

## About Orbits of Satellites and Planets

### Personal introduction

The mathematics of the ellipse, parabola and hyperbola (the ‘conic sections’) was a favourite subject on which to spend many lessons for the more advanced schoolboys and schoolgirls in my younger days. Indeed I had to buy an entire schoolbook on the subject (part III of Brown & Manson’s *Elements of Analytical Geometry*). The topic was more complicated than the mathematics of circles but simple enough that exact results could often be deduced. As it happens, the possible orbits of a body under the gravitational influence of another are conic sections, so textbooks on Dynamics for students always had sections on orbital motion, largely as mathematical exercises decades before any satellites were launched. Indeed generations of would-be physicists were brought up with the works of Routh, Lamb or Loney knowing much more about orbits than any modern Honours physics student.

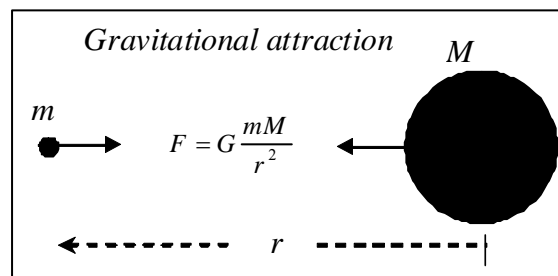
The first artificial satellite (Sputnik) was launched in 1959 and over the next two decades mankind went to the Moon and probes were launched to all the nearby planets. At last, you may think, ‘orbits’ became a subject of real current interest. Maybe it did, but over the same period extensive conic section teaching fell out of the advanced school syllabus and many details about orbits fell out of most physicist’s degree programme. Even basic orbital physics has now become a specialist topic, something one tends to learn only if there is a special need, not a topic of general interest. Yet ‘orbits’ have never been more relevant to topics of everyday life, from GPS satellites, environmental sensing and imaging satellites, space probes to interests in new moons discovered around solar system planets and new planets discovered around other stars. The intention of this piece is to say something about the principles involved, highlight some results about orbits and their generality, introduce a little of the vocabulary used without indulging in mathematical proofs. As I write this introduction I have no idea how this piece will turn out, for there is the electronic equivalent of a blank piece of paper in front of me.

Some time later: my ‘paper’ is no longer blank and I’ll present 15 useful facts about orbits (the ‘**Results**’) and give illustrations of their relevance. I’ll end by introducing a number of other topics without going into much detail about them.

### The ingredients

The first ingredients are two orbiting bodies: a mass  $m$ , orbiting another mass  $M$  under the influence of their mutual gravity described by Newton’s law, namely that the force each exerts on the other is  $F = G \frac{mM}{r^2}$ , the force being directed along the line between them. The symbol  $r$  represents the distance between the

bodies and  $G$  is the ‘universal gravitational constant’ whose value in SI units is  $6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ . Newton’s law is strictly true for ‘particles’ of no significant size. Clearly even a mathematician would acknowledge that a planet or the Sun isn’t a ‘particle’. However, Newton himself deduced that the gravitational effect of a uniform spherical body was the same as that of a particle of the same mass located at its centre, so long as one was beyond the



surface of the body. This result greatly simplifies talking about satellites orbiting even close to the Earth.

Gravity is usually described as a ‘central force’ because the direction of the force is towards the centre of the attracting ‘particle’. The spherical result also hints that small irregularities in the shape of the Earth will make real orbits slightly different from their ideal shape, which is true and an analysis of these differences has told us some facts about the internal structure of the Earth. Satellites around other planets (and they’ve orbited all the planets out to and including Saturn) provide equivalent information away from the Earth. The spherical result also hints that working out the orbit of a probe around a very irregular body such as a comet or any of the smaller asteroids will be a complicated problem, which it is.

The second ingredient in the presentation here is the simplification that the mass of the orbiting body is small compared with the mass of the parent body. This is obviously true for satellites and is even true for planets orbiting the Sun. The Sun’s mass is a third of a million times greater than the Earth’s mass. This simplification just makes the maths simpler but doesn’t alter basic results. For instance the mutual orbits of binary stars are in most respects similar to planetary orbits but one has to be more careful about defining quantities such as the frame of reference for the orbit.

The third set of ingredients needed to describe orbits are Newton’s laws of motion and concepts that derive from them. Basically ‘ $\mathbf{F} = m\mathbf{a}$ ’ where  $\mathbf{a}$  is the acceleration of the body. From the acceleration can be derived velocity  $v$ , and such detail as distance travelled, period of the orbit, etc.

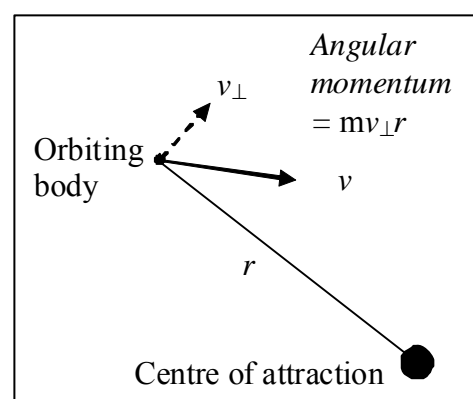
If you look back at the form of the gravitational force  $F$  you’ll see that the mass  $m$  of the body occurs on both sides of Newton’s law of motion ‘ $\mathbf{F} = m\mathbf{a}$ ’ and hence we can deduce our first result.

**Result 1** *The orbit of a body doesn’t depend on its mass.* Hence the mathematics relevant to orbital motion is usually written for unit mass (1 kg) and I’ll do that in later sections. Of course the energy required to put something into orbit does depend on its mass but once initial conditions of velocity and location are specified then the mass of the orbiting body doesn’t come into determining what the orbit will be.

The initial velocity of the orbiting body defines a line in space and this along with the centre of the attracting body defines a plane. Hence:

**Result 2** *The orbit always lies in a plane containing the centre of attraction.* For example, Earth satellites always orbit in a plane containing the centre of the Earth. Geostationary satellites therefore must lie above the equator. It’s not possible to have a satellite stationary above Aberdeen (latitude  $57^\circ$  N) or, indeed, any place not on the equator since the orbit required would be in a plane that doesn’t contain the centre of the Earth.

The *angular momentum* of a body about a point is the product of three quantities: the mass of the body ( $m$ ), the distance of the body from the point ( $r$ ) and the



component of the body's velocity perpendicular to the line from the point ( $v_{\perp}$ ). To change the angular momentum of a body there must be a twist on it, as measured from the point. Taking the obvious 'point' as the centre of attraction, then another result immediately follows from the central nature of gravitational attraction, namely result 3.

**Result 3** *The angular momentum of an orbiting body about the centre of attraction is constant.* This implies that orbital motion is in fact simpler than many other kinds of motion. We shan't make the deduction but Kepler's second law about a planet (or comet or asteroid) sweeping out equal areas in equal times follows from this result.

The constancy of angular momentum played an important role in the formation of the solar system from a rotating cloud of gas and dust. The outer parts of the cloud were orbiting around the inner material but the whole cloud was slowly pulled together by its own gravity. Collapse parallel to the axis of spin would have converted the cloud to a disk and collapse perpendicular to the axis of spin was less easy for it required an increase in rotational speed of the cloud as it shrunk because of conservation of angular momentum. This is the reason the solar system is pretty flat.

#### *The energy of an orbiting body*

Orbiting bodies have two kinds of energy: kinetic energy (abbreviated *KE*) and potential energy (*PE*). The kinetic energy is just  $\frac{1}{2}mv^2$  as you would expect and hence depends on how fast the body is travelling. The potential energy is  $PE = -\frac{GM}{r}$  and hence depends on how far from the centre of attraction ( $r$ ) the body is. The total energy  $E$  is just the sum of the two, i.e.  $E = KE + PE$ .

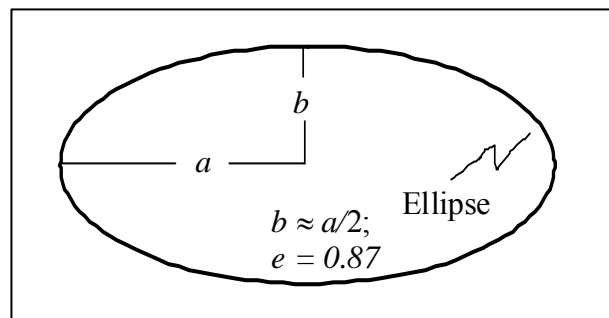
**Result 4** *If no force other than the central force acts on the orbiting body, then its total energy is constant.* This is just a statement of the law of conservation of energy. As the body changes its distance from the centre of attraction then it must change its speed and there is a conversion between potential energy and kinetic energy.

**Result 5** a. *If the total energy ( $E$ ) is positive, the orbit is a hyperbola.* The hyperbola is an open curve and the body will fly off 'to infinity'.

b. *If the total energy is zero, the orbit is a parabola.* The parabola is also an open curve and the body will eventually fly off.

c. *If the total energy is negative, then the orbit is an ellipse.* The ellipse is a closed curve so the body will circulate around the centre of attraction. This is the most interesting case. The magnitude of the gravitational *PE* is greater than the magnitude of the kinetic energy for all elliptically orbiting bodies.

An ellipse is a body whose shape is described by two parameters, slightly more complicated than a circle, which just needs one parameter, its radius. There is a choice of which parameters to use for an ellipse but a very common pair is its major and minor axes, denoted  $2a$  and  $2b$ . See the accompanying figure. Ellipses can vary from circular (the two axes are the same length) to very long



and thin ( $a \gg b$ ). A useful quantity to designate how fat or thin an ellipse is is its *eccentricity*, always represented by the symbol  $e$ .  $e = 0$  represents a circle and  $e$  near 1 a very thin ellipse. Venus' orbit is almost circular ( $e = 0.0067$ ); Halley's comet has a highly eccentric orbit ( $e = 0.967$ ).  $b$ ,  $a$  and  $e$  are related:  $b^2 = a^2(1 - e^2)$ . It's not obvious but a parabola has  $e = 1$  and a hyperbola has  $e > 1$ .

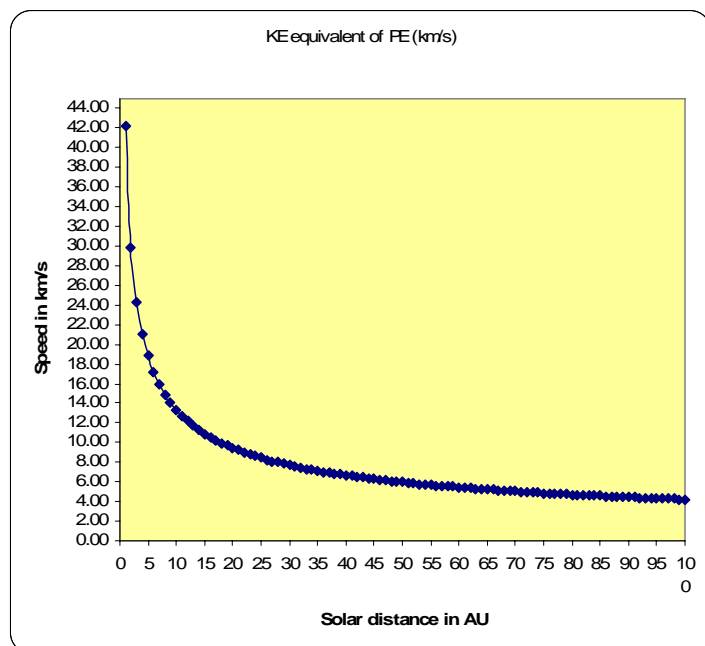
I said above that as a body varies its distance from the centre of attraction, the balance between  $PE$  and  $KE$  changes. Going towards the centre of attraction, the potential energy decreases and the kinetic energy increases. Since  $PE$  doesn't depend on direction, we have another result:

**Result 6** *The speed of a body depends on how far it is from the centre of attraction not on which direction it is going at that distance.*

The speed that must be given to a body to put it into a parabolic orbit (in which it will never return) is known as *the escape velocity*. In this case  $E = 0$  and the  $KE = -PE$ . i.e.  $\frac{1}{2}v_{exc}^2 = GM/r$ .

**Result 7** *The escape velocity  $v_{exc}$  is given by  $v_{exc}^2 = 2GM/r$  where  $r$  is the initial distance of the body from the centre of attraction.* This is the minimum speed necessary to ensure the body will not enter a returning orbit. Giving it more than this speed will put it in a hyperbolic orbit that will also ensure it won't return.

The potential energy has a large negative value for objects no further from the Sun than the planets. The Excel graph here shows the speed an object needs to have in  $\text{km s}^{-1}$  for its kinetic energy to match its potential energy in size. If that were to happen, the total energy would be zero and the body would have enough energy to escape from the Sun's gravitational pull. The points are plotted from 1 AU (the distance of the Earth from the Sun) to 100 AU, about twice the greatest distance of Pluto.

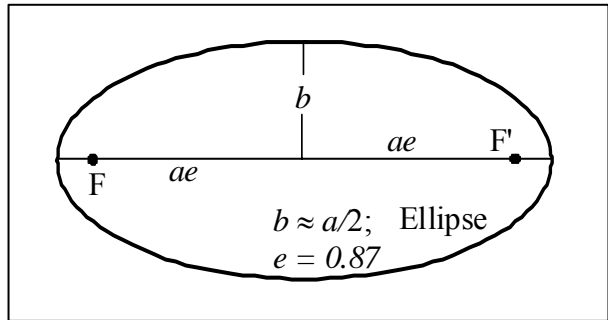


*The focus of the orbit*

It almost seems 'obvious' that the centre of attraction should be the centre of the orbit but like a good many 'obvious' implications, this one is wrong.

**Result 8** *The centre of attraction lies at the focus of the orbit, a special point in relation to a conic section.* I'm going to concentrate on ellipses. An ellipse has two foci  $F$  and  $F'$  each a distance  $ae$  from the centre along the major axis (see the nearby figure). Since  $e$  is zero for a circle, then a circular orbit does indeed have the centre of attraction at the centre of the circle. For Halley's comet, the focus of the ellipse is very near the end of the major axis and

this does make sense in that the comet is known to go out a long way from the Sun, come in close to the Sun, swing round and then go out again. If the ellipse in the diagram represents an orbit, then the centre of attraction is at one of the foci, F or F'. One consequence of this is that the Earth is not at the same distance from the Sun in winter and summer. In fact it is closer to the Sun in northern hemisphere winters.



*What orbit will a body follow?*

Suppose a body (it could be a satellite, a spacecraft, an asteroid chunk or even a planet) is launched with velocity  $v$  in some direction at a distance  $d$  from the centre of attraction, what orbit will the body follow? Earlier it was said (Result 1) that the orbit didn't depend on the body's mass so **from here on I'll consider a body of unit mass**. Finding the answer is not quite straightforward. Even a good physics student given this question 'blind' would struggle to work out exactly how to manipulate the mathematics most effectively to come up with the answer. The initial conditions define the angular momentum (called  $h$  in many texts) of the body. Anyway, I'm going to quote the answer.

**Result 9** 
$$a = -\frac{GM}{2E}, \quad e^2 = 1 + \frac{2Eh^2}{(GM)^2}.$$

That's it. At least these 2 parameters give the shape of the orbit. Notice that the extent of the ellipse, the semi-major axis  $a$ , is determined only by the body's energy. To find the orientation of the ellipse in space needs a bit more work, which we shan't do here. The relationships above give the major axis and the eccentricity of the orbit in terms of the initial angular momentum  $h$  per unit mass and the energy  $E$  per unit mass. I'm most interested in elliptical orbits for which, as we've seen, the total energy  $E$  is negative. Using the relationship above, the minor axis parameter  $b$  is therefore given by  $b^2 = -\frac{h^2}{(2E)}$ .

For a body of unit mass travelling in a circular orbit with velocity  $v$  the  $PE = -v^2$  and the  $KE = \frac{1}{2}v^2$ . Hence the total energy  $E = -\frac{1}{2}v^2$ .

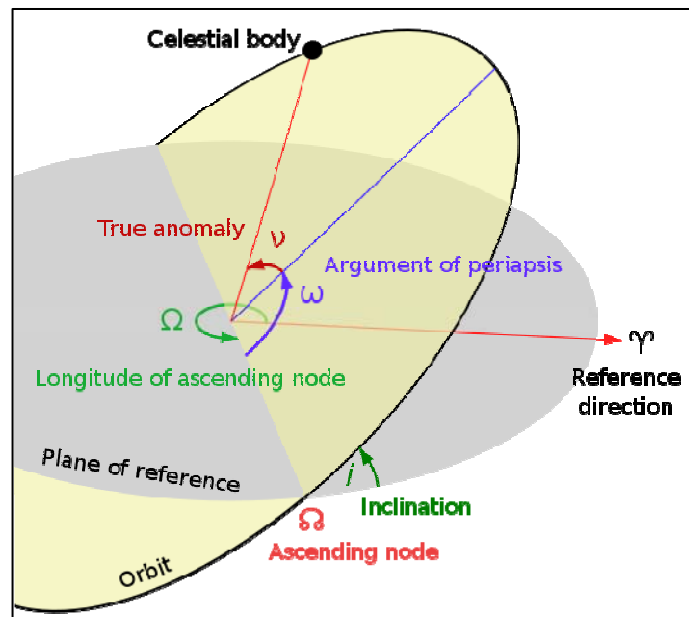
**Result 10** *For a circular orbit the kinetic energy is equal to half the magnitude of the potential energy.*

The kinetic energy is therefore quite high in relation to the potential energy. Most of the planets have pretty circular orbits (in addition to the example of Venus given above, Mercury is the least circular with  $e = 0.2056$ ; Earth  $e = 0.0167$ , Mars  $e = 0.0935$ , Jupiter  $e = 0.0489$ , Saturn  $e = 0.0565$ , Uranus  $e = 0.0457$ , Neptune  $e = 0.013$ ). Increasing the speed of a body in circular orbit, if that were possible, by a factor of  $\sqrt{2}$  will double its kinetic energy and hence make its total energy zero. This will be enough to turn the orbit into a parabola, taking the orbiting body off to infinity.

Another implication of this result is that since the potential energy is less in magnitude further from the centre of attraction, the further away a body is the slower it will be going. This is why the further out planets from the Sun travel more slowly in their orbit. More distant Earth satellites, which need more energy and hence fuel to put them into orbit, have less speed than lower satellites, which seems a bit counter-intuitive.

**Result 11** *6 parameters are needed to describe the position of a body in orbit, known as the 'orbital elements'. This is mainly a geometrical statement. However, one of the implications is that if a new body is seen in the sky such as a possible asteroid or trans-Neptunian object, then its orbit can be deduced from 3 observations of its declination and right ascension (essentially its celestial latitude and longitude) made with a modest time in between them, say a couple of weeks. Of course the observations will have been made from the moving, rotating platform of the Earth so some mathematics must first be done to find the equivalent observations relative to a plane containing the Sun before the orbital parameters can be found. Gauss showed how to do this in the 19<sup>th</sup> century. Taking more observations or a longer time between them will give the orbital elements to higher accuracy. A related question is how do NASA and ESA find the orbits of their space-probes to pin-point accuracy when the probes are too far away to be seen? I'm leaving this question for your exploration!*

The adjacent diagram from Wikimedia Commons will make it clearer why 6 elements are needed and what they are. We start with a reference plane in which there is a reference direction. The orientation of the plane of the orbit has to be defined in space. The orbit intersects the reference plane in two points, called the *ascending node* and *descending node*. These nodes turn out to be a central concept when trying to understand the occurrence of eclipses of the Sun and Moon. They are also important in understanding if an asteroid in an orbit that crosses the Earth's is likely to collide with the Earth. Anyway, defining the orbital plane in space takes 2 parameters, one to define its *inclination* ( $i$ ) relative to the reference plane and the next to define the angle between the reference direction and the line to the ascending node ( $\Omega$  in the diagram).



Next, the shape of the orbit is defined by the elliptical constants  $a$  and  $e$  as discussed earlier. The direction of the major axis (glorified with the words the *augment of the periapsis*,  $\omega$  in the diagram) must be defined to determine the orientation of the orbit within its plane. The final element is the so-called *epoch* of the body in the orbit, basically how far around the orbit the particle is (designated the *true anomaly*,  $\nu$  (Greek 'nu') in the diagram) the angle measured from the major axis. Phew.

**Result 12** *The period of revolution  $T$  (in planetary terms, the body's year) can be shown to be given by  $T^2 = 4\pi \frac{a^3}{GM}$ .*

This result puts into mathematical symbols Kepler's third law. Kepler deduced his 'law' from the half dozen special cases for which he had planetary data but Newton showed that it was firmly based on the mathematics of elliptical orbits. Only one orbital parameter is involved, namely the semi-major axis of the elliptical orbit. Hence all bodies with the same value of  $a$  have the same period whatever the eccentricity of their orbits.

In terms of dynamical parameters:

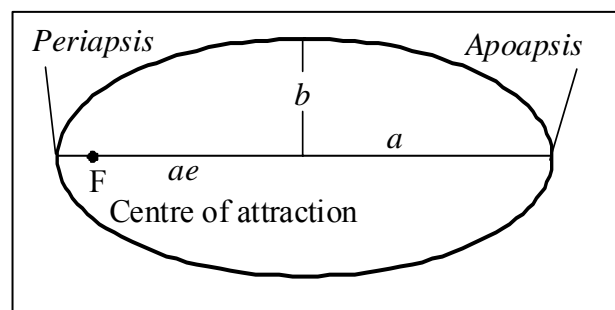
**Result 13**  $T^2 = 4\pi \frac{(GM)^2}{(-2E)^3}$ , showing that the period depends only on the total energy of the orbiting body. Only one dynamical parameter is involved.

*The distances of closest approach and furthest out*

When the orbiting body is closest or furthest from the centre of attraction, it is travelling at right angles to the radius from the centre. These points are called the *apses* of the orbit. The *periapsis* is when the body is closest, the *apoapsis* when the body is furthest. [When the centre of attraction is the Sun, the words *perihelion* and *aphelion* are used for the nearest and furthest points; when the centre of attraction is the Earth, the words *perigee* and *apogee* are used; in relation to a star, *periastron* and *apastron*]. This is rather a lot of Greek to get across a simple idea.

**Result 14** From the picture of the ellipse, the *periapsis distance from the centre of attraction is  $a(1 - e)$  and the apoapsis distance  $a(1 + e)$ .*

For a planet, these distances strongly determine how much solar heating it will get and the change in solar radiation and heating that will take place during a planetary year. For a satellite, the periapsis may be important in determining the drag it will experience in the upper atmosphere and hence its lifetime before it comes to Earth.



*Will it crash and burn?*

Satellites in low Earth orbit, say 1000 km high, have to be in a nearly circular orbit even if they change their height by several hundred km. What counts is their distance from the Earth's centre and since that is over 7000 km then a few hundred km change in distance is still a nearly circular orbit. In short, they must have been given a kinetic energy that is about half the magnitude of their negative potential energy, which at 7000 km is  $-GM/7 \times 10^6 = -3.4 \times 10^6 \text{ J kg}^{-1}$ . Half this magnitude implies a speed of  $1.85 \text{ km s}^{-1}$  or about  $6600 \text{ km h}^{-1}$ . You can see why most satellites are launched from near the equator because the rotation of the Earth gives points near the equator a speed of about  $1600 \text{ km h}^{-1}$ , which is at least a start. Many Earth monitoring satellites are in low-Earth orbit and if they aren't given the necessary speed they will indeed crash and burn. Another aspect is that satellites in low-Earth orbit don't have to lose much of their speed due to slight drag of the very thin extended Earth's

atmosphere to have their orbital parameters changed enough to become slightly non-circular. They will then enter the lower atmosphere and burn up.

Objects in orbit around the Sun are not only launched by mankind but are launched as a result of interactions within the belts of asteroids, the Kuiper belt and the Oort cloud. If an object's orbit is conspicuously elliptical the object will head in towards the Sun. Is it likely any will crash and burn in the Sun, or at least disintegrate near the Sun? The answer is 'yes', as can be seen using the information in earlier sections. Actually, the answer is also 'yes' from images sent back by the Soho probe that has been looking almost unblinkingly at the Sun for over a decade. These images show that over a year quite a number of comets that hadn't been spotted before plunge into the outer reaches of the Sun.

**Result 15** *If an object is 'launched' in the solar system so that most of its energy is its potential energy, then its perihelion distance will be approximately  $h^2/2GM$ . ( $h$  is its angular momentum per unit mass about the Sun).*

The result can be found by some manipulation of the relationships already given. If this distance is less than about 20 million km then a comet or man-made object isn't likely to survive such a close approach to the Sun. Icy bodies in the Kuiper belt at a distance of 50 AU from the Sun have orbital velocities about  $4.2 \text{ km s}^{-1}$ . Most of this speed must be removed by some interaction or collision if a fragment is going to end up by going close to the Sun. This doesn't happen often but it does happen several times a year.

### **A miscellany of more advanced issues**

Here are 4 further 'topics', two particularly relevant to planets and moons and two relevant to satellites and space-probes.

#### *The influence of other planets*

If the orbits of all bodies in a solar system were independent then the trajectories of the planets and moons would all be fixed by the initial conditions that launched them. However, gravitational attraction is present between every two bodies and so every planet, for example, feels a slight influence of all the other planets. Newton's law of gravity lets one work out what the relative size of the interplanetary forces are and they are all very small compared with the force of the Sun. Even Jupiter, whose mass is greater than all the other planets put together, is less than 1/1000 times the mass of the Sun and at its nearest to the Earth it is more than 4 times further than the Sun. The net result of the pulling of the other planets is that the energy of a planet in its orbit varies only a little over time, the changes averaging out to zero. Since the major orbital axis ( $a$ ) is determined only by the planet's energy then this doesn't change much. However, the change in eccentricity produced by the changing angular momentum of a planet as the other planets speed it up or slow it down do alter the shape of the orbit. The Earth's orbit, for example, can vary from completely circular ( $e = 0$ ) to an eccentricity of about 4 times the present value. This has a very important effect on climate changes on a timescale of many tens of thousands of years, for it alters the focus of the orbit in relation to its centre and hence the variation in how far the Earth is from the Sun. In addition the orbital inclination can vary by about 3 degrees and the whole orbit precesses around in space. This last effect is another influence on climate because the Earth's rotation axis is more or less fixed in space and hence the precession of the orbit changes the times during the year when the Earth is nearest and furthest from the Sun. Our meteorology course

says more about these climatological effects. They are not little effects but are thought to be the main reasons for the approximately periodic appearances of ice-ages.

### *Resonance of orbits*

Orbital resonance is quite common in the solar system. It can happen when more than two bodies interact over a long period of time. The periods of Neptune and Pluto, for example, are in a resonance very close to 2:3. The periods of Jupiter's moons Io, Europa and Ganymede are in resonance close to 1:2:4. The conspicuous gap in Saturn's rings is because of a 1:2 resonance in the period of a body within the gap and the moon Mimas. There are Kirkwood gaps in the asteroid belt for asteroids whose orbital periods resonate with that of Jupiter in several ways, the 1:2 resonance being clearest. There is even a very close 8:13 resonance between the orbits of Earth and Venus. Is this last one co-incidence? It's actually hard to tell because working out exactly the result of many bodies interacting over a few billion years is not possible. The special feature of resonance is that it is more stable than non-resonance so that once a resonance is set up, it tends to persist.

### *The gravitational sling-shot*

Throw a ball against a hard wall and if it's made of suitable stuff it will come off with almost the same speed but in a different direction. A little energy will be lost in the bounce, none gained. Swing a planet round the Sun and it comes out the other side moving in a different direction but will have the same energy and same speed when it's at the same distance from the Sun. No energy is gained. But do the same with a probe around a planet that itself is in orbit and the energy of the probe will change because the manoeuvre is similar to hitting a ball with moving bat or racket. A bowler in cricket could not bowl a ball all the way to the boundary line without it touching the ground yet a well-timed hit by the batsman could see the ball disappear at pace and with height into the stands. The extra energy given to the ball comes from the moving bat. Obvious, really. Likewise, a space-probe swung around the back of a planet is dragged forward by the moving planet and given extra speed. This is the gravitational sling-shot. The New Horizons probe on its way to Pluto and beyond as I write will reach the dwarf planet several years earlier than it would have done on the energy it was given by the launch rocket, because of a gravitational sling-shot from Jupiter. The energy acquired by the probe comes from the orbital energy of the planet. Events like this happen in nature too. When a comet comes close to a planet its energy and orbit are altered.

A probe can be slowed by swinging it round the front of an orbiting planet. The effects are opposite from those you might expect from a ball and racket because the racket exerts a moving repulsive force but the planet exerts a moving attractive force. That aside, the effects are just as real. The sling-shot works and is used for most space-probe missions to distant planets. NASA's Messenger probe to Mercury needed multiple planetary encounters to get it into orbit around Mercury and take off much of the speed it acquired from the gravitational attraction of the Sun as it hurtled inwards towards Mercury.

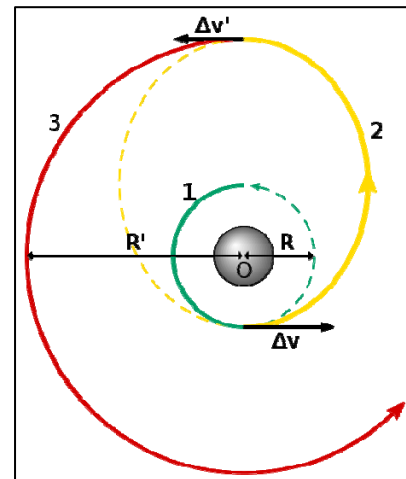
### *Changing satellite orbits*

If you want to put a satellite into a medium or high orbit (e.g. a geostationary satellite at over 40,000 km from the Earth's centre), then blasting it all the way up with a rocket and then releasing it would be hugely wasteful of energy because it takes a lot of energy to put a rocket that high and the once the rocket has released the satellite the rocket is useless. It doesn't fall

back to Earth because it has acquired a similar orbital speed to the satellite. In fact it would pose a hazard as ‘space junk’. The technique is to put the satellite into low-Earth orbit and give the satellite some propulsive ability of its own, or for larger transfers an attached transfer orbit stage, that is capable of raising it to a higher orbit. The simplest but quite frequently required case is to change from a circular low-Earth orbit to a circular medium- or high-Earth orbit. This is done by a Hohmann transfer orbit, a technique devised by Hohmann as early as 1925, long before any artificial satellites existed.

The Hohmann transfer orbit is an elliptical orbit whose periapsis is the lower orbital radius and whose apoapsis is the higher radius. Of course you now know what this means (!) but a diagram always helps. In the lower orbit (1 in the nearby diagram), a burst of rocket power gives the satellite an extra speed  $\Delta v$  necessary to put it into the elliptical transfer orbit (2 in the diagram). When the satellite reaches the apoapsis another burst of speed  $\Delta v'$  is given to take it out of its elliptical orbit and put it into the (higher energy) circular orbit required (3 in the diagram). If all this makes sense then you can congratulate yourself on reading this document well.

The results given earlier even allow us to work out quite easily the parameters  $a$  and  $e$  of the transfer orbit. Using the symbols in the diagram,  $R$  is the radius of the lower orbit and  $R'$  the radius of the higher orbit. Result 14 gives  $a(1 - e) = R$  and  $a(1 + e) = R'$ , from which the eccentricity  $e = (\rho - 1)/(\rho + 1)$ , where  $\rho = R'/R$ , the ratio of the two orbital radii. It also follows that  $a = (R' + R)/2$ , which is clear from looking at the diagram. Given  $a$  and  $e$  you could work out the two bursts in speed  $\Delta v$  and  $\Delta v'$  from the energy relationships given earlier.



Orbital transfer can take place accidentally if you are not careful. Suppose you want to dock with a space station orbiting 1 km in front of you in the same circular orbit. Giving yourself some more speed ‘to catch up’ quickly will alter your energy and hence move you into an orbit with a larger value of  $a$ . You will no longer be in the same orbit as the station and will miss it. Manoeuvres like docking are trickier in orbit than similar ones on Earth.

The point of the above is not really to turn anyone into a space scientist but to show that the subject is based on understandable physics, it makes sense, it’s relevant to astronomy, space science and indeed to what’s happening in the world around us. In fact space science is worth approaching £10 billion per year to the UK economy, including the building of satellites, the contributions to space exploration programs and the sale of knowledge and services earned from remote sensing and satellite communications. Knowing some of the science of orbits should not be the preserve of specialists only. We all have a vested interest in the success of space science and many technically literate people are employed in this business.

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