Selection, Trade, and Employment:
the Strategic Use of Subsidies

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Abstract: We study how the interaction between economic openness and competitive selection affects the effectiveness of employment (and entry) subsidisation. Within a two-country heterogeneous-firms model with endogenous labour supply, we find that optimal employment subsidies are always positive even though they can have *pro-* or *anti-competitive* effects on industry selection depending on whether the economy is open or not. We also find that *selection* effects resulting from international competition and *fiscal* externalities may imply that non-cooperative policies entail under-subsidisation of employment. Whilst always having pro-competitive selection effects on the industry, entry subsidies are shown to be less effective in raising employment and welfare than employment subsidies.

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1. Introduction

In recent years, welfare state reforms have tended to be characterised by a shift in emphasis from the use of passive labour market policies to that of active labour market policies (ALMPs). These programmes, often combined with reductions in employment protection within the *flexicurity* model, consist of interventions aimed at reducing search frictions (e.g. public employment services) and increasing employability (e.g. training schemes), but also of direct job creation measures such as wage and employment subsidies.[1] This type of subsidies accounted on average for about 25% of total ALMPs in the OECD in 2003 and their use intensified during the Great Recession. In addition, whilst they have often been introduced to support specific types of workers (such as the young or the long-term unemployed), they have increasingly been perceived as a means to accelerate job recovery[2] and demands for targeting them towards specific types of firms (as opposed to types of workers) and/or sectors have abounded.[3]

The literature on the assessment of the effectiveness of ALMPs typically adopts partial equilibrium approaches in which the focus is placed on microeconomic incentives (for individual workers, e.g. in seeking work, and for individual firms, e.g. in hiring). These policies, however, have implications that go beyond individual agents' behaviour and affect aggregate performance via aggregation effects that start from the industry level. In addition to being influenced by the extent of international openness, these effects also work through complex channels that are shaped by competitive selection forces within industries – and that thus turn out to be an important determinant of the aggregate general equilibrium impact of policy.

The fact that exposure to international competition enhances competitive selection within industries and the role of the latter in determining aggregate productivity and growth are now acknowledged by policy makers: whilst helping firms to protect and create jobs in a globalised environment by supporting their cost competitiveness is increasingly seen as an important complement to growth strategies, there is also an awareness that '*productivity growth requires constant reallocation of resources … from less to more efficient produces*' (Blanchard *et al*., 2014).

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[1] These policies are central to the "European Employment Strategy" to address structural unemployment and to increase labour participation and are a cornerstone of the Social Investment model of the welfare state. See Andersen and Svarer (2012) for a discussion of the Danish case. The 2013 EU Annual Growth survey, available at http://ec.europa.eu/europe2020/making-it-happen/annual-growth-surveys/index_en.htm, encourages the member states to step up ALMP.

[2] The use of employment and wage subsidies to raise employment during the recent recession was endorsed by the ILO-IMF (2010) Conference on "The Challenges of Growth, Employment and Social Cohesion" and have recently been advocated by the IMF (2013). Employment subsidy schemes were introduced, e.g. in Germany (*Kurzarbeitergeld*, see OECD, 2009), and in Ireland, while Japan augmented her existing Employment Adjustment Subsidy Programme by multiple stimulus packages (Kluve, 2010).

[3] For instance, Marzinotto *et al.* (2011) suggest that unused EU structural funds could be used to target wage subsidies to promote job creation in the exportable sector as a means to reducing external debt burdens. In a similar vein, the Irish Exporter Association argued that "*the* [2009] *Employment Subsidy Scheme Second Round is too little*" and that "*spreading the* [use of the] *Employment Subsidy Fund will inevitably dilute … [its] impact ... to support ... exports, ... key route to balancing the Exchequer and driving the economy out of recession*" http://www.irishexporters.ie/section/TheEmploymentSubsidySchemeSecondRoundistoolittle.

In this paper we aim to investigate how the interaction between economic openness and competitive selection shapes the effectiveness of ALMPs and governments' incentives in adopting them. In particular, we ask whether employment subsidies can help achieve the type of reallocations that can lead to aggregate productivity and employment growth, and how their effectiveness is affected by international policy spill-overs. To address these issues, we focus on the optimal determination of employment subsidies in a two-country two-sector model characterised by firm heterogeneity (which, in the presence of free-entry and exit, results in an endogenous determination of the marginal and average productivity of the industry), and endogenous labour supply (which endogenises the aggregate level of employment).

From a theoretical perspective, an employment subsidy can be justified if it corrects distortions that render the market equilibrium suboptimal. Dating back to the pioneering work of Pigou (1933) and Kaldor (1936), an extensive literature has examined the impact of employment subsidies and the taxation required to finance them. A significant strand of this literature, however, does not rely on general equilibrium frameworks characterised by imperfectly competitive goods markets,[4] and/or limits the analysis to closed economy settings.[5] A notable exception is Molana *et al.* (2012) who study the role of employment subsidies as fiscal stimuli in the open economy.[6] They show that the effectiveness of subsidies in raising the level of economic activity is shaped by a country's trade openness and that governments can act strategically in setting subsidies in the presence of international policy externalities. However, their paper does not allow for heterogeneity across firms and hence cannot account for the role of competitive selection in determining these effects.

Our research is also related to a strand of the literature that highlights the impact of intra-industry reallocations on aggregate performance. Di Giovanni and Levchenko (2013) find that the size composition of industries interacts with trade openness in determining aggregate output volatility. Several studies document how misallocations across heterogeneous production units can affect aggregate productivity and the transmission of shocks (e.g., Baily *et al.*, 1992; Restuccia and Rogerson, 2010). Of particular interest is the fact that different firms exhibit different cyclical patterns of net job creation (Moscarini and

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[4] The use of employment subsidies to raise employment has been advocated in Johnson (1980), Jackman and Layard (1980), Layard and Nickell (1980), Dreze and Malinvaud (1994), Phelps (1994) and Artis and Sinclair (1996) among others. Search and matching models, based on Mortensen and Pissarides (1994), provide an alternative framework within which the effects of employment subsidies have been analysed – for examples, see Mortensen and Pissarides (2003), Boone and van Ours (2004) and Cardullo and van der Linden (2006) among others. More recently, Brown *et al.* (2011) have constructed a Markov model of a labour market to compare the effectiveness of different employment subsidies.

[5] Within a simple closed economy macroeconomic model with monopolistically competitive markets *à la* Dixit and Stiglitz (1977), Fleurbaey (1998) shows that employment subsidies financed by profit taxation take the economy closer to the Walrasian equilibrium by countering the negative impact of market power on the level of economic activity.

[6] The open economy literature has typically focused on trade policy instruments – see, e.g. Brander and Spencer (1985), Venables (1987). Bettendorf and Heijdra (2004) analyse the use of production subsidies (in the presence of import tariffs) but their analysis is limited to the case of a small open economy and abstracts from distortionary taxes on labour income. More recently, Bilbiie *et al.* (2008) have studied the effectiveness of labour, sales and other subsidies as counter-cyclical stabilisation policy tools in raising employment and output within dynamic stochastic general equilibrium models, but do not allow for intra-industry selection effects.

Postel-Vinay, 2012; Elsby and Michaels, 2013). These papers, however, do not consider the interaction between competitive selection on the one hand, and labour market policies aimed at increasing employment and trade openness on the other.

Another (still fairly small) strand of the literature to which our work is related concerns the effects of policy on competitive selection. Demidova and Rodriguez-Clare (2009) focus on the effects of trade policy in a small open economy, whilst Felbermayr *et al*. (2013) consider non-cooperative tariff policies within a two-country setting. Contrary to our model, both of these papers assume a one sector economy and an exogenous labour supply, and their focus is not on employment creation policies.[7] Pflüger and Suedekum (2013) develop a two-country model to analyse strategic interaction between governments in setting entry subsidies financed via lump-sum taxation.

This paper extends the model developed by Molana *et al*. (2012) to allow for intra-industry productivity heterogeneity among firms. Molana *et al*. (2012) show that employment subsidies in the presence of monopolistic distortions affect the endogenous level of employment. By relaxing the assumption of symmetry in firms' producitivy, in this paper we show that employment subsidies also have an impact on aggregate efficiency via reallocation effects across countries, sectors, and firms within sectors. Ultimately, by subsidising employment, the government controls the selectivity of competition and contributes to correcting the market distortion (arising from differences in mark-up between the monopolistic good on the one hand and leisure, an outside good, and the imported varieties on the other) that results in an under-consumption of the differentiated good. Crucially, international openness alters the effects of the policy on selection and aggregate productivity. Whilst in autarky the optimal employment subsidy, by softening competition, has *anti-competitive effects* on the economy, in the open economy it has *pro-competitive effects* and results in a higher average productivity of firms. International spillovers, consisting of *selection* and *fiscal externalities*, lead to non-cooperative and cooperative policy equilibria that are characterised by positive subsidies. Whether the non-cooperative solutions entail levels of subsidisation that exceed or are short of those characterising the cooperative outcome hinges, however, on the nature of the externality between countries which in turn depends on how the subsidy is targeted.

Reforms of product markets – particularly aimed at facilitating entry – are considered as an effective means to increase aggregate productivity and employment.[8] Our analysis of an entry subsidy reveals that whilst it always has pro-competitive selection effects on the industry, it is less effective in raising employment and welfare than an employment subsidy. This is due to the fact that the latter enables the government to tackle the monopolistic distortion more directly.

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[7] In a recent paper, Haaland and Venables (2014) derive optimal domestic sales subsidies, import tariffs and export subsidies in a two sector model of a small open economy. By allowing for labour supply in the monopolistic sector to be flexible or fixed, the model allows to generalise results obtain via special cases in the literature.

[8] As Blanchard *et al*. (2014) state, "*Structural reform in product markets – particularly lowering barriers to entry of new firms – is likely to produce a larger growth payoff than reform in labor markets*".

The rest of the paper is organised as follows. Section 2 analyses the closed economy case. Section 3 extends the model to a two-country setting, and Section 4 analyses the strategic subsidy games between governments. Section 5 examines the role of trade liberalisation and productivity shocks. Section 6 compares the impact of employment and entry subsidies, and Section 7 concludes the paper.

2. Closed economy

Consider an economy consisting of two sectors, one imperfectly and one perfectly competitive, respectively producing a horizontally differentiated good and a homogeneous commodity. Labour supply is endogenous and a government employment subsidy in the differentiated sector is financed via proportional income taxation.

2.1. Demand and technology

The population of consumers is characterised by a representative household with N identical members which are either employed or unemployed. We assume that an employed worker is required (by legislation) to supply a fixed number of work hours (which is normalised to unity), a fraction h of household members are employed, and the total household income is equally shared amongst its members (unemployed members are 'insured' in this sense even if there is no unemployment benefit per se; see e.g., Andolfatto, 1996; Merz, 1995). The corresponding utility function and the budget constraint, written at the household level, are

$$U = \left(\frac{A}{1-\beta}\right)^{1-\beta} \left(\frac{Y}{\beta}\right)^{\beta} - \frac{\theta N h^{1+\delta}}{1+\delta}, \quad 0 < \beta < 1, \delta > 0, \theta > 0, N > 0, \tag{1}$$

$$P_A A + P_Y Y = N(1-t)wh, \tag{2}$$

where A and P_A are quantity consumed and price of the homogenous commodity, Y and P_Y are the quantity consumed and price of the differentiated good, w is the wage rate, and t is the proportional income tax rate.[9] The aggregate labour supply function and the demand functions for the two goods are, respectively,

$$L^s = Nh = N\left(\frac{(1-t)w}{\theta P}\right)^{1/\delta}, \quad A = \frac{(1-\beta)(1-t)wL^s}{P_A} \text{ and } Y = \frac{\beta(1-t)wL^s}{P_Y}, \tag{3}$$

where $P = P_A^{1-\beta} P_Y^{\beta}$ is the consumer price index. Y is assumed to be a CES bundle of differentiated varieties with 'dual' price index P_Y, respectively given by

$$Y = \left(\int_{i \in M} y(i)^{1-1/\sigma} di\right)^{\frac{1}{1-1/\sigma}} \text{ and } P_Y = \left(\int_{i \in M} p(i)^{1-\sigma} di\right)^{\frac{1}{1-\sigma}}, \tag{4}$$

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[9] We concentrate on proportional income taxation since it accounts for the bulk of tax revenue from the personal sector in advanced industrial economies.

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where M is the set of available varieties, $y(i)$ and $p(i)$ are the quantity consumed and the price of variety i respectively, and $\sigma > 1$ is the constant elasticity of substitution between varieties. The demand for each variety is then

$$y(i) = Y\left(\frac{p(i)}{P_Y}\right)^{-\sigma}, \qquad i \in M . \tag{5}$$

The homogenous good is produced under perfectly competitive conditions using a constant returns to scale technology with a unit labour requirement of one, i.e. $L_A = A^s$, where L_A and A^s denote the labour demand and the quantity supplied by this sector, respectively. Given the assumed technology, the zero-profit condition and free mobility of labour across the two sectors imply $w = P_A$. We use this good as the numeraire and normalise $P_A = 1$, which in turn implies $w = 1$ and $P = P_Y^\beta$.

In the differentiated good sector, each firm employs labour as the only input to produce one variety of the good using a linear technology with increasing returns to scale. Dropping the variety indicator i and distinguishing firms by their productivity parameter $\varphi \in [1, \infty)$, the labour requirement to produce and market a quantity y of the good is $l(\varphi) = \alpha + \dfrac{y(\varphi)}{\varphi}$, where α is the fixed labour requirement. A firm's profit is $\pi(\varphi) = p(\varphi) y(\varphi) - (1-s) l(\varphi)$, where $s \in [0,1)$ is the employment subsidy rate that the firm receives from the government. Profit maximisation under standard monopolistically competitive assumptions then yields the familiar mark-up rule: $p(\varphi) = \dfrac{\sigma(1-s)}{(\sigma-1)\varphi}$. Given this, and revenue $r(\varphi) = p(\varphi) y(\varphi)$, operating profits are: $\pi(\varphi) = r(\varphi)/\sigma - (1-s)\alpha$.

As in Melitz (2003), before they can set up and start producing, a large pool F of identical potential entrants each pay a fixed entry sunk cost f, measured in terms of the numeraire good, that enables them to draw a productivity parameter φ from a common population with a known p.d.f. $g(\varphi)$, defined over support $\varphi \in [1, \infty)$ with a continuous cumulative distribution $G(\varphi)$. A firm's survival in the market will depend on the magnitude of its φ in relation to the threshold φ_c which satisfies $\pi(\varphi_c) = 0$ and defines the marginal firms; firms with $\varphi \in [1, \varphi_c)$ will not enter since they would make a loss, while those with $\varphi \in [\varphi_c, \infty)$ will make non-negative profits. Prior to entry, therefore, it is known that a fraction $G(\varphi_c)$ of F will be unsuccessful, while a fraction $M \equiv (1 - G(\varphi_c)) F$ will succeed and start production. Thus, ex-post, M is the mass of varieties available to consumers. We can therefore redefine the p.d.f. of the surviving firms over $\varphi \in [\varphi_c, \infty)$ by $\mu(\varphi) = \dfrac{g(\varphi)}{1 - G(\varphi_c)}$,

which can then be used to obtain a measure of the aggregate productivity of the industry as the weighted average of operating firms' productivity levels $\varphi \in [\varphi_c, \infty)$,[10]

$$\tilde{\varphi} = \left(\int_{\varphi_c}^{\infty} \varphi^{\sigma-1} \mu(\varphi) d\varphi \right)^{\frac{1}{\sigma-1}}.$$ (6)

Using $p(\tilde{\varphi}) / p(\varphi_c) = \varphi_c / \tilde{\varphi}$, $y(\tilde{\varphi}) / y(\varphi_c) = (\tilde{\varphi} / \varphi_c)^{\sigma}$ and $r(\varphi) = p(\varphi) y(\varphi)$, we obtain

$$\frac{r(\tilde{\varphi})}{r(\varphi_c)} = \left(\frac{\tilde{\varphi}}{\varphi_c} \right)^{\sigma-1} \quad \Rightarrow \quad r(\tilde{\varphi}) = \alpha\sigma(1-s) \left(\frac{\tilde{\varphi}}{\varphi_c} \right)^{\sigma-1}.$$

All the relevant variables can then be written in terms of φ_c and $\tilde{\varphi}$. In particular, the industry price level, operating profits and labour demand are respectively given by

$$P_Y = M^{1/(1-\sigma)} \frac{\sigma(1-s)}{(\sigma-1)\tilde{\varphi}},$$ (7)

$$\pi(\tilde{\varphi}) = \alpha(1-s) \left[\left(\frac{\tilde{\varphi}}{\varphi_c} \right)^{\sigma-1} - 1 \right],$$ (8)

$$l(\tilde{\varphi}) = \alpha \left((\sigma-1) \left(\frac{\tilde{\varphi}}{\varphi_c} \right)^{\sigma-1} + 1 \right).$$ (9)

Finally, the indirect per capita utility can be written as

$$u \equiv \frac{U}{N} = \theta \frac{\delta h^{1+\delta}}{1+\delta},$$ (10)

which is monotonically increasing in h. Thus, maximising u is equivalent to maximising h.

2.2. General equilibrium and policy analysis

Entry continues until the expected *net* entry profit is zero, i.e. $M\pi(\tilde{\varphi}) - Ff = 0$, which we write as

$$M\left(r(\tilde{\varphi}) - (1-s)l(\tilde{\varphi}) \right) - Ff = 0.$$ (11)

The aggregate market clearing conditions for the labour, differentiated good and homogeneous good markets are, respectively

$$L_A + Ml(\tilde{\varphi}) = L^s,$$ (12)

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[10] To see this, define $\tilde{\varphi} = \left(\int_{\varphi} \varphi^{-1}(y(\varphi)/y(\tilde{\varphi})) \mu(\varphi) d\varphi \right)^{-1}$ and note that the weight $y(\varphi)/y(\tilde{\varphi})$ is given by $(\varphi/\tilde{\varphi})^{\sigma}$ which can be substituted back in the definition of $\tilde{\varphi}$ to obtain (6).

$$Mr(\tilde{\varphi}) = \beta(1-t)L^s, \tag{13}$$

$$A + fF = A^s, \tag{14}$$

where $A = (1-\beta)(1-t)L^s$, and $A^s = L_A$.

Finally, the government budget constraint is[11]

$$sMl(\tilde{\varphi}) = tL^s. \tag{15}$$

The above equations complete the model, which consists of 14 equations and 14 unknowns: F, M, L^s, L_A, $l(\tilde{\varphi})$, A, A^s, P_Y, $r(\tilde{\varphi})$, $y(\tilde{\varphi})$, $\pi(\tilde{\varphi})$, $\tilde{\varphi}$, φ_c and either the tax rate t or the subsidy rate s.[12] In order to obtain explicit solutions, we adopt the Pareto distribution and let

$$G(\varphi) = 1 - \varphi^{-\gamma} \quad \text{and} \quad g(\varphi) = \gamma\varphi^{-(1+\gamma)}, \quad \varphi \in [1, \infty), \tag{16}$$

where the shape parameter γ provides an inverse measure of dispersion.[13] Then, $1 - G(\varphi_c) = \varphi_c^{-\gamma}$ and (17) imply

$$\tilde{\varphi}^{\sigma-1} = \left(\frac{\gamma}{1+\gamma-\sigma}\right)\varphi_c^{\sigma-1}. \tag{18}$$

Making use of (18), we rewrite (11) as

$$M\alpha(1-s)\left(\frac{\sigma-1}{1+\gamma-\sigma}\right) - Ff = 0. \tag{19}$$

Given (16), $M \equiv (1 - G(\varphi_c))F$ implies $M = \varphi_c^{-\gamma}F$ which can then be substituted into (19) to obtain the equilibrium value of the productivity cut-off,

$$\varphi_c = \left(\frac{\alpha(\sigma-1)(1-s)}{f(1+\gamma-\sigma)}\right)^{1/\gamma}. \tag{20}$$

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[11] We assume that the government sets a uniform subsidy rate common to all firms in the industry. This assumption reflects the fact that, due to the informational requirements of firm-specific intervention, governments often choose to use fairly 'blunt' policy instruments, targeted to 'groups' or categories of agents. Note, however, that with CES preferences (where firms' mark-up, and hence the monopolistic distortion addressed by the subsidy, does not depend on their productivity) the subsidy should be the same for all firms – contrary to the case of firm-specific market power where it is well known that first-best policies depend on firms' productivity – see e.g., Leahy and Montagna (2001) and Nocco *et al*. (2014).

[12] Note that in general equilibrium one of the market clearing conditions, e.g. the homogeneous goods market equilibrium in (14), can be obtained from the rest and is therefore redundant.

[13] In the Pareto distribution, both mean and variance are negatively related to the shape parameter γ. Thus, the smaller is γ, the higher is the average firm's efficiency and the higher is the productivity dispersion. To obtain meaningful results we impose $\gamma > \sigma - 1$.

As is clear from (20), $\frac{\partial \varphi_c}{\partial \gamma} < 0$: the minimum productivity required to survive in equilibrium

is positively related to the degree of heterogeneity between firms. Moreover, $\frac{\partial \varphi_c}{\partial s} < 0$: a

higher subsidy *softens competition,* making it easier to survive in equilibrium.

For a given *s*, and treating *t* as endogenous, the model can be solved (see the Appendix for details) to express all endogenous variables in terms of *s*. The corresponding equilibrium tax rate is given by

$$t = \frac{\beta(1+\gamma\sigma-\sigma)s}{\beta(1+\gamma\sigma-\sigma)s+\gamma\sigma(1-s)} < s. \tag{21}$$

The indirect utility function in (10) is a monotonic function of *h* which, from (3), is in

turn given by $h = \left(\frac{1-t}{\theta P_Y^\beta}\right)^{1/\delta}$. Thus, a subsidy affects welfare via three channels: the tax rate

and, through P_Y, the mass of varieties and the average industry productivity. Using (21) and

the solution for P_Y to evaluate *h*, we obtain the solution $h(s)$ which can be shown to be

strictly concave in *s*. Thus, given that $\frac{\partial \varphi_c}{\partial s} < 0$ also holds, it is welfare-improving to subsidise

employment and to *soften* competitive selection in the monopolistic sector. Furthermore, $h(s)$ reaches a unique maximum at

$$s^{opt}(\gamma) = \frac{(1-\beta)\gamma}{\gamma\sigma-\beta(1+\gamma\sigma-\sigma)} > 0, \tag{22}$$

where $\frac{ds^{opt}}{d\gamma} > 0$. Hence: the lower is the degree of productivity heterogeneity between firms,

the larger is the optimal subsidy since, at higher values of γ, the subsidy has a lower marginal

effect. Consistently, by substituting (22) into (20), it can be easilty verified that $\frac{\partial \varphi_c^{opt}}{\partial \gamma} < 0$,

i.e. the optimal value of the productivity cut-off is lower (and so is the optimal average productivity in the industry) the more homogeneous are firms.

It is instructive to point out that the homogenous firms scenario analysed by Molana *et al.* (2012), that can be considered as a useful benchmark for the present analysis, can be obtained in our model by letting $\gamma \to \infty$, which corresponds to the case in which all firms draw the same productivity level with probability one, which occurs for $\gamma \to \infty$. As is clear

from equation (22), $\lim\limits_{\gamma\to\infty} s^{opt} = \frac{1}{\sigma}$ and $\lim\limits_{\gamma\to\infty} \varphi_c^{opt} = 1$. Given the firm's optimal price rule,

$p(\varphi) = \dfrac{\sigma(1-s)w}{(\sigma-1)\varphi}$, this implies that when firms are homogenous $p\left(\varphi_c^{opt}\big|_{\gamma\to\infty}\right) = w$ holds.

Thus, as discussed in Molana *et al.* (2012), with homogenous productivities, the optimal subsidy eliminates the mark-up margin $\sigma/(\sigma-1)$, fully correcting the monopolistic distortion. More generally, the extent to which the subsidy addresses this distortion is directly related to the size of γ, i.e. it increases in the degree of homogeneity of firms. The monopolistic distortion, due to the fact that the differentiated good is priced at a mark-up while leisure and the homogenous good are not, results in a wedge between the marginal rate of substitution and the marginal rate of transformation between leisure and the homogenous good on the one hand and the differentiated good on the other. Consequently, the market outcome is characterised by a sub-optimal level of the consumption of the differentiated product and an excessive consumption of the homogenous good and leisure. In contributing to correct this distortion, the subsidy reduces the share of employment in the homogenous good sector[14]

$$\frac{L_A}{L} = \frac{(1-\beta)\gamma\sigma}{\gamma\sigma + \beta(1+\gamma\sigma-\sigma)\left(\dfrac{s}{1-s}\right)}, \tag{23}$$

which is negatively related to *s*.

Intuitively, by reducing firms' costs and making it easier for them to survive in equilibrium, the subsidy *softens competition* and this works towards increasing entry. As shown in the Appendix, the mass of firms is concave in *s*, but reaches a maximum at a subsidy level that exceeds the value that maximises employment. Thus, increasing the subsidy up to its optimum level expands the mass of firms in the industry, which contributes towards raising welfare and aggregate employment (and will also have procompetitive effects that partially offset the initial competition softening effects of the subsidy). However, as is clear from (21), an increase in subsidy raises the tax rate, which reduces labour supply and welfare. In addition, the lower average productivity in the industry contributes to offsetting the initial price-reducing effect of the subsidy. Taken together, these forces underpin the concavity of $h(s)$.[15]

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[14] See Bilbiie *et al.* (2008) and Molana *et al.* (2012) for further discussion. The effects of cost heterogeneity on the optimality of the market solution has been examined by Dhingra and Morrow (2015) for the CES and VES cases and by Nocco *et al.* (2014) for quasi-linear demands. Dhingra and Morrow (2015) show that in a one-sector-heterogeneous-CES world, the market solution corresponds to the first best – i.e. how the market allocates resources across firms does not matter (and the optimal policy is *laissez faire*). When, as in our case, mark-ups differ across sectors, the monopolistic distortion leads to inefficient market allocations that can be corrected by policy – to an extent that, as we show, depends on the degree of heterogeneity among firm productivities.

[15] Note that a higher value of γ is associated with a lower mass of firms and a lower aggregate employment. Thus, the skewedness of the productivity distribution matters in determining the effectiveness of the subsidy policy i.e. $L^{opt} = L\left(\gamma, s^{opt}(\gamma)\right) \Rightarrow \dfrac{dL^{opt}}{d\gamma} = \dfrac{\partial L^{opt}}{\partial\gamma} + \dfrac{\partial L^{opt}}{\partial s^{opt}}\dfrac{ds^{opt}}{d\gamma}$, where $\dfrac{dL^{opt}}{d\gamma} < 0, \dfrac{d^2 L^{opt}}{d\gamma^2} > 0$. Hence, ceteris paribus, a fall in productivity heterogeneity (i.e. a higher value of γ) will result in a lower aggregate employment in equilibrium, *despite a higher optimal subsidy rate*.

In sum, by reducing the selectivity of competition in the monopolistic industry, the subsidy triggers a reallocation of resources across the two production sectors, away from leisure and – within the monopolistic sector – away from the most efficient and towards less efficient firms. Despite its anti-competitive effects, an optimally chosen subsidy leads to welfare gains.

3. A two-country setting

In this section we extend the model to a two-country setting. Both economies (home and foreign) are characterised by the same consumer preferences and technologies discussed in the autarkic model above. The homogenous good (that we retain as the *numeraire*) is freely traded whilst the differentiated good is traded at a per-unit iceberg trade cost, $\tau > 1$. We shall denote the foreign country's variables by an asterisk and focus the discussion of the two-country model on the home country.

The differentiated product aggregator and its price index are now respectively given by

$$Y = \left(\int_{i \in M} y_d(i)^{1-1/\sigma} di + \int_{i \in M_x^* \subseteq M^*} y_x^*(i)^{1-1/\sigma} di \right)^{\frac{1}{1-1/\sigma}}, \; P_Y = \left(\int_{i \in M} p_d(i)^{1-\sigma} di + \int_{i \in M_x^* \subseteq M^*} p_x^*(i)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}, \quad (24)$$

where the subscripts d and x refer to domestically consumed and exported varieties, respectively – thus, e.g., y_x^* and p_x^* are the quantity and price of foreign exported varieties consumed in the home country. Demand for the domestic and foreign varieties of the differentiated good are respectively given by

$$y_d(i) = Y \left(\frac{p_d(i)}{P_Y} \right)^{-\sigma}, \qquad y_x^*(i) = Y \left(\frac{p_x^*(i)}{P_Y} \right)^{-\sigma}. \quad (25)$$

The possibility of trade implies that firms in the monopolistic sector will have to decide after entry whether to produce for the domestic market only or whether to also export. In addition to the fixed entry cost f and the fixed cost α_d required for production of y_d, an exporting firm also incurs a fixed cost α_x (also in terms of labour) for producing and marketing the output y_x it sells abroad. Given the higher complexity of operating in foreign markets, it is plausible to assume $\alpha_d < \alpha_x$.

As in the autarkic case, we shall assume that the government does not set firm-specific subsidies. However, the openness of the economy results in the possibility of broad categories of firms/activities to be targeted – e.g. consistent with the pressures for some form of employment support to be directed to exporters or 'high quality job' firms during the recent recession. Hence, we shall briefly examine an 'export-only' subsidy, s_x, for labour employed in the production for exports, and a 'domestic-only' subsidy, s_d for labour employed in production for domestic sales, in addition to the 'uniform' employment subsidy

case, $s = s_d = s_x$. A firm's profits from domestic and foreign sales are then given respectively by

$$\pi_d(\varphi) = p_d(\varphi) y_d(\varphi) - (1-s_d) w l_d(\varphi), \quad \pi_x(\varphi) = p_x(\varphi) y_x(\varphi) - (1-s_x) w l_x(\varphi), \quad (26)$$

where

$$l_d(\varphi) = \alpha_d + \frac{y_d(\varphi)}{\varphi}, \qquad l_x(\varphi) = \alpha_x + \frac{\tau y_x(\varphi)}{\varphi}. \tag{27}$$

Maximisation of (26) subject to the demand functions in (25) and labour requirements in (26) implies the following optimal price rules for a firm with productivity φ serving both markets:

$$p_d(\varphi) = \frac{\sigma(1-s_d)w}{(\sigma-1)\varphi}, \qquad p_x(\varphi) = \frac{\sigma(1-s_x)\tau w}{(\sigma-1)\varphi}. \tag{28}$$

3.1. The general equilibrium

The competitive selection process that follows entry will result in the emergence of two productivity cut-offs, defined by $\varphi_d = \sup\{\varphi : \pi_d(\varphi_d) = 0\}$ and $\varphi_x = \sup\{\varphi : \pi_x(\varphi_x) = 0\}$, that respectively correspond to the productivity of the marginal firms that survive in the domestic market and to that of the marginal exporters. Thus, the possibility of international trade, and the fact that trade is costly, will result in a partitioning between exporting and non-exporting firms. Only relatively more productive firms will afford to export and φ_x determines the partition between the two types of firm: for a given mass of entrants F, a mass $M \equiv (1 - G(\varphi_d))F = \varphi_d^{-\gamma}F$ of firms with productivity $\varphi \in [\varphi_d, \infty)$ will survive and produce for the domestic market and a subset of these, with mass $M_x \equiv (1 - G(\varphi_x))F = \varphi_x^{-\gamma}F$ and with productivity $\varphi \in [\varphi_x, \infty)$, will also produce and export to the foreign country. Following the same procedure as in autarky, for any given φ_d and φ_x we obtain the corresponding average productivities,

$$\tilde{\varphi}_d = \left(\frac{\gamma}{1+\gamma-\sigma}\right)^{1/(\sigma-1)} \varphi_d, \qquad \tilde{\varphi}_x = \left(\frac{\gamma}{1+\gamma-\sigma}\right)^{1/(\sigma-1)} \varphi_x. \tag{29}$$

The zero expected *net* entry profits condition implies that $M\pi_d(\tilde{\varphi}_d) + M_x\pi_x(\tilde{\varphi}_x) - P_A f F = 0$ always hold in equilibrium. The labour market clearing condition is

$$L_A + Ml_d(\tilde{\varphi}_d) + M_x l_x(\tilde{\varphi}_x) = L^s, \tag{30}$$

where, as in autarky, $L_A = A^s$ and $L^s = N\left(\dfrac{(1-t)w}{\theta P_A^{1-\beta} P_Y^\beta}\right)^{1/\delta}$. The balanced government budget

constraint and the trade balance equation are, respectively

$$w\left(s_d M l_d\left(\tilde{\varphi}_d\right) + s_x M_x l_x\left(\tilde{\varphi}_x\right)\right) = twL^s, \tag{31}$$

$$P_A\left(A^s - A - fF\right) + M_x r_x\left(\tilde{\varphi}_x\right) = M_x^* r_x^*\left(\tilde{\varphi}_x^*\right). \tag{32}$$

Finally, the model is closed by noting that, since the homogenous good is freely traded, $w = P_A = w^* = P_A^* = 1$. As shown in the Appendix, we reduce the model to 12 equations which can be solved to determine φ_d, φ_x, φ_d^*, φ_x^*, F, F^*, P_Y, P_Y^*, h, h^*, t, t^*. Using these equations, the following relationships can be shown to hold in general equilibrium between the two countries' productivity cut-offs:

$$\frac{\varphi_x}{\varphi_d^*} = \tau\left(\frac{1-s_x}{1-s_d^*}\right)^{\sigma/(\sigma-1)}\left(\frac{\alpha_x}{\alpha_d^*}\right)^{1/(\sigma-1)}, \qquad \frac{\varphi_x^*}{\varphi_d} = \tau^*\left(\frac{1-s_x^*}{1-s_d}\right)^{\sigma/(\sigma-1)}\left(\frac{\alpha_x^*}{\alpha_d}\right)^{1/(\sigma-1)}. \tag{33}$$

These imply that, for any given level of τ and τ^*, subsidy policies in the two countries trigger selection effects that will result in changes in the efficiency composition of the industry (and hence in market structure) in both countries.

3.2. The effects of employment subsidies

The model can be solved recursively to first determine the two countries' productivity cut-offs $\left(\varphi_d,\ \varphi_x,\ \varphi_d^*,\ \varphi_x^*\right)$ by considering four equations consisting of (33) above and the zero expected net profits of entry equations which can be written as (see the Appendix):

$$\begin{aligned}
\alpha_d\left(1-s_d\right)\varphi_d^{-\gamma} + \alpha_x\left(1-s_x\right)\varphi_x^{-\gamma} &= \frac{(1+\gamma-\sigma)f}{\sigma-1}, \\
\alpha_d^*\left(1-s_d^*\right)\varphi_d^{*-\gamma^*} + \alpha_x^*\left(1-s_x^*\right)\varphi_x^{*-\gamma^*} &= \frac{(1+\gamma^*-\sigma)f^*}{\sigma-1}.
\end{aligned} \tag{34}$$

We now impose full symmetry, i.e. $\left(\alpha_d,\ \alpha_x,\ f,\gamma,\tau\right) = \left(\alpha_d^*,\alpha_x^*,f^*,\gamma^*,\tau^*\right)$, on (32) and (33). Focussing on a uniform subsidy (by letting $s \equiv s_x = s_d$ and $s^* \equiv s_x^* = s_d^*$) and allowing for the two countries' subsidies to differ (i.e. $s \neq s^*$), we obtain the solutions for the productivity cut-offs for the home country:

$$\varphi_d = \left(\frac{\alpha_d (\sigma-1)(1-s)}{f(1+\gamma-\sigma)} \right)^{1/\gamma} \left(\frac{\left(\frac{\alpha_x}{\alpha_d}\right)^{\gamma/(\sigma-1)-1} \tau^{\gamma} - \left(\frac{\alpha_x}{\alpha_d}\right)^{-[\gamma/(\sigma-1)-1]} \tau^{-\gamma}}{\left(\frac{\alpha_x}{\alpha_d}\right)^{\gamma/(\sigma-1)-1} \tau^{\gamma} - \left(\frac{1-s^*}{1-s}\right)^{\gamma\sigma/(\sigma-1)-1}} \right)^{1/\gamma} , \tag{35}$$

$$\varphi_x = \left(\frac{\alpha_x (\sigma-1)(1-s)}{f(1+\gamma-\sigma)} \right)^{1/\gamma} \left(\frac{\left(\frac{\alpha_x}{\alpha_d}\right)^{\gamma/(\sigma-1)-1} \tau^{\gamma} - \left(\frac{\alpha_x}{\alpha_d}\right)^{-[\gamma/(\sigma-1)-1]} \tau^{-\gamma}}{\left(\frac{1-s^*}{1-s}\right)^{\gamma\sigma/(\sigma-1)-1} - \left(\frac{\alpha_x}{\alpha_d}\right)^{-[\gamma/(\sigma-1)-1]} \tau^{-\gamma}} \right)^{1/\gamma} . \tag{36}$$

These can be substituted into (33) to obtain the corresponding expressions for the foreign country. Inspections of the cut-offs reveals that with full symmetry and for any given s^*, a *unilateral* rise in s increases φ_d and φ_x^* (which rises even more than φ_d does) and reduces φ_x, and φ_d^*.[16]

Equations (35), (36) and the corresponding equations determining the foreign productivity cut-offs can then be used to obtain the mass of firms characterising the equilibrium. While this could not be done analytically, our extensive numerical analysis shows that an increase in uniform subsidy in the home country will lead to greater entry (F), a larger mass of surviving firms (M) characterised by a higher average efficiency ($\tilde{\varphi}_d$), and a larger extensive margin of export (M_x / M). It will also have the opposite effects on the foreign country, which experiences a reduced entry (F^*), a smaller mass of surviving firms (M^*) characterised by a lower average efficiency $(\tilde{\varphi}_d^*)$, and a smaller extensive margin of export (M_x^* / M^*).

Thus, *contrary to what happens in autarky*, a unilateral increase in uniform employment subsidy to all home firms has a *pro-competitive selection effect* on the monopolistic industry. To begin with, as in autarky, the policy has an anti-competitive effect on the monopolistic sector: by lowering labour costs, it will initially work towards a reduction of both the domestic and the export productivity cut-offs. By softening competition and making it easier to survive in the domestic market and to export, this effect will bring about entry – which, as noted for the autarkic cas, will partially offset the initial anticompetitive impact of the policy. The openness of the economy introduces another pro-competitive effect, however. As is reflected in the positive relationship between φ_d and φ_x^*, the reduction of φ_d triggered by the

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[16] This holds for the unilateral case as long as, for the given values of $\left(s^*,\alpha_d,\alpha_x\right)$, the value of τ is sufficiently large to ensure positivity of the denominators of (34) and (35). However, this qualification is not required for symmetric solutions since the denominators of (34) and (35) are positive for all $\tau \geq 1$ given that $\alpha_x > \alpha_d$ and $\gamma > \sigma -1$.

subsidy makes it easier for foreign exporters to penetrate the domestic market. This toughens the degree of import competition for domestic firms and exerts an upward pressure on φ_d. The net effect of the subsidy is an increase in the domestic productivity cut-off and in the size of the monopolistic industry. Moreover, the subsidy has an *adverse selection effect* on the foreign country, which results from an expenditure switching across countries, with consumption of imported varieties falling in favour of domestic ones – an effect that, together with the increase in the mass of home country exporters, underpins a contraction of the monopolistic sector in the foreign country.

4. Welfare and optimal policy in the symmetric case

Retaining the assumption of symmetry between countries, in this section we study the optimal policy. As in autarky, utility is monotonically increasing in employment – which we continue to use to proxy welfare. Given the complexity of the algebra involved, however, we now use numerical solutions to illustrate the optimal policy and its effects.[17]

4.1. Uniform subsidies

Letting $s \equiv s_x = s_d$ and $s^* \equiv s_x^* = s_d^*$, we indicate the solution for employment in the home and foreign country by $h(s^*, s)$ and $h^*(s^*, s)$ which can be shown to be concave in s (for any given s^*) and in s^* (for any given s), respectively. In Figure 1 we plot, for a few different values of s^*, sections of $h(s^*, s)$ in (s, h) space which show that, for all relevant values of $\overline{s}^*$, $h(\overline{s}^*, s)$ is strictly concave in s and has a unique maximum at some $0 < s < 1$. Thus, each country has a unilateral incentive to set a positive employment subsidy.

Figure 1. Sections of home country's welfare function $h(s^*, s)$ in (s, h) space

(for different values of s^*)
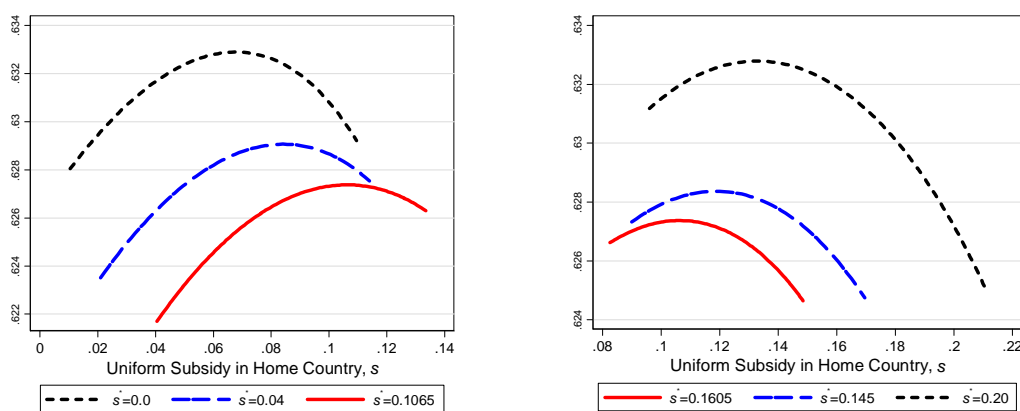
Note: The curves are truncated at each end to reduce the scale effect.

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The concavity of a country's welfare function with respect to its own subsidy stems from the positive *selection* effects of the policy discussed in the previous section together with negative *fiscal* effects: the welfare function exhibits a trade-off between the combined effect of the larger mass of firms and their higher average productivity (which implies that consumers gain both at the extensive and at the intensive margin because of a higher variety and lower average prices) and the higher tax rate required to finance the increase in the subsidy.[18]

Clearly, the externalities of a unilateral change in subsidy by the home country's government will affect the foreign country's policy incentives. When both governments are policy active, their reaction functions and the resulting Nash equilibrium can be obtained using the iterative numerical solution method by maximising (sequentially and in turn) $h(s^*, s)$ and $h^*(s^*, s)$, which are symmetric, holding the other country's subsidy constant. Our numerical analysis suggests that $\dfrac{\partial^2 h(s^*, s)}{\partial s \partial s^*} > 0$ and $\dfrac{\partial^2 h^*(s^*, s)}{\partial s \partial s^*} > 0$ which imply that the two countries' subsidies are strategic complements and the two reaction functions are upward sloping in (s^*, s) space. We also find that the welfare functions are saddle-shaped, implying that the policy externality is non-monotonic. To explain this, consider the effects of increases in the foreign subsidy rates on home welfare. As s^* increases, while the optimal value of s rises, the corresponding maximum value of $h(s^*, s)$ falls initially and then rises as s^* exceeds a threshold value. Figure 1 illustrates this property. The left panel shows the negative externality region for s^*: as $\bar{s}^*$ is raised, $h(\bar{s}^*, s)$ shifts down and its maximum moves to the right; thus whilst the policy choices exhibit strategic complementarity throughout, in this region the raising of s^* is an 'unfriendly move' by the foreign country. There is however a threshold for s^* at which the raising of s^* becomes a 'friendly move'. This is shown in the right panel of Figure 1: as $\bar{s}^*$ is raised, $h(\bar{s}^*, s)$ shifts up and its maximum still moves to the right. Figure 2 shows the home governments' reaction function $RF(s^*, s) = 0$ and the corresponding iso-welfare contours in the (s^*, s) space. While the reaction function is upward sloping in the (s^*, s) space, the shape and hierarchy of the iso-welfare loci change at the threshold level of s^* denoted by $\hat{s}^*$. This occurs at the intersection of the RF with the 45º degree line which, given the assumed symmetry, corresponds to the Nash equilibrium solution (s_N^*, s_N).

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[18] Other things equal, the increase in a country's tax rate is higher the larger is its mass of firms.

Figure 2. The iso-welfare contours and the reaction function for the home country with uniform employment subsidy
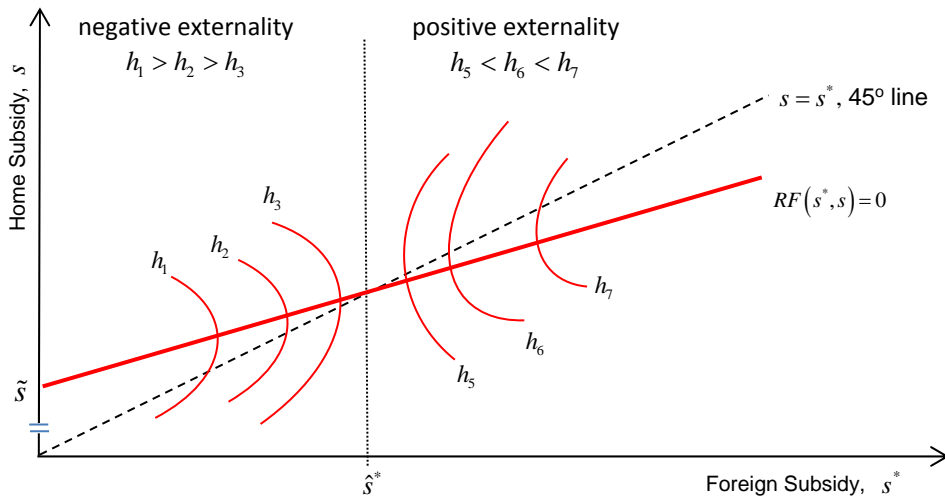
Note: (i) $\hat{s}^*$ is the threshold at which externality switches; (ii) $\tilde{s} > 0$ is the unilateral optimal subsidy when $s^* = 0$; (iii) *RF* is approximated by a straight line for simplicity to emphasise its monotonicity and slope relative to the 45º line.

Figure 3 depicts both governments' reaction functions. As can be seen from the figure, the cooperative solution $\left(s_C^*, s_C\right)$, in which the two governments jointly choose subsidies to maximise the sum of the two countries's welfare, lies above the Nash equilibrium, i.e. $\left(s_C^*, s_C\right) > \left(s_N^*, s_N\right)$, since it occurs in the positive externality region. Hence, the non-cooperative behaviour entails *under-subsidisation* from a global welfare point of view. Table A3 in the Appendix provides a comparison of the non-cooperative and cooperative solutions with the no-policy benchmark solution (see columns labelled "*Initial Case*").

Figure 3. Nash and cooperative solutions with uniform employment subsidy
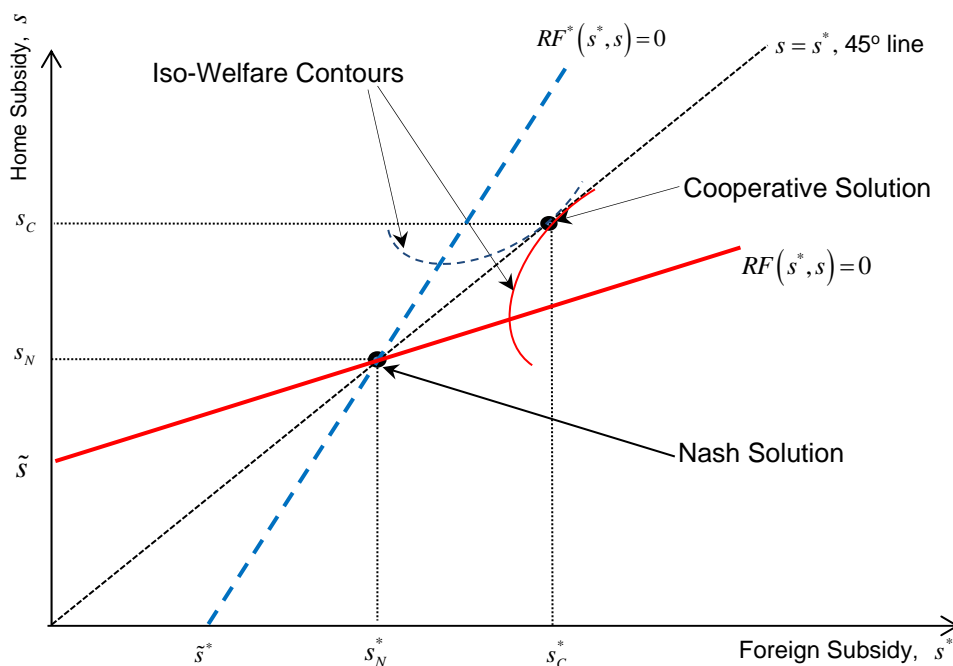
As discussed in the previous section, a unilateral increase in subsidy in one country has *negative selection* and *variety effects* on its competitor's industry that amount to an international reallocation of resources across countries within the monopolistic sector.[19] This can be seen by comparing the "*Employment Subsidy*" and "*No Policy Benchmark*" columns of Table A4 in the Appendix which show that a unilateral increase in s raises φ_d as well as φ_x^* and results in lower F^*, φ_d^*, and M_x^*. In addition, the policy has a positive *fiscal externality* on the foreign country: since an increase in s negatively affects the mass of foreign firms and exporters, it can be shown to reduce the foreign government's subsidy bill (for a given $\bar{s}^* > 0$), thus enabling the foreign government to reduce its tax rate for a given level of subsidy (or to increase the subsidy rate for a given tax rate). Overall, the trading partner initially experiences a fall in welfare that gives rise to an incentive to retaliate. Consistently, this retaliation by the foreign country has a welfare reducing (and anti-competitive) effect on the home country.

As is evident from Table A3, welfare at the Nash equilibrium is higher than in the no-policy case, even though both countries experience a worsening of their firms' productivity distribution (reflected in a fall in the domestic cut-off relative to the no-policy case). Up to $\left(s_N^*, s_N\right)$, there are negative policy externalities, as the negative selection and variety effects dominate the positive fiscal-spill-over effects. The Nash equilibrium occurs where the negative selection and variety effects are exactly offset by the positive fiscal spill-over effects. However, when setting their policies non-cooperatively, the two governments fail to internalise the fact that raising subsidies above the Nash equilibrium level would generate a positive externality since the negative selection and variety effects would then be dominated by the positive fiscal-spill-over effects. The cooperative behaviour, where these positive externalities are 'jointly exploited', leads to an equilibrium at $\left(s_C^*, s_C\right) > \left(s_N^*, s_N\right)$ which yields higher welfare level.

4.2 Targeted employment subsidies

Given that export performance is often seen as crucial to employment growth, we now briefly consider the effects of an 'export-only' employment subsidy (i.e. $s_x > 0$, $s_d = 0$). If used unilaterally, this policy has the same qualitative effects on the two countries' productivity cut-offs as a uniform subsidy: by increasing the domestic cut-off and reducing the export productivity cut-offs it has pro-competitive effects on the domestic industry and triggers a reallocation of resources towards relatively more efficient firms. However, since, by discriminating in favour of the export activity of firms, the 'export-only' subsidy is biased

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[19] These two effects combine to produce an increase in the foreign price index P_Y^*. This negative spill-over is partially mitigated by the fact that the average productivity of home country's exporters is lower as a result of the subsidy – and hence the average price of imported varieties in the foreign country is now higher, softening competition for foreign firms in their domestic market.

towards relatively more efficient firms in the industry, the reallocation effect is relatively stronger in this case.

As with the uniform subsidy, an employment subsidy targeted to exports has a negative efficiency spill-over and a positive fiscal spill-over effect on the trading partner. In this case, however, the latter is smaller (due to the relatively smaller subsidy bill arising from subsidising only exporting activities) and never dominates, so that the overall externality effect is always negative. This case is illustrated in Figure 4 where, similar to the uniform subsidy case, the reactions functions of the two countries are upward sloping.

Figure 4. Nash and cooperative solutions with export-only employment subsidy
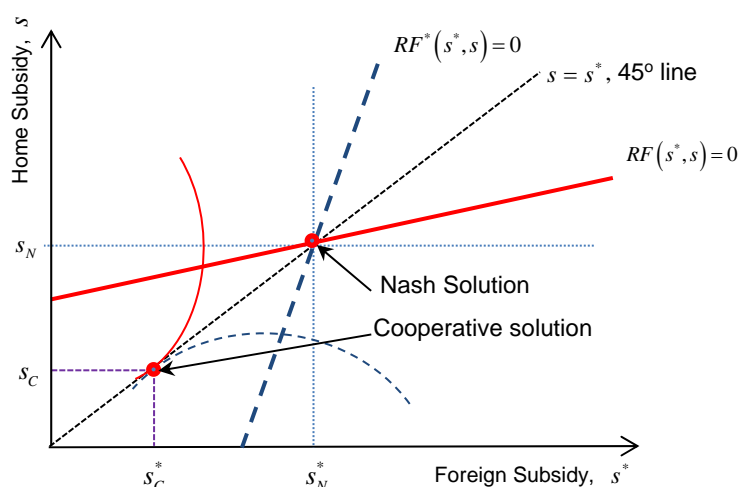
However, due to the monotonic nature of the inter-country negative externality, the cooperative solution now lies below the Nash equilibrium level of subsidy: by failing to internalise the negative externality, the non-cooperative behaviour of the governments entails over-subsidisation from a global welfare point of view.

It is also interesting to briefly consider the effects of a 'domestic-only' subsidy (i.e. $s_d > 0, \ s_x = 0$) where the employment subsidy is targeted towards the domestic operation of firms. This policy, which is clearly biased towards relatively less efficient firms, softens selection in the home market (i.e. contrary to the previous two cases, it *reduces* the domestic productivity cut-off) and thus reallocates resources away from more efficient and towards relatively less efficient firms. In so doing, this policy achieves the strongest 'home-market effect' and leads to the largest negative externality on the foreign country (via its strongest market stealing effect). In this case, as shown in Figure 5, the two governments' reaction functions are downward sloping, since the subsidies are strategic substitutes and the negative externality induces governments to over-subsidise when acting non-cooperatively, with the cooperative solution lying below the Nash equilibrium.

Figure 5. Nash and cooperative solutions with domestic-only employment subsidy
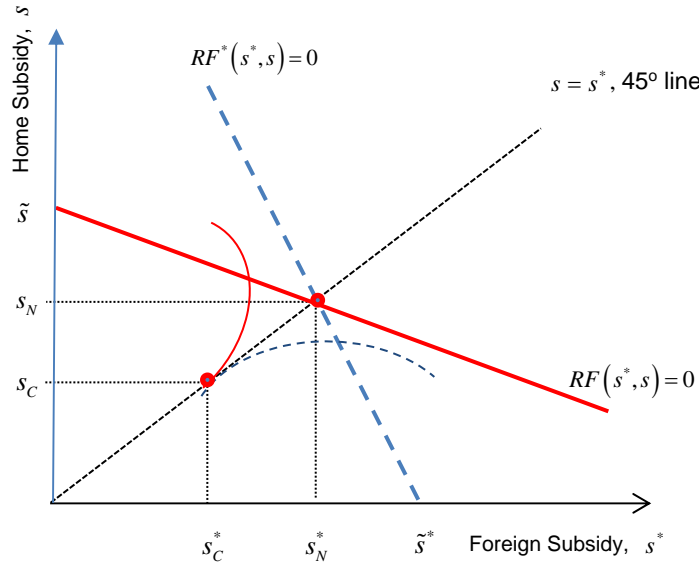
5. Trade liberalisation and productivity shocks

As is well established in the literature, in this type of model trade liberalisation (i.e. a reduction in trade costs) typically has pro-competitive effects on an industry, and reallocates resources towards more efficient firms. In our model, trade liberalisation also strengthens the pro-competitive effects of an employment subsidy.

Figure 6 illustrates the effects of a 5% reduction in trade costs on the effectiveness of unilateral increases in uniform employment subsidy by the home government when the foreign government is policy inactive: as the graph in the left panel shows, increases in s raise ϕ_d and this effect is stronger the lower are trade costs.

Figure 6. Impact of unilateral uniform employment subsidy policy by home country ($s^* = 0$) as trade costs fall by 5%

Home domestic productivity cut-off $\phi_d\left(s^*, s\right)$

Home welfare $h\left(s^*, s\right)$
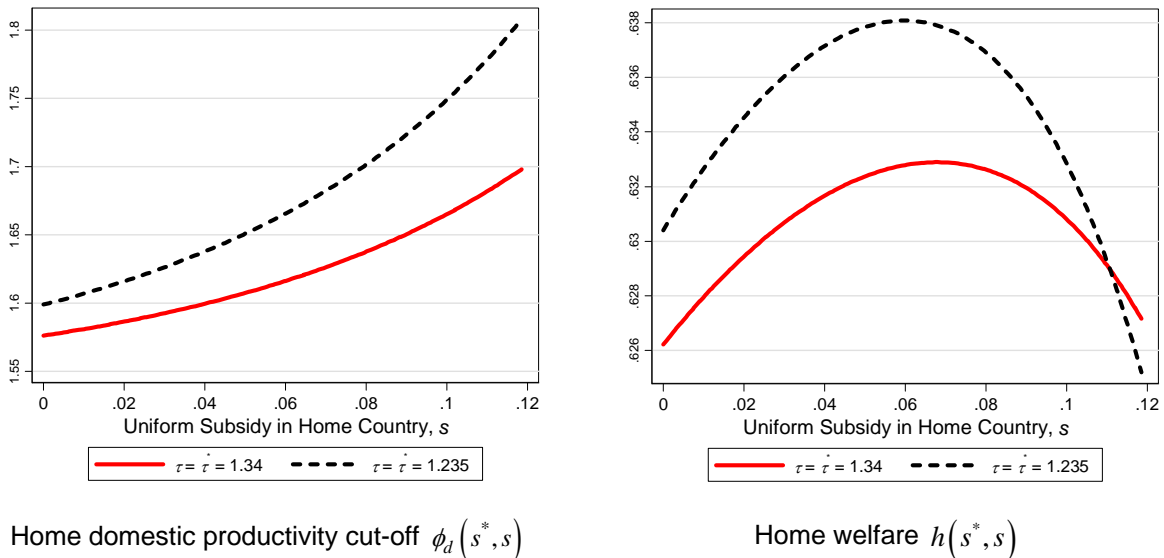
Thus, the standard competitive selection forces triggered by trade liberalisation strengthen the pro-competitive effects of the subsidy discussed earlier. As a result, the unilateral optimal subsidy is lower and its welfare effects are higher at lower trade costs, as illustrated in the right panel of Figure 6.

As can be seen from Table A3, trade liberalisation also results in a lower Nash equilibrium subsidy and hence in a much higher degree of under-subsidisation relative to the cooperative solution – therefore suggesting much stronger policy externalities beyond the Nash equilibrium when trade barriers are lower. This is because whilst the Nash equilibrium subsidy falls in τ, the optimal subsidy in the symmetric cooperative case is unaffected by trade costs. To see this, consider that the equilibrium solutions for subsidy and tax levels in the cooperative case cannot be distinguished from the corresponding solutions in the autarkic case given by equations (19), (20) and (21) – see also Table A3 for our numerical solutions. Specifically, the (reduced form) objective functions are $h\left(s^{*},s;\tau^{*},\tau\right)=h^{*}\left(s^{*},s;\tau^{*},\tau\right)\equiv\tilde{h}\left(s;\tau\right)$ which can be written as $\tilde{h}\left(s\right)=\kappa\left(\tau\right)h\left(s\right)$ where $h\left(s\right)$ is the corresponding autarkic (reduced form) employment equation and $\kappa\left(\tau\right)$ is a monotonically decreasing function with $\kappa'<0$, $\kappa\left(1\right)>1$ and $\kappa\left(\infty\right)=1$.

Our discussion of the autarkic case implies that, whilst the existence of heterogeneity in the CES framework does not alter the qualitative arguments for employment subsidisation, it affects the size of the optimal policy – with the optimal subsidy increasing in γ. Table A3 reports the effects of a 5% reduction in γ on the Nash and cooperative equilibria. A fall in γ corresponds to an increase in the degree of heterogeneity of firms' productivity, and can hence be thought of as a positive productivity shock. In the no-subsidy equilibrium this leads, as we would expect, to an increase in welfare and to a reallocation of resources towards the monopolistic sector driven by an increase in both domestic and export productivity cut-offs that raise the average productivity in that sector. With policy active governments, the same productivity shock results in the non-cooperative equilibrium being characterised by higher subsidies. Thus, contrary to the autarkic and cooperative case, the non-cooperative equilibrium is characterised by a positive relationship between the optimal subsidy and the degree of heterogeneity of firms the opposite relationship.[20] The degree of under-subsidisation relative to the cooperative solution would also be higher at lower values of γ – i.e. with more efficient productivity distributions, subsidising above the Nash level would generate larger positive policy externalities which non-cooperative policies fail to internalise.

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[20] Consistently, we have verified numerically that if the home country had a productivity distribution characterised by a higher heterogeneity than its trading partner (i.e. $\gamma<\gamma^{*}$), a unilateral subsidy when the foreign government is not policy active would be more effective in raising welfare and employment and hence the optimal subsidy would be lower than when countries' productivity distributions are symmetric.

6. Entry subsidies: a comparison

Given the role of entry in facilitating reallocations towards more efficient producers, the reduction of entry barriers is seen as an effective way to increase aggregate productivity and employment. To this end, governments implement policies (ranging from simplifying red tape procedures to start-up grants) to support entrepreneurship and induce the setting up of new firms.

In this section we briefly examine the effect of entry subsidies on aggregate productivity and employment against those of employment subsidies discussed above. In order to allow for a direct comparison between the two types of subsidies, we modify our model to replace employment subsidies s with an ad-valorem entry subsidy v (i.e. proportional to a firm's entry cost f) which is again financed via proportional income taxation. Thus, setting $s = 0$ in the autarkic model developed in Section 2 and introducing v instead, the government budget constraint in equation (15) becomes

$$v f F = t L^s . \tag{37}$$

Since the subsidy reduces the effective cost of entry, the expected zero-profit entry condition in (11) is now given by

$$M \left(r(\tilde{\varphi}) - l(\tilde{\varphi}) \right) - (1-v) f F = 0 . \tag{38}$$

Note that because the entry subsidy does not affect firms' marginal conditions, the average industry revenues and profits are not affected by the subsidy.

The autarkic productivity cut-off is now given by

$$\varphi_c = \left(\frac{\alpha(\sigma - 1)}{f(1+\gamma-\sigma)(1-v)} \right)^{1/\gamma} , \tag{39}$$

which is increasing in v. Thus, as in Pflüger and Suedekum (2013) and contrary to the employment subsidy case discussed above, an entry subsidy has a *pro-competitive* effect, and this effect is stronger the higher is the degree of heterogeneity among firms (i.e. the lower is γ). The welfare function in (10) can be shown to be strictly concave in v with the corresponding optimal entry subsidy in autarky given by

$$v^{opt} = \frac{\gamma(1-\beta)}{\gamma\sigma - \beta(\sigma-1)} , \tag{40}$$

which is positive and decreasing in γ. Hence, in contrast to the employment subsidy case, the more homogenous are firms the lower are the optimal entry subsidy and industry average productivity. A comparison between equations (22) and (40) and the corresponding welfare levels reveals that $v^{opt} < s^{opt}$ and $h(v^{opt}) < h(s^{opt})$: ceteris paribus, (i) the optimal entry subsidy rate is smaller than the optimal employment subsidy rate, and (ii) the optimal employment subsidy is associated with higher employment and welfare levels. To see this consider that the procompetitive effects of the entry subsidy can be shown to result in a fall in the mass of
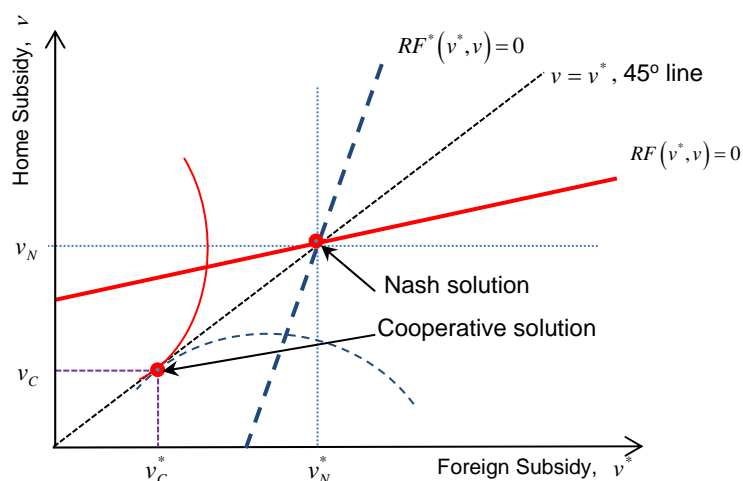
surviving firms.[21] Instead, an increase in the mass of firms contributes to explain why the optimal employment subsidy, despite its anti-competitive effects on the industry in autarky, leads to higher levels of welfare than an entry subsidy. More generally, underpinning these results is the fact that an employment subsidy offers a more direct way, than an entry subsidy, to tackle the monopolistic distortion in this model.[22]

Moving to the two-country setting, the solutions for the unilateral, Nash and cooperative policy equilibria are given in Table A4. As can be seen from the table, governments have a unilateral incentive to subsidise the entry of firms. However, contrary to autarky, the unilateral (i.e. when the trading partner's government is policy inactive) optimal entry subsidy rate is higher than the unilateral optimal employment subsidy rate – but its associated level of welfare continues to be lower than that achieved via an employment subsidy.

In a two-country world, both subsidies have procompetitive effects on the industry but these are stronger with an entry subsidy that leads to a larger entry. The resulting tougher selection forces lead to a smaller increase in the equilibrium mass of firms in the industry (and even in a fall in the extensive margin of exports). It is the smaller increase in product variety that drives the lower welfare achieved with this type of subsidy.

The reaction functions of the two governments when they are both policy active, which are illustrated in Figure 8, are upward sloping in (v^*,v) space with the Nash equilibrium entailing a positive entry subsidy rate. The Nash equilibrium, however, lies above the cooperative solution, hence non-cooperative behaviour leads to over-subsidisation in this case.

Figure 8. Nash and Cooperative solutions with entry subsidy

The intuition behind these results is consistent with that provided by Pflüger and Suedekum (2013): an increase in entry subsidy by one government has a selection and a fiscal externality effect on its trading partner. Whilst the latter is positive, the former is negative and dominates. This gives an incentive to retaliate that results in the non-cooperative equilibrium being characterised by over-subsidisation from a global welfare point of view.[23] As is clear from Table A4, although the non-cooperative entry subsidy is larger than the corresponding employment subsidy, the former leads to a lower level of welfare than the latter. Thus, despite its direct (and hence stronger) pro-competitive effects, an entry subsidy is less effective in increasing employment and welfare than an employment subsidy.

7. Conclusions

Employment subsidies are an important component of active labour market polices and their use by governments has increased in recent years in an attempt to raise (or restore) employment levels in the face of an adverse economic climate. This paper has studied their effects within a general equilibrium framework characterised by an endogenous level of employment and cost heterogeneity among firms.

We have shown that intra-industry competitive selection is an important channel in the transmission of the effects of employment subsidies on the level of economic activity and aggregate efficiency. Importantly, and arguably counterintuitively, international openness alters the nature of the effects of the subsidy on intra-industry selection: whilst the subsidy has an anti-competitive effect in autarky, it has pro-competitive effects in the open economy (akin to those of trade liberalisation) which result in higher average productivity and in a larger extensive margin of export.

Given the implications of the policy for market entry, aggregate efficiency and welfare, and in light of the international externality effects of the policy, governments have an incentive to use employment subsidies strategically. When governments subsidise firms uniformly, we show that international spillovers consist of both *selection* and *fiscal externalities* that result in non-cooperative and cooperative policy equilibria that are characterised by positive subsidies. Whether the non-cooperative solutions entail levels of subsidisation that exceed or fall short of those characterising the cooperative outcome depends, however, on the nature of the externality between countries – which in turn is affected by how the subsidy is targeted.

Importantly, despite stronger pro-competitive effects on industry, an entry subsidy is shown to be *less* effective in increasing employment and welfare than an employment subsidy. This is because it offers a less direct way to tackle the monopolistic distortion than does an employment subsidy.

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[23] As can be seen from Table A4, although qualitatively the nature of the international spill-over effects of an entry subsidy are similar to those of an employment subsidy, the latter has a stronger negative externality.

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Appendix

Table A1. Notation used in the model setup (for autarky and the home country only)

Description	Notation
Fixed cost of production of the differentiated good (closed economy)	α
Fixed cost of production of the differentiated good (domestic & export)	α_d & α_x
Budget share of Y & A	β & $1-\beta$
Labour supply elasticity (inverse of real wage elasticity of supply)	δ
Productivity distribution shape parameter (Pareto)	γ
Firm Level Productivity (differentiated good sector)	φ
productivity cut-off for marginal firms (closed economy)	φ_c
productivity cut-offs for marginal firms (non-exporting & exporting)	φ_d & φ_x
Average productivity (closed economy)	$\tilde{\varphi}$
Average productivity (non-exporting & exporting)	$\tilde{\varphi}_d$ & $\tilde{\varphi}_x$
Scale coefficient of labour supply	θ
CES elasticity of substitution	σ
Profit of a firm producing the differentiated good (closed economy)	π
Profit of a firm producing the differentiated good (domestic & export)	π_d & π_x
Iceberg trade cost for exporting firms	τ
Per capita demand for homogenous good	a
Aggregate demand for homogenous good	A
Aggregate supply of homogenous good	A^s
Mass of entrants	F
Employment ratio	h
Fixed entry cost	f
Labour requirement for producing the differentiated good (closed economy)	l
Labour requirement for producing the differentiated good (domestic & export)	l_d & l_x
Aggregate labour supply (employment)	$L^s = Nh$
Consumer population size	N
Mass of varieties of differentiated good produced (mass of surviving firms)	M
Mass of varieties of differentiated good produced and exported	M_x
Consumer price index	P
price of the homogeneous good	P_A
CES price index for Y	P_Y
Variety prices set by a firm producing the differentiated good (closed economy)	p
Variety prices set by a firm producing the differentiated good (non-exporting & exporting)	p_d & p_x
Revenue of a firm producing the differentiated good (closed economy)	r
Revenue of a firm producing the differentiated good (domestic & export)	r_d & r_x
Labour subsidy received by differentiated good producers	s
Labour subsidy received by differentiated good producers (domestic & export)	s_d & s_x
Entry subsidy	v
Income tax rate	t
Wage rate	w
Demand for a variety of differentiated good (closed economy)	y
Domestic and foreign demand for a domestically produced variety of differentiated good	y_d & y_x
Aggregate demand for differentiated good (CES)	Y
Total and per capita utility	u

A1. Solution of the closed economy model

Use $\quad \tilde{\varphi}^{\sigma-1} = \left(\dfrac{\gamma}{1+\gamma-\sigma}\right)\varphi_c^{\sigma-1}\quad$ to $\quad$ write $\quad r(\tilde{\varphi}) = \alpha\sigma(1-s)\left(\dfrac{\tilde{\varphi}}{\varphi_c}\right)^{\sigma-1}\quad$ and

$l(\tilde{\varphi}) = \alpha\left[(\sigma-1)\left(\dfrac{\tilde{\varphi}}{\varphi_c}\right)^{\sigma-1}+1\right]\quad$ respectively $\quad$ as $\quad r(\tilde{\varphi}) = \alpha\sigma(1-s)\left(\dfrac{\gamma}{1+\gamma-\sigma}\right)\quad$ and

$l(\tilde{\varphi}) = \alpha\left(\dfrac{1+\gamma\sigma-\sigma}{1+\gamma-\sigma}\right)$, which are then substituted in (13) and (15) to obtain

$M\alpha\sigma(1-s)\left(\dfrac{\gamma}{1+\gamma-\sigma}\right) = \beta(1-t)L$ and $sM\alpha\left(\dfrac{1+\gamma\sigma-\sigma}{1+\gamma-\sigma}\right) = tL$. For any given M and L

these solve for t, given by equation (20), and also imply $M = \dfrac{\beta(1+\gamma-\sigma)L}{\alpha\big(\beta(1+\gamma-\sigma)s+\gamma\sigma(1-s)\big)}$.

Substituting $w = P_A = 1$ and the expression for P_Y in (7), the labour supply function in (3) is written as

$$L = N\left(\frac{1-t}{\theta\left(M^{1/(1-\sigma)}\dfrac{\sigma(1-s)}{(\sigma-1)\tilde{\varphi}}\right)^{\beta}}\right)^{1/\delta},$$

which, upon replacing L using the result derived above, yields

$$M = N\left(\frac{\beta(1+\gamma-\sigma)}{\alpha\big(\beta(1+\gamma-\sigma)s+\gamma\sigma(1-s)\big)}\right)\left(\frac{1-t}{\theta\left(M^{1/(1-\sigma)}\dfrac{\sigma(1-s)}{(\sigma-1)\tilde{\varphi}}\right)^{\beta}}\right)^{1/\delta}.$$

Substituting for $\tilde{\varphi}$ and t using (17), (19) and (20), we obtain the following solutions:

$$M = \Omega\big(\beta(1+\gamma-\sigma)\big)^{-\frac{\delta(\sigma-1)}{\beta-\delta(\sigma-1)}}(1-s)^{-\left[\frac{(\sigma-1)(\gamma+\beta(1-\gamma))}{\gamma[\beta-\delta(\sigma-1)]}\right]}\big[\gamma\sigma(1-s)+\beta(1+\gamma\sigma-\sigma)s\big]^{\frac{(1+\delta)(\sigma-1)}{\beta-\delta(\sigma-1)}},$$

$$L = \alpha\Omega\big(\beta(1+\gamma-\sigma)\big)^{-\left[\frac{\beta}{\beta-\delta(\sigma-1)}\right]}(1-s)^{-\left[\frac{(\sigma-1)(\gamma+\beta(1-\gamma))}{\gamma[\beta-\delta(\sigma-1)]}\right]}\big[\gamma\sigma(1-s)+\beta(1+\gamma\sigma-\sigma)s\big]^{\frac{\beta+(\sigma-1)}{\beta-\delta(\sigma-1)}},$$

where $\quad \Omega = \left(\dfrac{\alpha}{\Lambda N}\right)^{\frac{\delta(\sigma-1)}{\beta-\delta(\sigma-1)}}(\gamma\sigma)^{-\frac{(\sigma-1)}{\beta-\delta(\sigma-1)}}\quad$ and $\quad \Lambda = \theta^{-1/\delta}\rho^{\beta/\delta}\big((\alpha/f)(\sigma-1)/(1+\gamma-\sigma)\big)^{\beta/\gamma\delta}$.

Inspection of these reveals that both M and L are concave in s, with L reaching a maximum at a lower value of s than does M. It is straightforward to show that the value of s in equation (21) maximises L and hence welfare U which is a monotonically increasing function of L.

Note that equation (22) is obtained using (12), the expressions for L and M and $l(\tilde{\varphi}) = \alpha\left(\dfrac{1+\gamma\sigma-\sigma}{1+\gamma-\sigma}\right)$ obtained above.

Table A2. Two-country model setup

No	Description	Equation for the home country
(A1)	Consumer's utility function	$U = \left(\dfrac{a}{1-\beta}\right)^{1-\beta}\left(\dfrac{y}{\beta}\right)^{\beta} - \dfrac{\theta h^{1+\delta}}{1+\delta}$
(A2)	Aggregate labour supply	$L^s \equiv Nh = N\left(\dfrac{(1-t)w}{\theta P}\right)^{1/\delta}$
(A3)	Aggregate demand for homogenous good	$A \equiv Na = \dfrac{N(1-\beta)(1-t)wh}{P_A}$
(A4)	Aggregate demand for differentiated good	$Y \equiv Ny = \dfrac{N\beta(1-t)wh}{P_Y}$
(A5)	Consumer price index	$P = P_A^{1-\beta}P_Y^{\beta}$
(A6)	Production function for the homogenous good	$A^s = L_A$
(A7)	Productivity distribution in the differentiated good sector	$G(\varphi) = 1 - \varphi^{-\gamma}$ and $g(\varphi) = \gamma\varphi^{-(1+\gamma)}$, $\varphi \in [1,\infty)$
(A8)	Mass of varieties of differentiated good (mass of surviving firms)	$M \equiv (1 - G(\varphi_d))F$; $M = \varphi_d^{-\gamma}F$, $\varphi \in [\varphi_d, \infty)$
(A9)	Mass of varieties of differentiated good exported	$M_x \equiv (1 - G(\varphi_x))F$; $M_x = \varphi_x^{-\gamma}F$, $\varphi \in [\varphi_x, \infty)$
(A10)	Average productivity cut-offs are proportional to marginal firms' cut-offs	$\tilde{\varphi}_d^{\sigma-1} = \left(\dfrac{\gamma}{1+\gamma-\sigma}\right)\varphi_d^{\sigma-1}$, $\tilde{\varphi}_x^{\sigma-1} = \left(\dfrac{\gamma}{1+\gamma-\sigma}\right)\varphi_x^{\sigma-1}$
(A11)	Productivity distribution of the surviving firms in the differentiated good sector	$\mu_d(\varphi) = \dfrac{g(\varphi)}{1-G(\varphi_d)}$, $\mu_x(\varphi) = \dfrac{g(\varphi)}{1-G(\varphi_x)}$
(A12)	CES aggregation of differentiated goods consumed (domestically produced and imported)	$Y = \left(\displaystyle\int_{\varphi\in[\varphi_d,\infty)} M\mu_d(\varphi)y_d(\varphi)^{1-1/\sigma}\,d\varphi + \int_{\varphi\in[\varphi_x^*,\infty)} M_x^*\mu_x^*(\varphi)y_x^*(\varphi)^{1-1/\sigma}\,d\varphi\right)^{\frac{1}{1-1/\sigma}}$
(A13)	CES price index for Y	$P_Y = \left(\displaystyle\int_{\varphi\in[\varphi_d,\infty)} M\mu_d(\varphi)p_d(\varphi)^{1-\sigma}\,d\varphi + \int_{\varphi\in[\varphi_x^*,\infty)} M_x^*\mu_x^*(\varphi)p_x^*(\varphi)^{1-\sigma}\,d\varphi\right)^{\frac{1}{1-\sigma}}$
(A14)	Demand for a domestically produced differentiated good facing a firm with a given productivity	$y_d(\varphi) = Y\left(\dfrac{p_d(\varphi)}{P_Y}\right)^{-\sigma}$, $\varphi \in [\varphi_d, \infty)$
(A15)	Demand for an imported differentiated good facing a firm with a given productivity	$y_x^*(\varphi) = Y\left(\dfrac{p_x^*(\varphi)}{P_Y}\right)^{-\sigma}$, $\varphi \in [\varphi_x^*, \infty)$
(A16)	Labour requirement for producing the differentiated good by a firm with a given productivity for its domestic production	$l_d(\varphi) = \alpha_d + \dfrac{y_d(\varphi)}{\varphi}$, $\varphi \in [\varphi_d, \infty)$

Table A2 continued

No	Description	Equation for the home country
(A17)	Profit and revenue of a firm with a given productivity for its domestic production	$\pi_d(\varphi) = r_d(\varphi) - (1 - s_d) w l_d(\varphi),$ $r_d(\varphi) = p_d(\varphi) y_d(\varphi); \ \varphi \in [\varphi_d, \infty)$
(A18)	Price set by a firm with a given productivity for its domestic production	$p_d(\varphi) = \dfrac{\sigma(1 - s_d) w}{(\sigma - 1)\varphi}, \quad \varphi \in [\varphi_d, \infty)$
(A19)	Labour requirement for producing the differentiated good by a firm with a given productivity for its export production	$l_x(\varphi) = \alpha_x + \dfrac{\tau y_x(\varphi)}{\varphi}, \quad \varphi \in [\varphi_x, \infty)$
(A20)	Profit and revenue of a firm with a given productivity for its export production	$\pi_x(\varphi) = r_x(\varphi) - (1 - s_x) w l_x(\varphi),$ $r_x(\varphi) = p_x(\varphi) y_x(\varphi); \ \varphi \in [\varphi_x, \infty)$
(A21)	Price set by a firm with a given productivity for its export production	$p_x(\varphi) = \dfrac{\sigma(1 - s_x)\tau w}{(\sigma - 1)\varphi}, \quad \varphi \in [\varphi_x, \infty)$
(A22)	Aggregating domestic price of differentiated good	$\displaystyle \int_{\varphi \in [\varphi_d, \infty)} M \mu_d(\varphi) p_d(\varphi)^{1-\sigma} \, d\varphi = M p_d(\tilde{\varphi}_d)^{1-\sigma}$
(A23)	Aggregating imported price of differentiated good	$\displaystyle \int_{\varphi \in [\varphi_x^*, \infty)} M_x^* \mu_x^*(\varphi) p_x^*(\varphi)^{1-\sigma} \, d\varphi = M_x^* p_x^* (\tilde{\varphi}_x^*)^{1-\sigma}$
(A24)	CES price index for differentiated good in terms of aggregates	$P_Y = \left(M p_d(\tilde{\varphi}_d)^{1-\sigma} + M_x^* p_x^*(\tilde{\varphi}_x^*)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$
(A25)	Aggregating revenue of domestic sales of differentiated good	$\displaystyle \int_{\varphi \in [\varphi_d, \infty)} M \mu_d(\varphi) r_d(\varphi) \, d\varphi = M r_d(\tilde{\varphi}_d)$
(A26)	Aggregating revenue of exports of differentiated good	$\displaystyle \int_{\varphi \in [\varphi_x, \infty)} M_x \mu_x(\varphi) r_x(\varphi) \, d\varphi = M_x r_x(\tilde{\varphi}_x)$
(A27)	Aggregating profit of domestic sales of differentiated good	$\displaystyle \int_{\varphi \in [\varphi_d, \infty)} M \mu_d(\varphi) \pi_d(\varphi) \, d\varphi = M \pi_d(\tilde{\varphi}_d)$
(A28)	Average profit for domestic sales	$\pi_d(\tilde{\varphi}_d) = \dfrac{r_d(\tilde{\varphi}_d)}{\sigma} - (1 - s_d) w \alpha_d$
(A29)	Aggregating profit of exports of differentiated good	$\displaystyle \int_{\varphi \in [\varphi_x, \infty)} M_x \mu_x(\varphi) \pi_x(\varphi) \, d\varphi = M_x \pi_x(\tilde{\varphi}_x)$
(A30)	Average profit for exports	$\pi_x(\tilde{\varphi}_x) = \dfrac{r_x(\tilde{\varphi}_x)}{\sigma} - (1 - s_x) w \alpha_x$
(A31)	Labour used in the differentiated sector for domestically used production	$\displaystyle \int_{\varphi \in [\varphi_d, \infty)} M \mu_d(\varphi) l_d(\varphi) \, d\varphi = M l_d(\tilde{\varphi}_d)$
(A32)	Labour used in the differentiated sector for export production	$\displaystyle \int_{\varphi \in [\varphi_x, \infty)} M_x \mu_x(\varphi) l_x(\varphi) \, d\varphi = M_x l_x(\tilde{\varphi}_x)$
(A33)	Labour used in the differentiated sector for export production	$\displaystyle \int_{\varphi \in [\varphi_x, \infty)} M_x \mu_x(\varphi) l_x(\varphi) \, d\varphi = M_x l_x(\tilde{\varphi}_x)$

A2. Derivation of the equations of the two-country model used in the numerical analysis

Using equations (A16) and (A18), (A17) in Table A2 can be written as $\pi_d = \frac{r_d}{\sigma} - (1 - s_d) w \alpha_d$. Then the zero profit condition of marginal non-exporting and exporting firms in home and foreign country imply

$$\varphi = \varphi_d \Rightarrow \pi_d(\varphi_d) = 0 \Rightarrow \quad r_d(\varphi_d) = \sigma(1 - s_d) w \alpha_d \tag{E1}$$

$$\varphi = \varphi_d^* \Rightarrow \pi_d^*(\varphi_d^*) = 0 \Rightarrow \quad r_d^*(\varphi_d^*) = \sigma(1 - s_d^*) w^* \alpha_d^*, \tag{E1*}$$

$$\varphi = \varphi_x \Rightarrow \pi_x(\varphi_x) = 0 \Rightarrow \quad r_x(\varphi_x) = \sigma(1 - s_x) w \alpha_x, \tag{E2}$$

$$\varphi = \varphi_x^* \Rightarrow \pi_x^*(\varphi_x^*) = 0 \Rightarrow \quad r_x^*(\varphi_x^*) = \sigma(1 - s_x^*) w^* \alpha_x^*. \tag{E2*}$$

The zero expected profit of entry for the home and foreign country require

$$M \pi_d(\tilde{\varphi}_d) + M_x \pi_x(\tilde{\varphi}_x) - F P_A f = 0, \tag{E3}$$

$$M^* \pi_d^*(\tilde{\varphi}_d^*) + M_x^* \pi_x^*(\tilde{\varphi}_x^*) - F^* P_A^* f^* = 0. \tag{E3*}$$

The balanced government budget constraints (equating the subsidy bill with tax revenue) for the home and foreign country are

$$s_d w M l_d + s_x w M_x x_x = N t w h, \tag{E4}$$

$$s_d^* w^* M^* l_d^* + s_x^* w^* M_x^* l_x^* = N t^* w^* h^*. \tag{E4*}$$

The CES price indices in (A13) and the aggregations in (A22) and (A23) imply

$$P_Y = \left(M \left(p_d(\tilde{\varphi}_d) \right)^{1-\sigma} + M_x^* \left(p_x^*(\tilde{\varphi}_x^*) \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}, \tag{E5}$$

$$P_Y^* = \left(M^* \left(p_d^*(\tilde{\varphi}_x^*) \right)^{1-\sigma} + M_x \left(p_x(\tilde{\varphi}_x) \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{E5*}$$

Per-capita labour supplies, given by (A2), are

$$h = \left(\frac{(1-t) w}{\theta P_Y^\beta} \right)^{1/\delta}, \tag{E6}$$

$$h^* = \left(\frac{(1-t^*) w^*}{\theta P_Y^{*\beta}} \right)^{1/\delta}. \tag{E6*}$$

The other equilibrium conditions which should hold are the labour market equilibrium conditions,

$$L_A + M l_d + M_x l_x = N h, \tag{E7}$$

$$L_A^* + M^* l_d^* + M_x^* l_x^* = N h^*, \tag{E7*}$$

the global market equilibrium condition for the homogenous good

$$P_A \left(A + F f \right) + P_A^* \left(A^* + F^* f^* \right) = P_A A^s + P_A^* A^{*s}, \tag{E8}$$

and the global trade balance,

$$P_A \left(A^s - A - F f \right) + M_x r_x \left(\tilde{\varphi}_x \right) = M_x^* r_x^* \left(\tilde{\varphi}_x^* \right). \tag{E9}$$

It can be shown that (E8) and (E9) are satisfied – by the implication of Walras law – if (E1) to (E7*) hold.

Finally, since the homogeneous good is competitively produced under constant return to scales condition, is freely traded and is used as numeraire, we have $P_A = P_A^* = 1$, $w = P_A = 1$ and $w^* = P_A^* = 1$. Taking account of the latter normalisations and making some substitutions to eliminate as many variables as possible, equations in (E1)-E(6*) can be shown to be expressed in the form of (E1)′-E(6*)′ below which satisfy labour resource conditions in (E7)-(E7*) and can be solved to determine the 12 unknowns $\left(\varphi_d, \varphi_x, \varphi_d^*, \varphi_x^*, P_Y, P_Y^*, F, F^*, h, h^*, t, t^* \right)$ on the assumption that the subsidy rates are treated as exogenous policy instruments.

$$\left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} N \beta \left(1 - t \right) h \left(P_Y \right)^{\sigma - 1} \left(\varphi_d \right)^{\sigma - 1} \left(1 - s_d \right)^{-\sigma} = \sigma \alpha_d \tag{E1)′}$$

$$\left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} N \beta \left(1 - t^* \right) h^* \left(P_Y^* \right)^{\sigma - 1} \left(\varphi_d^* \right)^{\sigma - 1} \left(1 - s_d^* \right)^{-\sigma} = \sigma \alpha_d^* \tag{E1*)′}$$

$$\left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} N \beta \left(1 - t^* \right) h^* \left(P_Y^* \right)^{\sigma - 1} \left(\varphi_x \right)^{\sigma - 1} \tau^{1-\sigma} \left(1 - s_x \right)^{-\sigma} = \sigma \alpha_x \tag{E2)′}$$

$$\left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} N \beta \left(1 - t \right) h \left(P_Y \right)^{\sigma - 1} \left(\varphi_x^* \right)^{\sigma - 1} \tau^{*1-\sigma} \left(1 - s_x^* \right)^{-\sigma} = \sigma \alpha_x^* \tag{E2*)′}$$

$$\left(1 - s_d \right) \varphi_d^{-\gamma} \left(\left(\frac{\gamma}{1 + \gamma - \sigma} \right) \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \frac{N \beta \left(1 - t \right) h}{\sigma} \left(P_Y \right)^{\sigma - 1} \left(\varphi_d \right)^{\sigma - 1} \left(1 - s_d \right)^{-\sigma} - \alpha_d \right) +$$
$$\left(1 - s_x \right) \varphi_x^{-\gamma} \left(\left(\frac{\gamma}{1 + \gamma - \sigma} \right) \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \frac{N \beta \left(1 - t^* \right) h^*}{\sigma} \left(P_Y^* \right)^{\sigma - 1} \left(\varphi_x \right)^{\sigma - 1} \tau^{1-\sigma} \left(1 - s_x \right)^{-\sigma} - \alpha_x \right) = f \tag{E3)′}$$

$$\left(1 - s_d^* \right) \varphi_d^{*-\gamma} \left(\left(\frac{\gamma}{1 + \gamma - \sigma} \right) \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \frac{N \beta \left(1 - t^* \right) h^*}{\sigma} \left(P_Y^* \right)^{\sigma - 1} \left(\varphi_d^* \right)^{\sigma - 1} \left(1 - s_d^* \right)^{-\sigma} - \alpha_d^* \right) +$$
$$\left(1 - s_x^* \right) \varphi_x^{*-\gamma} \left(\left(\frac{\gamma}{1 + \gamma - \sigma} \right) \left(\frac{\sigma}{\sigma - 1} \right)^{1-\sigma} \frac{N \beta \left(1 - t \right) h}{\sigma} \left(P_Y \right)^{\sigma - 1} \left(\varphi_x^* \right)^{\sigma - 1} \tau^{*1-\sigma} \left(1 - s_x^* \right)^{-\sigma} - \alpha_x^* \right) = f^* \tag{E3*)′}$$

$$s_d \varphi_d^{-\gamma} F\left(\left(\frac{\gamma}{1+\gamma-\sigma} \right)\left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} N\beta(1-t)h(P_Y)^{\sigma-1}(\varphi_d)^{\sigma-1}(1-s_d)^{-\sigma} + \alpha_d \right) +$$

$$s_x \varphi_x^{-\gamma} F\left(\left(\frac{\gamma}{1+\gamma-\sigma} \right)\left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} N\beta(1-t^*)h^*\left(P_Y^*\right)^{\sigma-1}(\varphi_x)^{\sigma-1}\tau^{1-\sigma}(1-s_x)^{-\sigma} + \alpha_x \right) = Nth \quad \text{(E4)}'$$

$$s_d^* \varphi_d^{*-\gamma} F^*\left(\left(\frac{\gamma}{1+\gamma-\sigma} \right)\left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} N\beta(1-t^*)h^*\left(P_Y^*\right)^{\sigma-1}\left(\varphi_d^*\right)^{\sigma-1}\left(1-s_d^*\right)^{-\sigma} + \alpha_d^* \right) +$$

$$s_x^* \varphi_x^{*-\gamma} F^*\left(\left(\frac{\gamma}{1+\gamma-\sigma} \right)\left(\frac{\sigma}{\sigma-1} \right)^{-\sigma} N\beta(1-t)h(P_Y)^{\sigma-1}\left(\varphi_x^*\right)^{\sigma-1}\tau^{*1-\sigma}\left(1-s_x^*\right)^{-\sigma} + \alpha_x^* \right) = Nt^*h^* \quad \text{(E4}^*)'$$

$$P_Y = \left(\frac{\sigma}{\sigma-1} \right)\left(\left(\frac{\gamma}{1+\gamma-\sigma} \right)F(\varphi_d)^{-(1+\gamma-\sigma)}(1-s_d)^{1-\sigma} + \right.$$

$$\left. \left(\frac{\gamma}{1+\gamma-\sigma} \right)F^*\left(\varphi_x^*\right)^{-(1+\gamma^*-\sigma)}\tau^{*1-\sigma}\left(1-s_x^*\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \quad \text{(E5)}'$$

$$P_Y^* = \left(\frac{\sigma}{\sigma-1} \right)\left(\left(\frac{\gamma}{1+\gamma-\sigma} \right)F^*\left(\varphi_d^*\right)^{-(1+\gamma^*-\sigma)}\left(1-s_d^*\right)^{1-\sigma} + \right.$$

$$\left. \left(\frac{\gamma}{1+\gamma-\sigma} \right)F(\varphi_x)^{-(1+\gamma-\sigma)}\tau^{1-\sigma}(1-s_x)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \quad \text{(E5}^*)'$$

$$h = \left(\frac{1-t}{\theta P_Y^\beta} \right)^{1/\delta} \quad \text{(E6)}'$$

$$h^* = \left(\frac{1-t^*}{\theta P_Y^{*\beta}} \right)^{1/\delta} \quad \text{(E6}^*)'$$

The above 12 equations are used as in our numerical analysis. To see the recursive nature of the model and to derive equations (32) and (33), note that (E1)′ and (E1*)′ can be used to eliminate $\left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} N\beta(1-t)h(P_Y)^{\sigma-1}$ and $\left(\frac{\sigma}{\sigma-1} \right)^{1-\sigma} N\beta(1-t^*)h^*\left(P_Y^*\right)^{\sigma-1}$ from (E2)′-(E3*)′ to obtain $\left(\frac{\varphi_x}{\varphi_d^*} \right)^{\sigma-1} = \tau^{\sigma-1}\left(\frac{1-s_x}{1-s_d^*} \right)^{\sigma}\left(\frac{\alpha_x}{\alpha_d^*} \right)$ and $\left(\frac{\varphi_d}{\varphi_x^*} \right)^{\sigma-1} = \tau^{*1-\sigma}\left(\frac{1-s_d}{1-s_x^*} \right)^{\sigma}\left(\frac{\alpha_d}{\alpha_x^*} \right)$ which yield (32), and $\left(\alpha_d(1-s_d)\varphi_d^{-\gamma} + \alpha_x(1-s_x)\varphi_x^{-\gamma} \right)\left(\frac{\sigma-1}{1+\gamma-\sigma} \right) = f$ and $\left(\alpha_d^*\left(1-s_d^*\right)\varphi_d^{*-\gamma^*} + \alpha_x^*\left(1-s_x^*\right)\varphi_x^{*-\gamma^*} \right)\left(\frac{\sigma-1}{1+\gamma^*-\sigma} \right) = f^*$ which are in (33).

Table A3. Comparing the equilibrium solutions of the two-country model for different cases: the role of γ and τ

Variables	Benchmark Equilibrium with no Subsidy			Cooperative Equilibrium with Uniform Subsidy			Nash Equilibrium with Uniform Subsidy		
	Initial Case $\gamma = 3.4$ $\tau = 1.3$	γ falls by 5% $\gamma = 3.23$ $\tau = 1.3$	τ falls by 5% $\gamma = 3.4$ $\tau = 1.235$	Initial Case $\gamma = 3.4$ $\tau = 1.3$	γ falls by 5% $\gamma = 3.23$ $\tau = 1.3$	τ falls by 5% $\gamma = 3.4$ $\tau = 1.235$	Initial Case $\gamma = 3.4$ $\tau = 1.3$	γ falls by 5% $\gamma = 3.23$ $\tau = 1.3$	τ falls by 5% $\gamma = 3.4$ $\tau = 1.235$
s	0	0	0	0.141	0.138	0.141	0.1065	0.108	0.096
t	0	0	0	0.093264	0.089964	0.093264	0.069499	0.069564	0.062392
h	0.626221	0.666732	0.630406	0.627455	0.667994	0.631649	0.627373	0.667929	0.63151
φ_d	1.576315	1.802935	1.59898	1.507403	1.721922	1.529077	1.524962	1.740256	1.552213
φ_x	2.624806	3.002163	2.529419	2.510056	2.867262	2.418839	2.539295	2.897793	2.455439
F	32.31272	36.21378	32.52871	29.35685	33.01826	29.55308	30.12235	33.75514	30.55257
M	6.876808	5.395827	6.594781	7.27327	5.707304	6.974983	7.174764	5.638442	6.851928
M_x	1.214627	1.039349	1.386744	1.284653	1.099346	1.466693	1.267254	1.086082	1.440817
P_Y	0.208905	0.178603	0.205455	0.183934	0.158001	0.180896	0.190042	0.16248	0.188731
$L_d = Ml_d\left(\tilde{\varphi}_d\right)$	289.9721	297.2097	278.0799	306.6896	314.3662	294.1118	302.5359	310.5733	288.923
$L_x = M_x l_x\left(\tilde{\varphi}_x\right)$	102.4336	114.4976	116.9487	108.3391	121.107	123.6911	106.8718	119.6458	121.5089
$\left(L_d + L_x\right)/L^s$	0.626625	0.6175	0.626625	0.661448	0.651912	0.661448	0.652575	0.644109	0.649921

- See Table A4 for the case of unilateral policy by the home country when the foreign country is policy inactive.

- The parameter values used in the calibrations are: $N = 1000$, $\beta = 0.8$, $\delta = 2$, $\theta = 8.9245$, $\sigma = 3.8$, $\alpha_d = 2.5$, $\alpha_x = 5.0$ and $f = 3.36$.

- The solutions were obtained using the MCP/PATH engine in GAMS; the robustness of these solutions are confirmed by extensive sensitivity analyses. The solution values for all other variables are available from the authors on request.

Table A4. Comparing the Optimal Policy Values of Variables in Different Cases: the role of different subsidies

Variables	No Policy	Unilateral Policy		Cooperative Policy		Nash Policy	
	Benchmark	Employment Subsidy, s	Entry Subsidy, v	Employment Subsidy, s	Entry Subsidy, v	Employment Subsidy, s	Entry Subsidy, v
s, v	0	0.0675 [0]	0.165 [0]	0.141	0.063	0.1065	0.2205
t	0	0.061378 [0]	0.042784 [0]	0.093264	0.011522	0.069499	0.04675
h	0.626221	0.632891 [0.61641]	0.631018 [0.618848]	0.627455	0.62637	0.627373	0.625145
φ_d	1.576315	1.62377 [1.523872]	1.718958 [1.536813]	1.507403	1.606774	1.524962	1.696141
φ_x	2.624806	2.307872 [2.972826]	2.559028 [2.862328]	2.510056	2.675524	2.539295	2.824333
F	32.31272	44.19013 [18.26889]	48.69677 [22.43769]	29.35685	34.09604	30.12235	39.44735
M	6.876808	8.502408 [4.362014]	7.719658 [5.205555]	7.27327	6.799197	7.174764	6.544062
M_x	1.214627	2.572795 [0.449714]	1.99547 [0.62825]	1.284653	1.200919	1.267254	1.155856
P_y	0.208905	0.187957 [0.217316]	0.194055 [0.21518]	0.183934	0.205777	0.190042	0.197616
$L_d = Ml_d(\tilde{\varphi}_d)$	289.9721	216.9724 [37.92585]	168.2846 [52.9824]	306.6896	101.2775	302.5359	97.47715
$L_x = Ml_x(\tilde{\varphi}_x)$	102.4336	575.4906 [221.8574]	493.7969 [272.483]	108.3391	387.9768	106.8718	373.4184
$(L_d + L_x)/L^s$	0.626625	0.909304 [0.359918]	0.78254 [0.44031]	0.661448	0.619405	0.652575	0.59733

▪ The columns in the Unilateral Policy refer to the case in which the home country acts unilaterally while the foreign country remains policy inactive. Figures in square brackets are the corresponding values for the foreign country.
▪ The parameter values used in the calibrations are as reported in Table A3, with $\gamma = 3.4$ and $\tau = 1.3$.
▪ The solutions were obtained using the MCP/PATH engine in GAMS; the robustness of these solutions are confirmed by extensive sensitivity analyses.