yourself to see if they are right. Please report any errors spotted! I may regret saying this but if anyone on the course has any question you think could be answered in a like way, then e-mail it to me with your accompanying comments.

1 *Is it worth transporting hydrogen from Jupiter to Earth to use as a fuel?*

Answer: No!

At the visible surface of Jupiter the strength of gravity  $g = 25 \text{ m s}^{-2}$  and the distance  $r$  of the surface from the planet’s centre is  $71 \times 10^4 \text{ km}$ . The work required to lift 1 kg of hydrogen against Jupiter’s gravity so that it won’t fall back again is just  $gr = 1800 \text{ MJ}$ . [This is the same as the KE of 1 kg given the escape velocity at Jupiter of  $60 \text{ km s}^{-1}$ ].

The energy available from burning hydrogen in oxygen is  $143 \text{ MJ kg}^{-1}$ . End of story. In burning the hydrogen you get back less than one tenth of the energy it takes to liberate the hydrogen from Jupiter’s atmosphere. There are other energy factors involved such as the KE the hydrogen gains in falling the distance of Jupiter from the Sun to the distance of the Earth from the Sun and in falling to Earth from the upper atmosphere but even if these energies could be harvested they don’t make up for the initial extraction effort.

If you want energy from burning hydrogen then much the best option is to generate the energy necessary to electrolyse water from a renewable resource and then separate water into its components of oxygen and hydrogen. In the process you generate the oxygen needed to burn the hydrogen and hence won’t be depleting the world’s stock of oxygen when the hydrogen is burnt to produce water again. Sustainability in action. Bringing hydrogen or methane from the outer solar system, even from places more favourable than Jupiter, won’t be good for the Earth.

2 *Will a wind turbine generate useful energy for a Mars colony?*

Answer: No!

The density of the atmosphere at the surface of Mars is about  $0.018 \text{ kg m}^{-3}$ , compared with that of the Earth's atmosphere of just over  $1.2 \text{ kg m}^{-3}$ . [See the blue-panel section on the course web-page on 'planetary ballooning'].

The power,  $P$ , available from a wind turbine is  $P = \frac{1}{2} \alpha \rho \pi r^2 v^3$ , where  $\alpha$  is an efficiency factor determined mainly by the design,  $\rho$  is the density of the atmosphere, the crucial factor here,  $r$  is the radius of the blades and  $v$  the velocity of the wind. [This can be deduced from first year mechanics but for a statement see e.g. Wikipedia article 'Wind turbine'].

On Earth it takes a decent sized turbine to generate 1 kW of electricity, with a blade diameter of some 2 m in a wind speed of  $10 \text{ m s}^{-1}$ . For the same wind speed on Mars the turbine needs to have 10 times the radius (i.e. be gigantic), since the atmosphere is only about  $1/100^{\text{th}}$  as dense. It's true that gravity is only 38% of its value on Earth and that tall structures will be easier to make but against that the only natural resources are stone! Into the bargain, the wind on Mars isn't nearly as regular as we are accustomed to.

*Conclusion:* forget about wind turbines on Mars.

### 3 Could you cycle at 10 mph ( $4.4 \text{ m s}^{-1}$ ) on the surface of Venus?

Answer: No!

The aerodynamic drag force  $F_d$  of an object moving through a fluid is given by  $F_d = \frac{1}{2} \rho v^2 C_d A$ , where  $\rho$  is the density of the fluid,  $v$  is the speed relative to the fluid,  $C_d$  is a dimensionless number called the drag factor, which depends on the shape of the object and its texture and  $A$  is the cross-sectional area of the object at right angles to the flow. [See, e.g. the Wikipedia article 'Drag coefficient'].

For a person on a bike  $C_d$  is about 0.9;  $A \approx 1 \text{ m}^2$ ;  $\rho \approx 1.2 \text{ kg m}^{-3}$  on Earth and hence for  $v = 4.4 \text{ m s}^{-1}$  the drag force  $F_d$  is about 10 N. In short, when cycling at a modest pace on Earth you don't notice very much wind force against you.

On Venus,  $\rho \approx 70 \text{ kg m}^{-3}$  [See the blue-panel section on the course web-page on 'planetary ballooning']. Hence at the same speed the force against you is about 700 N. This is as big as many people's weight on Earth and larger than their weight on Venus (where gravity is 91% of Earth's gravity). There is no way you could power yourself to travel as fast as  $4.4 \text{ m s}^{-1}$ . Even at normal walking pace, the effort on Venus you would need to expend would be the same as walking straight into a near gale on Earth. Another comparison can be made by using the drag area figure ( $C_d A$ ) for a typical car of  $0.8 \text{ m}^2$ . A drag of 700 N is generated on Earth by a car travelling at  $38 \text{ m s}^{-1}$ , equivalent to  $137 \text{ km h}^{-1}$  or 86 mph. In other words, you'd need pretty well all the power of an average car going near its top speed, burning oxygen that isn't readily available on Venus at a good rate, to reach a speed of 10 mph.

*Conclusion:* powered movement over the surface of Venus is going to be very slow. Another conclusion is that a wind of 10 mph, if it occurs, would lift a person clean off their feet. This conclusion is re-inforced when you remember that it is about  $450 \text{ }^\circ\text{C}$  on the surface and hence anyone 'out there' would be wearing a very cumbersome furnace resistant protection suit.

4 *Will a car with its doors and windows sealed float on the surface of Venus?*

Answer: no, but a caravan would!

A Toyota Aygo is  $3.4 \text{ m} \times 1.6 \text{ m} \times 1.5 \text{ m}$  and has an unladen mass of 835 kg. If it were purely rectangular it would displace a volume of atmosphere of  $3.4 \times 1.6 \times 1.5 \text{ m}^3 = 8.16 \text{ m}^3$ . However, it's not rectangular. There's appreciable clearance space below the body and the bonnet is below the windscreen, obviously. Say it displaces  $5 \text{ m}^3$  of atmosphere and hence the upthrust on it will be equal to the weight of atmosphere displaced, namely  $\rho g V = 70 \times 8.9 \times 5 = 3115 \text{ N}$ . The weight of the car is  $mg = 835 \times 8.9 = 7532 \text{ N}$ . The weight is a bit more than twice the upthrust and hence a small car like this won't float, even though it is a light car.

What counts is clearly the mass in comparison with the atmospheric  $\rho V$ , the mass of an equal volume of atmosphere. A VW Caravelle has a volume of about  $18 \text{ m}^3$  and hence  $\rho V$  of 1260 kg compared with its mass of 2310 kg. It won't float either.

A modest caravan would have a volume of  $25 \text{ m}^3$  giving it a  $\rho V$  of 1750 kg and a mass of 800 kg. Hence it will clearly float over the surface of Venus.

People won't float but what about a craft like the space shuttle? From NASA's specified dimensions, a rough estimate of the volume it displaces is  $5.8 \times 10^3 \text{ m}^3$  and its unladen mass is 75,000 kg. Hence its  $\rho V$  factor is about 400,000 kg, over 5 times its mass. A craft like that has no chance of landing on the surface of Venus unless atmosphere is let into the craft as it descends. Lift-off becomes easy. Just pump out the native atmosphere within and it will float up.

In summary, objects designed to leave the Earth need to be as light as possible to reduce the need for fuel and hefty rockets yet objects designed to rest on Venus' surface need to be of average density greater than that of the atmosphere.

*Conclusion:* It will be very difficult to take large objects to the surface of Venus. Objects such as living quarters will need to be assembled, secured to the ground to prevent them floating away and then have the native atmosphere removed. They have to be strong enough not to collapse under an atmospheric pressure 90 times that on Earth (if their internal pressure is made similar to Earth's atmospheric pressure) and survive a standing temperature of 450 °C. It's a challenge too great for mankind at present.

5 *If you can exert enough muscular effort in a space suit to launch yourself upwards at a speed of  $1.5 \text{ m s}^{-1}$  what is the smallest spherical stony asteroid you can stand on such that you will return to the ground again after such a leap?*

Answer: about 2 km diameter

On Earth, an initial velocity  $u$  of  $1.5 \text{ m s}^{-1}$  will result in your centre of mass rising a height given by  $u^2/2g = 0.33 \text{ m}$ , not much without a spacesuit but probably a challenge in one. The question boils down to how large an asteroid has an escape velocity of  $1.5 \text{ m s}^{-1}$ ? If you are to return to the ground, the asteroid needs to be a little bit larger.

On the surface of a spherical body of radius  $r$ , the escape velocity  $v$  is when the kinetic energy  $\frac{1}{2}mv^2$  equals the gravitational potential energy (measured from infinity) of  $mgr$ .  $g$  is the local strength of gravity. i.e.  $v^2 = 2gr$ .

The local value of  $g$  is the value such that  $mg = GmM/r^2$ , where  $M$  is the mass of the asteroid. The mass is the volume  $\frac{4\pi r^3}{3}$  times the density  $\rho$ . Putting this together makes  $g = 4G\pi r\rho/3$ . Hence the escape velocity  $v^2 = 8G\pi\rho r^2/3$ . Writing this the other way around gives the radius  $r$  for a given escape velocity as  $r = \sqrt{3/8\pi\rho G}v$ .

Using  $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-1}$  and choosing  $\rho = 3000 \text{ kg m}^{-3}$ , with  $v = 1.5 \text{ m s}^{-1}$  the numbers pan out as  $r = 1.16 \times 10^3 \text{ m}$ , or 1.16 km. The mass  $M$  of such an asteroid is  $\frac{4\pi r^3 \rho}{3} \approx 2 \times 10^{13} \text{ kg}$ .

*Conclusion:* the figures are a stunning illustration of how weak a force gravity is. It takes the combined gravitational attraction of 20,000 million tonnes of stone to just prevent an astronaut leaping up and not disappearing off into space.

If you jumped up with a velocity of  $1.5 \text{ m s}^{-1}$  on an asteroid only a little bigger, you would travel many km into space before returning to the ground – pretty scary! I suspect it would be difficult to walk to a desired place on an asteroid just 2 km in diameter because the slightest upward velocity will cause you to leave the ground and travel a considerable distance.

- 6 *If the asteroid in the previous answer (radius 1.16 km) were rotating, what period of rotation would give an object just sitting on the equator enough speed to reach the escape velocity?*

Answer: about 1 h 20 min.

If the asteroid rotates in period  $T$  then it has a rotational velocity  $\omega \text{ rad s}^{-1}$  such that  $T = 2\pi/\omega$ . The speed of an object a distance  $r$  from the rotation axis is  $\omega r$ . On the equator,  $r$  is the radius and the escape velocity  $v$  is  $1.5 \text{ m s}^{-1}$ . Hence  $\omega = v/r = 1.5/1.16 \times 10^3 = 1.3 \times 10^{-3} \text{ rad s}^{-1}$ . This gives a rotation period of  $4.86 \times 10^3 \text{ s}$  or 1.35 h.

*Conclusion:* a modest rotation is enough to sweep rocks and boulders off the parts of the surface that are furthest from the rotation axis for a small asteroid.

- 7 *If you can lift a weight of 100 N in your spacesuit ( $\equiv$  a mass of 10 kg on Earth), how big a mass can you lift while standing on the asteroid of 1.16 km radius of the previous questions?*

Answer: a boulder about 4 m in diameter

You can lift a mass  $mg = 100 \text{ N}$ . Given  $g = 4G\pi r\rho/3 = 9.7 \times 10^{-4} \text{ m s}^{-2}$ , this gives a mass of  $1 \times 10^5 \text{ kg}$ , or 100 tonnes. A spherical boulder of this mass will have a radius of  $(3m/(4\pi\rho))^{1/3} = 2 \text{ m}$ . i.e. you could lift a boulder about twice your height.

- 8 *If you are standing on a surface in the outer solar system where the temperature is  $-200^{\circ}\text{C}$  and the outer skin of your space suit has the same temperature, at what rate is energy leaking into space from your suit?*

Answer: less than 1 W

The energy loss  $E$  in  $\text{J m}^{-2} \text{s}^{-1}$  from a body due to radiation is  $E = \alpha\sigma T^4$  where  $T$  is its absolute temperature,  $\sigma$  is Stefan's constant of  $5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$  and  $\alpha$  is a dimensionless number that gives the 'emissivity' of the body, the fraction of blackbody radiation that it emits.

If the space suit at a temperature of 73 K ( $-200^{\circ}\text{C}$ ) of say  $3 \text{ m}^2$  area were surrounded by space at a temperature of 3 K then the radiation loss would be  $\alpha \times 3(73^4 - 3^4) = 4.8\alpha \text{ W}$ . Since half of the surroundings are at the same temperature as the spacesuit, the loss rate is only half this or  $2.4\alpha \text{ W}$ . If the emissivity of the spacesuit is a third or less, then the rate of emission of energy will be less than 1 W. The next question shows that spacesuit outer temperatures need to be much higher to radiate the heat produced by an astronaut.

- 9 *Estimating the metabolic activity of an astronaut working in space to be 200 W, what emissivity should the spacesuit of  $3 \text{ m}^2$  surface area have if its outer surface is at a temperature of  $20^{\circ}\text{C}$  and it radiates at a rate that matches this activity?*

Answer: 0.16

Our astronaut working on a modest task might generate 200 W (about twice the basic metabolic rate of a fit person not yet middle-aged). If the emissivity of the spacesuit (in the infra-red where it emits its radiation) is  $\alpha$  and its surface area is  $3 \text{ m}^2$ , then the energy emitted by the spacesuit at temperature 293 K ( $20^{\circ}\text{C}$ ) is such that  $200 = 3\alpha\sigma 293^4$ . Hence  $\alpha = 200/(3 \times 5.67 \times 10^{-8} \times 293^4) = 0.16$ .

*Conclusion:* You might think that freezing to death is the hazard in the cold of space but overheating can be a serious problem. If the emissivity of the spacesuit is less than the figure above then for that rate of working it can't radiate away heat fast enough. If the outside of the suit cools down more, then the same applies or if the astronaut works harder there will be overheating within the suit. In reality, keeping the temperature inside the suit at a comfortable level is not an easy technological problem to solve.

- 10 *How large must solar panels be at the distances of the solar system planets and their moons to generate 1 KW of electricity when held perpendicular to the Sun's rays?*

Answer: there is a table at the end of the following calculation

Thinking of 'space' applications, we'll ignore atmospheric absorption for simplicity. We'll find the answer for the distance of the Earth from the Sun and use the inverse square law of radiation intensity to convert the area needed near the Earth to other distances. For distances closer than the Earth we'll take account of the reduced efficiency because of higher operating temperatures.

We need a figure for the efficiency of solar panels. Various web-sites quote numbers but it's not always clear what input spectrum is being assumed. We'll assume that 50% of the Sun's radiation is useful in activating the solar cell and that the solar panel is 15% efficient at converting this radiation to electricity. (You'll find web-sites discussing solar cells over 40% efficient but that is not the norm). There is also a decrease in efficiency with increasing temperature that will affect solar cells much nearer the Sun than the Earth. At the distance of Mercury, we'll take it that the cells are 50% less efficient. With these facts and a solar flux of radiation at the Earth of about  $1300 \text{ W m}^{-2}$  then we get a solar panel size at the Earth's distance of about  $10 \text{ m}^2$ . We'll take this as the baseline size.

Planet	Dist (AU)	Area ( $\text{m}^2$ )
Mercury	0.39	3
Venus	0.72	7.5
Earth	1.0	10
Mars	1.52	23
Jupiter	5.2	270
Saturn	9.6	920
Uranus	19.2	3700
Neptune	30.1	9000

For this calculation, planetary moons are at the same distance as planets. Pluto and the outer solar system are at 40 AU and beyond. In reality, solar panels degrade in efficiency in space due to bombardment by high energy particles so as time goes on the requirement gets bigger. A half-life of 30 years is not an unreasonable figure in the inner solar system.

*Conclusion:* enormous solar panels are needed in the outer solar system to generate significant power. Remember that the communications signal from a distant source decreases in strength with the square of the distance away of the source so more power is needed to communicate with the outer solar system. A better technology for the outer solar system may be to create enormous solar reflectors (much cheaper than enormous solar panels) to heat a central unit that generates electrical power by a standard turbine/alternator arrangement, or some variation of this. The technology of the moment is to put a nuclear-powered source in probes to the outer solar system.

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