Wages, Cohort Effects and Unemployment Dynamics

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Abstract

This paper analyses a model in which firms cannot pay discriminate based on year of entry to a firm, and develops an equilibrium model of wage dynamics and unemployment. The model is developed under the assumption of worker mobility, so that workers can costlessly quit jobs at any time. Firms on the other hand are committed to contracts. We solve for the dynamics of wages and unemployment, and show that real wages exhibit a downward stickiness. In periods with adverse shocks the wage does not fall sufficiently to clear the labour market.

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Introduction

This paper develops a model in which firms cannot pay discriminate based on year of entry to a firm—there are no "cohort effects"— and develops an equilibrium model of wage dynamics and unemployment. The model is developed under the assumption of worker risk aversion, and also mobility, so that workers can costlessly quit jobs at any time. Firms on the other hand are risk neutral and are committed to contracts. Because of equal treatment of workers, firms have to trade-off the desire to insure their risk-averse workers against the need to respond to market conditions to not only prevent their workers from quitting but also to take advantage of states of the world where labour is cheap. We solve for the dynamics of wages and unemployment, and show that real wages exhibit a downward stickiness, due to the desire to insure incumbent workers. The equal treatment assumption prevents firms from cutting wages for new entrants, so that in periods with adverse shocks the wage does not fall sufficiently to clear the labour market.

We argue that even our rudimentary model, when fed sectoral productivity shocks from the post-war U.S. economy, gives a reasonably good account of U.S. unemployment movements.

The idea that internal equity considerations can play a part in wage rigidity is by no means novel. Truman Bewley has argued recently that it is a key feature constraining wage cuts for new hires in recessions. In his story, cutting wages of incumbents will have such a negative impact on their morale that firms will avoid it under all but extreme circumstance; at the same time while new hires may be willing to work at a lower wage than that paid to incumbents, paying them less would disrupt internal equity and so their wages will be set at the same level as incumbents' (controlling for experience, etc.):

> New employees, in contrast, feel it is inequitable to be paid according to a scale lower than the one that applied to colleagues that were hired earlier. For this reason, downward pay rigidity for new hires exists only because the pay of existing employees is rigid. ()

Bringing in workers at a *higher* pay than incumbents is even more problematic (thus while—in contrast to the primary sector—he found evidence that new hires are sometimes paid a lower rate than incumbents in the secondary sector, even there, paying new hires *more* than incumbents is deemed to be very disruptive REFERENCE).

Assuming that Bewley's account is accurate, it raises the important question of how forward looking firms take into account the fact that such constraints may arise in the future: for example, a firm, anticipating this downward wage rigidity, may temper wage increases in better times. Or in more generality, and supposing that firms can offer long-term contracts, the firm must take into account these equal treatment constraints which will prevent it bringing in new hires at a low wage in downturns, and also prevent the firm hiring at a higher wage than that offered to incumbents when the labour market is tight. To our knowledge, the dynamic implications of equal treatment have not been analyzed elsewhere. In our model however the desire to prevent incumbent wages from falling arises from to the desire to insure workers, rather than directly from worker moral considerations.

The linking of the pay of new hires to that of incumbents means that wage rigidity also has real allocational implications. Obviously wage rigidity for incumbents need not imply deviations from Walrasian outcomes so long as hiring is at the Walrasian level (in our model workers only separate for exogenous reasons). We show however that (under certain conditions) firms hire up to the point where the real wage equals the marginal product of labour; to the extent then that wages do not correspond to Walrasian levels hiring will be inefficient and in fact we show that this occurs only in the direction of wages being too high leading to inefficiently low employment and an excess supply of labor.

The paper builds on the seminal contribution of (hereafter BDN). They develop three versions of a model of labour contracting where a risk-neutral firm potentially offers insurance to risk-averse employees. In the spot market model, wages are determined solely by the value of a worker's marginal product, in the full-commitment contracting model, wages are constant but the level is determined by the worker's outside opportunity at the point at which he/she joins the firm, and in a version with no worker commitment (perfect mobility), where the worker is free to quit at any point, they show that the wage follows a ratchet like process, rising whenever the labour market is tighter than hitherto (since the worker joined the firm), but staying constant otherwise; hence the current wage is determined by the tightest labour market during a worker's tenure. They test these three models against each other on U.S. data. Perhaps surprisingly, the latter model appears to perform much better than the other two. Subsequent research cite: McDonaldW99,Grant03,ShinS02 has largely confirmed these results over different periods and using different datasets. Although the economic environments are distinct, our theoretical model deviates from theirs essentially only in the imposition of equal treatment.

In BDN, without the equal treatment assumption, each worker is treated independently, and the partial equilibrium analysis then boils down to a two-player game. It follows that the labour market must always clear, since at the point of hiring there are no restrictions on wages. The downward wage rigidity in their perfect mobility model provides insurance to the worker but does not directly affect employment decisions. Here, by contrast, we do not allow firms to treat each worker separately, but each new cohort of hires must fit into an existing wage structure. Even though we find that the characterization of optimal contracts is in a number of respects similar to that in BDN, the implications for employment are very different, as there will be episodes of involuntary unemployment. We also show that although very robust, the estimated business cycle effect on wages (i.e., through the minimum unemployment rate) in their estimations cannot explain very much of the movement of wages over the sample we look at (an extension of the one they examine). On the other hand, wage movements predicted by our model can explain much of this.

There is a small amount of other theoretical work in this specific area, following . showed in a model of repeated interactions with discounting that when mobility costs are low and enforcement costs are high a self-enforcing wage contract also exhibits sticky wages. In a self-enforcing wage contract, wages are structured so that neither employer nor employee have an incentive to deviate from the contract because the long-term benefits to adhering from the contract exceed any short-term gain to be had by deviating. The contract is then sustained by the threat of reverting to some worse equilibrium outcome. In a self-enforcing contract wages are sticky but may rise when demand for labour increases and fall when demand for labour weakens. The key result is that the wage itself follows a simple Markov process: the current wage depends only on the current state of demand and the wage last period. The model has been extended by who introduces matching frictions and produces simulations to show that the model is consistent with recent data from US manufacturing industry. An interesting feature of this model is that a worker who leaves a firm cannot get a job elsewhere immediately (an assumption in BDN), nor is thrown for ever onto a spot market (Thomas-Worrall), but has to wait to be matched with another firm. This potential loss of utility while unmatched provides sufficient leverage to allow firms to offer some insurance, even when firms can also quit the relationship.

The model

The model is as follows. There is an infinite horizon $t = 1, 2, 3 \ldots,$ and a single consumption good each period. All workers are assumed to be identical, apart from the date of entry into the economy (we abstract from any tenure or experience effects on productivity). Workers are risk averse with per period utility function $u(w),\ u' > 0, u'' < 0,$ where w is the income/consumption received within the period; crucially, it is assumed that they cannot make credit market transactions. There is no disutility of work, but hours are fixed so that workers are either employed or unemployed. Assume that if workers are not employed in a period, they receive some low consumption level $\underline{c} \geq 0.$ (Thus we only consider the extensive margin in the labour input, and moreover assume an infinitely inelastic labour supply above $\underline{c}$.) There is a large (but fixed) number of identical risk-neutral firms. The firm has a diminishing returns technology $f(N,s)$ with $\partial f / \partial N > 0, \partial^2 f / \partial N^2 < 0,$ where N is labour input and s is the current productivity shock (the sole source of fluctuations). It is assumed that a firm must always employ some (minimum measure of) workers each period. Workers and firms discount the future with respective factors $\beta_w, \beta_f \in (0,1).$ Workers have a probability $\delta \in (0,1)$ of survival each period, receiving zero utility in perpetuity if they exit (die). Those who die are replaced by the same number of new entrants. Moreover, there are a large number of workers relative to the number of firms, and we normalize the ratio of workers to firms to be one each period. We assume that the Walrasian wage is always greater than the unemployment consumption level: $\partial F / \partial N(1, s_t) > \underline{c}$ all t.

The state of nature (productivity) s_t follows a Markov process, with initial value $s_1,$ and countable state space S, but assume that from any state s only a finite number of states $r \in S$ are reachable next period with transition probabilities: $\pi_{sr} > 0$. Let $h_t \equiv (s_1, s_2, \ldots, s_t)$ be the history at t. While the firm is committed to contracts, workers are not (although we relax this later). The labour market offers a worker currently looking for work (at the start of t) a utility (discounted to t) of $\chi_t = \chi(h_t)$. We assume symmetry between the situation of a worker who is currently employed and one who is searching for work, by assuming that a worker who quits their current employer at t gets $\chi(h_t)$. Thus a firm must offer at least $\chi(h_t)$ to prevent its workers from quitting, and this is also the minimum utility that must be offered to hire: We assume that the firm can hire any number of workers by offering at least χ_t (and cannot hire otherwise). So the labour market is modelled as being competitive.

Our strategy will be to construct an equilibrium under the working hypothesis that firms hire each period (so they replace at least some of those who die), and then later we will find a restriction on parameters under which hiring does indeed always occur. This working hypothesis will also imply that we can ignore layoffs, but formally we will state the optimization problem *imposing* no layoffs, to avoid complicating the statement of the problem. Then we shall construct the hiring equilibrium as a solution to this problem. Finally it will follow that the hiring equilibrium is also a solution to a problem in which layoffs are permitted.

We work with a representative firm, and we shall use a $*$ superscript to denote equilibrium values. At the start of date 1, after s_1 is observed, firms commit to contracts $(w_t(h_t))_{t=1}^{\infty} = (w_1(h_1), w_2(h_2), w_3(h_3), \dots)$, which we assume are not binding on workers. *We assume equal treatment:* a worker joining subsequently, at τ after history h_τ, is offered a continuation of this same contract: $(w_\tau(h_\tau),\ w_{\tau+1}(h_\tau, s_{\tau+1}), w_{\tau+2}(h_\tau, s_{\tau+1}, s_{\tau+2}), \dots)$. (This is to be contrasted with the case where discrimination is permitted: in that case a worker joining at τ is offered a contract which in principle may be unrelated to that offered to previous cohorts.) Let $V_t(h_t)$ denote the continuation utility from t onwards from the contract:

$$V_t(h_t) = E\left[\sum_{t'=t}^{\infty} (\beta_w \delta)^{t'-t} u(w_{t'}(h_{t'})) \quad | \ h_t \right], \qquad \#$$

where E denotes expectation. Each firm also has a planned employment path $(N_t(h_t))_{t=1}^{\infty}$.

The problem faced by the firm is:

$$\max_{(w_t(h_t))_{t=1}^{\infty}, N_t(h_t)} E\left[\sum_{t=1}^{\infty} (\beta_f)^{t-1} (f(N_t(h_t)) - N_t(h_t) w_t(h_t)) \right] \qquad \text{Problem A}$$

subject to

$$V_t(h_t) \geq \chi(h_t) \qquad \#$$

for all $h_t,\ t \geq 1$, and

$$N_t(h_{t-1}, s) \geq \delta N_{t-1}(h_{t-1}) \qquad \#$$

for all h_{t-1}, all $s \in S$ with $\pi_{s_{t-1}s} > 0,\ t \geq 2$. (ref: partic) is the *participation constraint* that says that at any point in the future the contract must offer at least what a worker can get by quitting, while (ref: no layoff) imposes that the firm may not layoff workers.

The outside option is determined by the following in a symmetric equilibrium:

$$\chi_t = N_t^*(h_t) V_t^*(h_t) + (1 - N_t^*(h_t)) U_t(h_t) \qquad \#$$

where $U_t(h_t)$ is the discounted utility of a worker who is unemployed at t, i.e., the utility from the reservation wage, $u(\underline{c})$, plus future utility from the possibility of remaining unemployed or of getting a contract in the future (see below). There are two cases: if the labour market at time t clears, $N_t^*(h_t) = 1$ (all new entrants and any previously unemployed survivors are employed), then it must offer the utility offered by other firms. In symmetric equilibrium, other firms are offering an identical contract, and so it is the utility associated with this, $V_t^*(h_t)$, which must be offered. If, on the other hand, there is excess supply of labour, $N_t^*(h_t) < 1$, the outside opportunity will depend on the probability of getting a job if a worker quits. Recall we assume that a worker who quits, quits right after the current shock is observed, and has the same probability of getting a job as anyone else seeking a job: $N_t^*(h_t)$.

Necessary conditions for an optimal contract can be characterized with the help of a simple variational argument. *This is the central idea in understanding why there is a lower bound on the fall of real wages.* The idea is that even if the labor market is slack at $t+1$, the firm will not want to cut the wage too far because of the desire to insure incumbents. Once this point is reached, the wage will not fall faster no matter how low the supply price of outside workers. Suppose we are at h_t, let N_t and N_{t+1}^s denote the optimal employment levels after h_t and (h_t, s) respectively, and consider, starting from the optimal contract, reshuffling wages between t, and $t+1$ in state s, to backload them. Increase the wage at $t+1$ after state s by a small amount Δ, and cut the wage at t by x so as to leave the worker indifferent; do not change the contract otherwise:

$$\pi_{s_t s}\delta\beta_w u'(w_{t+1}(h_t,s))\Delta - u'(w_t(h_t))x \simeq 0.$$

This backloading satisfies all participation constraints since worker utility rises at $t+1$, and so from this point on constraints are satisfied, but also after h_t and earlier since utility is held constant over the two periods. The change in profits (viewed from h_t) is

$$-\pi_{s_t s}\beta_f N^s_{t+1}\Delta + N_t x \simeq -\pi_{s_t s}\beta_f N^s_{t+1}\Delta + \frac{\pi_{s_t s}\delta\beta_w u'(w_{t+1}(h_t,s))N_t\Delta}{u'(w_t(h_t))},$$

which is positive for Δ small enough unless

$$\frac{u'(w_{t+1}(h_t,s))}{u'(w_t(h_t))} \leq \frac{\beta_f N^s_{t+1}}{N_t\delta\beta_w}. \qquad \#$$

Since the change in profits cannot be positive by optimality of the original contract, (ref: *) must hold: marginal utility growth cannot exceed a certain amount. Conversely, the reverse argument (frontloading), which would be profitable if the strict version of (ref: *) holds, cannot be undertaken (only) if next period's participation constraint binds since utility falls at $t+1$, so the constraint would be violated. We summarize the necessary condition:

Lemma *In an optimal contract with perfect mobility, (ref: *) must hold; it can only hold strictly (<) if the participation constraint binds at* (h_t,s).

A way then to think about the evolution of an optimal contract is that there is a "target marginal utility growth rate":

$$\frac{u'(w_{t+1}(h_t,s))}{u'(w_t(h_t))} = \frac{\beta_f N^s_{t+1}}{N_t\delta\beta_w} \qquad \#$$

which will be maintained, unless a binding participation constraint at $t+1$ forces it to be lower. Put differently, this puts a lower bound on how fast real wages can decline, but a tight labor market at $t+1$ can imply that wage growth is not against this bound. Note that this lemma applies whether or not the firm is hiring at t or $t+1$.

It is instructive to compare this with the BDN model (in this context) which has symmetric discounting, so assume that $\beta_f = \beta_w$. The corresponding target (gross) "growth rate" in their model is 1: wages stay constant unless a binding participation constraint forces them to be higher. The only difference arises here because the term $N^s_{t+1}/N_t\delta$ reflects the number of new hires that will be made next period for each incumbent at t. The reason is the following: if discrimination is allowed (as in BDN) then each worker is treated independently, so the risk-neutral firm would like to fully insure each worker by holding wages constant. In the equal treatment model, wages would likewise be constant if the term $N^s_{t+1}/N_t\delta = 1$, that is, if none of the workers who die are replaced. In this case the firm is only having to deal with the incumbents, so this corresponds to the discrimination case. Whenever $N^s_{t+1}/N_t\delta > 1$, however, the firm is taking on additional workers at $t+1$ who will receive the same wage as the incumbents; hence the future wage is taken into account with a larger weight by the firm than by the incumbent worker, and this imparts a downward bias to the future wage in comparison with the discrimination case.

To proceed, assume provisionally that firms always hire (at all h_t) in equilibrium. Then employment is determined by a standard marginal productivity equation:

Lemma *If in a symmetric equilibrium hiring takes place at every h_t, then $N_t^*(h_t)$ satisfies*

$$\partial F(N_t^*(h_t), s_t)/\partial N = w_t^*(h_t). \qquad \#$$

Proof Suppose that $\partial F(N_t^*(h_t), s_t)/\partial N > w_t^*(h_t)$. It is feasible to increase current hiring holding the wage contract constant, and consider this as the only change to the firm's plan: An increase in current hiring by $\Delta > 0$, for Δ small enough, and holding the wage constant at $w_t^*(h_t)$, would lead to an increase in current profits, while holding employment at $t+1$ constant at $N_{t+1}^*(h_{t+1})$ in all states (so hiring falls by $\delta\Delta$), is feasible for Δ small enough given hiring is positive at $t+1$. Thus there is an increase in profits at t, and no change at other dates, contradicting profit maximization. A symmetric argument, using the fact that current hiring is positive so current hiring can be *reduced* by Δ, and that $t+1$ employment can be increased by $\delta\Delta$, rules out $\partial F(N_t^*(h_t), s_t)/\partial N < w_t^*(h_t)$.

Suppose that at some t, the participation constraint binds. Then there must be full employment and the wage is determined by marginal productivity at full employment:

Lemma *Consider a symmetric equilibrium in which hiring always occurs; then the participation constraint binds at h_t if and only if $N_t^*(h_t) = 1$; moreover if the constraint binds then $w_t^*(h_t) = \partial F(1, s_t)/\partial N$.*

Proof (i) Suppose first that the participation constraint binds,

$$V_t^*(h_t) = \chi(s_t), \qquad \#$$

and suppose contrary to the lemma that $N_t^*(h_t) < 1$. Under the hiring hypothesis, we know from Lemma ref: mp that $\partial F(N_t^*(h_t), s_t)/\partial N = w_t^*(h_t) > \underline{c}$ by the assumption on $\underline{c}$ and diminishing marginal productivity (i.e., $w_t^*(h_t) \leq \underline{c}$ would imply $N_t^*(h_t) > 1$). Likewise, at any t' it is not possible that $w_{t'}^*(h_{t'}) \leq \underline{c}$ since there is no feasible employment level ($N \leq 1$) for which $\partial F(N, s_t)/\partial N \leq \underline{c}$, and so Lemma ref: mp would be contradicted. Consequently, a worker who gets a job at t receives strictly more current utility than the utility from being unemployed, and in the future receives no less no matter when (or if) she would get a job if unemployed today, given that she would receive $w_\tau^*(h_\tau)$ regardless of when she was hired; consequently an unemployed worker is strictly worse off than an employed. Hence quitting at t will lead to a utility strictly less than $V_t^*(h_t)$ as there is a positive probability of unemployment. This contradicts (ref: particbind). The equilibrium wage follows directly from Lemma ref: mp. (ii) Now suppose that $N_t^*(h_t) = 1$. Since all workers are employed, $\chi(s_t)$ is defined to be equal to $V_t^*(h_t)$, so the participation constraint binds.

We define $\underline{w}_s^* = \partial F(1, s)/\partial N$, which in view of the above lemma is the equilibrium wage when the participation constraint binds in state s. Then we can summarize: in a symmetric equilibrium with hiring, if at $t+1$ the participation constraint isn't binding, wages are updated according to (ref: desired); if it is binding, then $w_{t+1}^* = \underline{w}_{s_{t+1}}^*$.

To proceed further, we shall put more structure on the problem. This will allow us to assert that the wage updating rule is of the following simple form: given w_t^* compute w_{t+1} under the hypothesis that the participation constraint at $t+1$ is not binding; if $w_{t+1} > \underline{w}_{s_{t+1}}^*$ then the hypothesis is confirmed and w_{t+1} is the equilibrium wage; otherwise the constraint *is* binding and the equilibrium wage will be at $\underline{w}_{s_{t+1}}^*$. The structure will also allow us to demonstrate sufficient conditions for the symmetric hiring equilibrium to exist.

Henceforth assume each firm has technology given by, at time t,

$$F(N, s_t) = M_t + a_t N^{1-\alpha}/(1-\alpha), \qquad \#$$

where $\alpha > 0$, $\alpha \neq 1$, $M_t \geq 0$ and for $\alpha < 1$, $M_t = 0$. (M_t, a_t) will evolve according to a Markov process, with $E[a_{t+1} \mid a_t] < (Max\{\beta_f, \delta\beta_w\})^{-1}$. Note that for $\alpha > 1$, F has an upper bound given by M_t, which given that we are modelling short-run production functions at the establishment or plant level, may be appropriate. We also assume henceforth that workers have per-period utility functions of the constant relative risk aversion family with coefficient $\gamma > 0$, $\gamma \neq 1$, described by $u(c) = c^{1-\gamma}/(1-\gamma)$. Finally we assume that $\alpha\gamma > 1$.

The "target" rate of wage growth (i.e., if unconstrained at $t+1$) is, from (ref: desired),

$$\frac{w_{t+1}}{w_t} = \left(\frac{\lambda N_t}{N_{t+1}}\right)^{\frac{1}{\gamma}}, \qquad \#$$

where $\lambda \equiv \frac{\delta\beta_w}{\beta_f}$. Under the hiring assumption, we also have that the marginal product of labour equals $a_t N_t^{-\alpha}$, so that using (ref: w=mp),

$$N_t = a_t^{\frac{1}{\alpha}} w_t^{-\frac{1}{\alpha}}. \qquad \#$$

Combining (ref: desiredmod) and (ref: empl) yields an equation for the evolution of wages if unconstrained at $t+1$:

$$\frac{w_{t+1}}{w_t} = \lambda^{\frac{\alpha}{\alpha\gamma-1}}\left(\frac{a_{t+1}}{a_t}\right)^{\frac{-1}{\alpha\gamma-1}} \equiv \xi\left(\frac{a_{t+1}}{a_t}\right). \qquad \#$$

Moreover if firms are constrained at $t+1$, then as $N_{t+1} = 1$, $w_{t+1} = \underline{w}^*_{s_{t+1}} = a_{t+1}$ (from Lemma ref: lemconstr). We can now state

Proposition *In a symmetric equilibrium with positive hiring, wages will satisfy*

$$w^*_{t+1} = \max\left\{\xi\left(\frac{a_{t+1}}{a_t}\right)w^*_t, a_{t+1}\right\}. \qquad \#$$

Proof We have just shown that w^*_{t+1} must equal one of the arguments of the max operator, depending on whether or not the participation constraint binds at $t+1$. Suppose first that $\xi(\frac{a_{t+1}}{a_t})w^*_t > a_{t+1}$, which given $\alpha\gamma > 1$, can be rewritten as $w^*_t > (a_t^{-1}a_{t+1}^{\alpha\gamma}\lambda^{-\alpha})^{1/(\alpha\gamma-1)}$. Suppose that the participation constraint binds at $t+1$ (so $w^*_{t+1} = a_{t+1}$ and $N_{t+1} = 1$) contrary to assertion. Lemma ref: lem1 implies that $\frac{w^*_{t+1}}{w^*_t} \geq \left(\frac{\lambda N_t}{N_{t+1}}\right)^{\frac{1}{\gamma}}$ with equality unless the participation constraint binds at $t+1$. Thus $a_{t+1}/w^*_t \geq \left(\lambda a_t^{\frac{1}{\alpha}} w_t^{*-\frac{1}{\alpha}}/1\right)^{1/\gamma}$, or equivalently $w^*_t \leq (a_t^{-1}a_{t+1}^{\alpha\gamma}\lambda^{-\alpha})^{1/(\alpha\gamma-1)}$. So we have a contradiction. Alternatively, suppose that $\xi(\frac{a_{t+1}}{a_t})w^*_t < a_{t+1}$, and suppose that $w^*_{t+1} = \xi(\frac{a_{t+1}}{a_t})w^*_t$. But this implies that labour demand exceeds unity, which is incompatible with equilibrium. Finally if $\xi(\frac{a_{t+1}}{a_t})w^*_t = a_{t+1}$, then whether the participation constraint either binds or does not, w^*_{t+1} equals this common value.

It should be stressed that (ref: update) must hold in a symmetric equilibrium in which hiring always takes place; i.e., it is a necessary condition. Using the above solution, the condition for hiring to occur at $t+1$ is

$$N^*_{t+1} = a_{t+1}^{\frac{1}{\alpha}} w_{t+1}^{*-\frac{1}{\alpha}} > \delta N^*_t = \delta a_t^{\frac{1}{\alpha}} w_t^{*-\frac{1}{\alpha}}. \qquad \#$$

We have not yet verified that if a solution satisfying (ref: update) is found which satisfies (ref: hiring), then this is an equilibrium. This is established in Appendix A. To do this, we shall consider a relaxed version of the problem faced by a potential deviant firm and show that this cannot improve on the putative equilibrium; it follows that a deviant cannot do better in a more constrained version.

Parameter values for which hiring equilibrium exists

Finally, we ask when will the hiring condition (ref: hiring) be satisfied, and when does the model predict outcomes other than spot market ones? The hiring condition requires $a_{t+1}^{\frac{1}{\alpha}} w_{t+1}^{*-\frac{1}{\alpha}} > \delta a_t^{\frac{1}{\alpha}} w_t^{*-\frac{1}{\alpha}}$; if firms are constrained at $t+1$ then $N = 1$ and hiring is positive; if they are not, then (ref: update1) holds, and after simplification the condition becomes

$$\frac{a_{t+1}}{a_t} > \lambda^{\frac{1}{\gamma}} \delta^{\frac{\alpha\gamma-1}{\gamma}} = \delta^{\alpha} \left(\frac{\beta_w}{\beta_f} \right)^{\frac{1}{\gamma}}. \qquad \#$$

Consequently, provided (ref: suff) holds for all states reachable with positive probability from (any) a_t that occurs with positive probability, the wage path, given by (ref: update), with associated employment levels given by (ref: empl), *is* an equilibrium. For $\beta_w = \beta_f$, condition (ref: suff) requires that the maximum rate of *fall* of productivity should be smaller than the exogenous turnover rate raised to the power of α.

To see when outcomes differ from spot outcomes, starting from full employment in some state a_t, we need the wage to fall by less than the spot wage. Thus we need, using $w_t = a_t$, from (ref: update1)

$$w_{t+1} = \lambda^{\frac{\alpha}{\alpha\gamma-1}} \left(\frac{a_{t+1}}{a_t} \right)^{\frac{-1}{\alpha\gamma-1}} a_t > a_{t+1}$$

which can be rewritten as

$$\frac{a_{t+1}}{a_t} < \lambda^{\frac{1}{\gamma}}. \qquad \#$$

Since $\delta^{\frac{\alpha\gamma-1}{\gamma}} < 1$ (from $\alpha\gamma > 1$), (ref: suff) and (ref: nonspot) are compatible: there exist shocks $\frac{a_{t+1}}{a_t}$ sufficiently small ("bad") that the contract wage does not fall enough to maintain full employment, but not sufficiently small that hiring falls to zero.

Appendix A

We establish that if a solution satisfying (ref: update) is found which satisfies (ref: hiring), then this is an equilibrium. We shall consider a relaxed version of the problem faced by a potential deviant firm and show that this cannot improve on the putative equilibrium; it follows that a deviant cannot do better in a more constrained version which allows for layoffs .

Consider the problem as formulated earlier, but in which the firm has no employment constraints, so that it solves Problem A without the constraint (ref: no layoff) (that is, it can costlessly reduce its workforce at any time, and only has to respect the participation constraints, which do not take into account layoffs, this despite the fact that a worker in calculating his utility from the contract should take into account the layoff possibility). In this problem, the same necessary conditions as before hold ((ref: desiredmod) and (ref: empl)), as the wage equals marginal product condition always holds given that employment can be chosen freely.

Proposition *An optimal contract in the relaxed problem evolves according to the following updating rule. With each history h_t is associated a minimum wage $\underline{w}(h_t)$ such that w_{t+1} is updated from w_t by*

$$w_{t+1} = \max\left\{ \xi\left(\frac{a_{t+1}}{a_t} \right) w_t, \underline{w}(h_{t+1}) \right\}, \qquad \#$$

where $w_1 = \underline{w}(h_1 = s_1)$.

Proof We define $\underline{w}_{h_t}$ to be the period t wage specified by an optimal contract starting at h_t which delivers exactly $\chi(h_t)$. This does not depend on N_{t-1} as N_t is chosen freely each period. We start by showing that $\underline{w}(h_t)$ is unique. Suppose otherwise: then there are two distinct contracts that deliver $\chi(h_t)$ to a worker, both of which satisfy participation constraints and yield the same profits. Construct a new contract from these two contracts by paying a wage at each period which delivers the average utility: if after h_t the worker receives respectively u^o and u' at t, then fix the wage to deliver $(u^o + u')/2$. Consider the static problem of maximizing profits given that workers receive utility u, so that $w = ((1-\gamma)u)^{1/(1-\gamma)}$. Substituting from (ref: empl) for N, yields profits of

$$M_t + \frac{a_t^{\frac{1}{\alpha}} \alpha((1-\gamma)u)^{-\frac{1-\alpha}{\alpha(1-\gamma)}}}{1-\alpha}. \qquad \#$$

As $\alpha\gamma > 1$, this is a strictly concave function of u. Thus the new contract gives the firm at least the average profit in each period, assuming it hires optimally in each period, and strictly more whenever wages differ, which by assumption they do at date 1. By construction, the new contract offers a discounted utility after h_t which is the average of the two original contracts', both of which satisfy the participation constraint after h_t so also the new contract must satisfy it. Thus the new contract is feasible and yields a higher profit: a contradiction. So $\underline{w}(h_t)$ is unique. Next note that in an optimal contract the participation constraint binds at the initial date ($t = 1$): $V_1(s_1) = \chi(s_1)$. If it did not, the firm would increase profits by cutting $w_1(s_1)$ holding the remainder of the contract fixed, and would still satisfy all participation constraints. So $w_1(s_1) = \underline{w}(s_1)$.

Next, we establish (ref: updatefin). Suppose that $\xi(\frac{a_{t+1}}{a_t})w_t > \underline{w}(h_{t+1})$ (see (ref: update1)). If the participation constraint at $t+1$ binds, $w_{t+1} = \underline{w}(h_{t+1})$. If we solve for N_{t+1} at $\underline{w}(h_{t+1})$ from (ref: empl), then it is easily checked (using $\xi(\frac{a_{t+1}}{a_t})w_t > \underline{w}(h_{t+1})$) that MU growth would be more than $\frac{\beta_f N_{t+1}}{N_t \delta \beta_w}$, which contradicts backloading from Lemma ref: lem1. Thus an optimal contract cannot involve the constraint holding, so by (ref: update1), $w_{t+1} = \xi(\frac{a_{t+1}}{a_t})w_t$. Conversely, if $\xi(\frac{a_{t+1}}{a_t})w_t \leq \underline{w}(h_{t+1})$, then if the constraint does not hold, by (ref: update1) $w_{t+1} \leq \underline{w}(h_{t+1})$. However $V_{t+1} > \chi_s$ by definition of the constraint not holding, so there must be some point τ in the future on some path h_τ, such that $w_\tau > \tilde{w}_\tau$ for the first time, where $(\tilde{w}_{t'})_{t'=t+1}^{\infty}$ corresponds to the contract, starting in state s, that delivers χ_s at $t+1$, *and* such that total discounted continuation utility from τ onwards is greater in the V_{t+1} contract. This implies that wage growth between $\tau - 1$ and τ is greater in the contract that delivers V_{t+1}; as growth rates are equal if both are unconstrained (from (ref: update1)), this can only be true if at least one participation constraint binds at τ. However the constraint for the V_{t+1} contract cannot bind as this would imply that minimum utility is obtained from τ onwards and thus the continuation utility could not be greater than in the χ_s contract. Thus the participation constraint binds in the χ_s contract and this reduces wage growth. However we have just shown above that a binding contract cannot reduce wage growth ($\xi(\frac{a_\tau}{a_{\tau-1}})\tilde{w}_{\tau-1} > \underline{w}_{s_\tau}$ implies that $\tilde{w}_\tau = \xi(\frac{a_\tau}{a_{\tau-1}})\tilde{w}_{\tau-1}$), a contradiction. So the constraint binds and $w_{t+1} = \underline{w}(h_{t+1})$.

The above result applies in a general environment. We next make the environment that of the putative symmetric hiring equilibrium, and verify that if a solution satisfying (ref: update) is found which satisfies (ref: hiring), then this is an equilibrium with relaxed constraints. To do this, we consider an individual firm facing a sequence of χ_t 's as described earlier, derived now assuming other firms follow (w_t^*, N_t^*). After any history h_t where there is full employment, it must be true that $\underline{w}(h_t) = \underline{w}_{s_t}^*$, that is, it is optimal for an individual firm when constrained to pay the same as all other firms. To see this, note that for the individual firm, the optimal wage updating rule is still (ref: update1), unless there is a binding participation constraint. If $\underline{w}(h_t) < \underline{w}_{s_t}^*$, then on any path of future realizations of

$(a_{t+1}, a_{t+2}, \dots)$, a path $(w_{t'})_{t'=t}^{\infty}$ starting at $\underline{w}(h_t)$ will stay below $(w_{t'}^*)_{t'=t}^{\infty}$ starting at $\underline{w}_{s_t}^*$ until the first occurrence of a binding constraint on the former path at say $\tilde{t} > t$ (which could conceivably raise the former path above the latter). But a binding constraint implies that continuation utility from $w_{\tilde{t}}$ equals $\chi_{\tilde{t}}$, and since the equilibrium contract must offer at least $\chi_{\tilde{t}}$ we can conclude that conditional on $(a_{t+1}, a_{t+2}, \dots, a_{\tilde{t}})$ occurring, and providing the firm is always hiring, the equilibrium contract offers strictly more. Since all histories after t can be partitioned into sets of this nature, unless no constraint ever binds in which case the equilibrium wage stays higher for ever, we can conclude that the equilibrium contract offers strictly more. This contradicts the fact that both wage contracts by assumption deliver the same utility χ_t. A symmetric argument rules out $\underline{w}(h_t) > \underline{w}_{s_t}^*$. It thus follows that the two contracts coincide.

Thus the highest deviation payoff in the relaxed version equals the putative equilibrium payoff, and it follows that no deviation could be more profitable in the more constrained version.

Existence of Equilibrium

From the previous section, it is only necessary to show that there exists a solution to the relaxed problem. First note that if T is finite, and given the continuity of profits in w_t, this is immediate as profits are bounded: if $\alpha > 1$ this is direct from the production function; otherwise $\alpha\gamma > 1$ implies $\gamma > 1$, and $w_t^* \geq \underline{c}$ implies a lower bound on χ_t, and hence on w_t, which in turn implies an upper bound on profits at t. Next, suppose that wages follow the spot process, $w_t = a_t$. Substituting in for $u_t = a_t^{1-\gamma}/(1-\gamma)$ in (ref: profit fn), we get per-period profits of ??? a_t. Since the putative equilibrium has $w_t^* \geq a_t$, actual profits in the equilibrium are bounded above by this amount. Under the assumption that $\delta_f < E[a_{t+1} \mid a_t]$, as $T \to \infty$, this yields an upper bound on expected discounted profits which converges to a finite number, say $\overline{\prod}$. Now suppose that when $T = \infty$, the relaxed problem has a policy for which profits do not converge. Clearly this requires that $\alpha < 1$, so that $\gamma > 1$. Then

Empirical Evidence

In this section we examine the evidence in support of our theory using both unbalanced panel data from the Panel Study on Income Dynamics (PSID) and macroeconomic data from the Bureau for Labor Statistics (BLS). We start by assessing the success of our theory in explaining unemployment using purely macroeconomic data. Then we examine the extent to which BDN's theory can explain the macroeconomic movements in wages using the PSID. Finally we return to the PSID and use it to assess the relative success of BDN's theory and our own in explaining macroeconomic wage movements over the cycle.

Macroecenomic Evidence: US Postwar Unemployment

In this subsection we assess how well our model fits US post war aggregate unemployment data from the BLS. In the one sector model studied above, unemployment falls to zero whenever the productivity shock is not too bad. Using a multisector model in which there each sector is subject to idiosyncratic productivity shocks we will obtain more realistic unemployment levels because it is less likely that all labour markets will simultaneously clear; moreover when the aggregate productivity shock is positive, there will be more sectors with low unemployment and consequently aggregate employment is likely to be lower. Naturally this exercise depends on how well correlated the sectoral shocks are.

In order to get some realistic calibrations, we use actual U.S. manufacturing industry multifactor productivity processes, as provided by the Bureau for Labor Statistics. This simultaneously fixes the degree of shock correlation, and also allows us to generate simulated unemployment and wage series which can be directly compared to the data. We make the extreme assumption that each sector is otherwise independent, so that the sectoral labor markets are completely segmented. As we shall see, even though the model is lightly parametrized (two degrees of freedom for wages and three for unemployment), feeding it these sectoral shocks leads to unemployment and wage simulations that correspond reasonably well to the data.

As equation xxx makes clear, given knowledge of the model's parameters, given an initial time period where there was full employment and given a TFP series it is possible to generate the sectoral "real wage" series that would be predicted by our theory. We note that we are able to solve the model on this basis because of the convenient property that the solution depends only on actual realizations of the random processes, and not on their distributions. It is then possible to derive the corresponding implications for unemployment (rates).

We generate separate predicted wage and unemployment series for 18 manufacturing sectors, and then aggregate using sectoral weights. For the empirical work below we add one further "residual" sector (the remainder of the nonfarm private economy). To implement our simulations we need to calibrate the rate of change in real wages when firms are unconstrained (and productivity is unchanged), $\lambda^{\frac{\alpha}{\alpha\gamma-1}}$, and γ and α, the parameters governing (relative) risk aversion and the curvature of the production function respectively. For the coefficient of relative risk aversion, γ, we use the value 1.2 which is in the standard range for simulations and for α we use 1.4. This translates to a short-run elasticity of demand for labor of 0.71. analyzed BLS manufacturing data for a similar period that we study, found a short-run demand elasticity ranging between close to zero and 0.71 with aggregate data, and of between -0.5 and -0.89 at the 4-digit industry level for manufacturing. In fact the wage solution depends only on two composite parameters, $\alpha\gamma$ and $\lambda^{\frac{\alpha}{\alpha\gamma-1}}$. Thus varying α and γ but keeping their product constant does not affect the solution for wages provided we hold $\lambda^{\frac{\alpha}{\alpha\gamma-1}}$ constant; the unemployment series will vary with $-1/\alpha$ however, as this measures the elasticity of labor demand by which $w_t/a_t > 1$ (i.e., the extent to which wages are too high for market clearing) translates into unemployment. Thus a lower value for α will magnify fluctuations in sectoral unemployment. We set $\lambda^{\frac{\alpha}{\alpha\gamma-1}}$ to be 0.98 (equivalently, $\lambda \approx .99$), which will lead to a distribution (depending on productivity shocks) of real wage declines when the constraint is not binding centred around 2% per year. Individual simulated wage series were generated for each of the 18 two digit manufacturing sectors for which TFP data are available from the BLS. For each sector a simulated unemployment series was generated as the difference between simnulated wages and TFP for that sector. An aggregate "manufacturing" unemployment index was then constructed as the weighted average of the individual sector series with weights given by value added shares. The results for actual aggregate unemployment and simulated "manufacturing" unemployment are graphed in Figure xxx. The simulated "manufacturing unemployment" series appears to do quite well despite the fact that the comparison is with *aggregate* unemployment; this is probably accounted for by the fact that manufacturing unemployment is volatile relative to other components and hence not much is lost in concentrating on manufacturing. In particular the volatility of actual unemployment is reasonably well matched as are the peaks and troughs of the actual series, although in the simulations get the timing of some of the peaks wrong. Since we use annual data, and given that unemployment usually lags output movements, it is not surprising that sometimes the actual peaks and troughs lag the simulated ones. It is possible that a richer model that generated sluggish unemployment responses via say adjustment costs and labor hoarding would improve matters in this regard.

BDN and Macroeconomic Movements in Real Wages

Several empirical studies (see Introduction) have largely confirmed the robustness of BDN's main empirical findings that the minimum rate of unemployment since hiring (henceforth we refer to this variable as just ”minu”) is a statistically important determinant of the current wage of an individual. In particular extends BDN's analysis (using six cohorts from the National Longitudinal Surveys) to cover the time period 1966 to 1998. He finds that the significance and importance of minu is broadly robust with respect to the addition of fixed time effects, the addition of extra control variates such as zzz and using subsamples selected on the basis of age, and sex, although current unemployment levels also have some explanatory power. Whilst minu is clearly statistically significant, as Grant points out, its importance in accounting for the time series variation in wages may not be great as the variation in $\min u$ over time is not very large. In this section we pursue this further, and using our own sample collected from the PSID we find that this variable accounts for only a small proportion of the time series macro wage movements that impinge on individuals in our sample.

We collected data from the PSID for the years 1968 to 1993 - encompassing the BDN years of 1976-84. This time period was one of immense macroeconomic activity. It spanned the two oil crises, the switch from fixed to floating exchange rates, financial deregulation and several business cycles. We collected data on private sector employees' hourly wage and a basic set of characteristics: gender, age, education, occupation, tenure (in months), race and state of residence. For the macro variates we use the annual CPI and monthly aggregate unemployment rates as reported by the BLS. Whilst we did not collect data on all of the BDN characteristics we believe we have the most important and most frequently recorded ones. Unlike BDN we do not exclude women and individuals who were in the workforce prior to 1947 - this reflects our desire to be as comprehensive as possible in order to be able to generate macroeconomic results using the data later on.

The differences in data collection make it impossible for us to replicate BDN's results exactly but we check whether the broad features of our sample are in line with theirs. Table 1 gives sample means and standard errors of our and BDN's main variates for the BDN years. The table shows that we have nearly 30% more data points than do BDN and that average wages in our sample are around 11% lower than in BDN. Both of these differences are largely though not wholly down to the inclusion of women (excluding women, for example, gives an average log wage less than 2% below BDN's). Average tenure is a little higher in BDN but minu is rather lower than in our data. We should expect some differences here as we did not adopt BDN's adjustment method. Instead we simply use the PSID variate ”number of months with current employer” without adjustment . Table 2 gives the results for a BDN style regression of log wages on characteristics (see below), minu (m_{it}) and BDN's two other ”competitor” labour market condition variates, namely, the unemployment rate at the date of hiring (u_{0it}) and the current unemployment rate (un_t) for the BDN years and our full sample. Despite some differences in data construction and characteristics used we see that our results are quite close to that found by BDN and particularly so for the key variate minu itself. In further regressions (not reported here but available on request), we like find that the results are qualitatively robust with respect to the addition of a trend and year dummies - minu is always negative, highly significant and with a coefficient value between -.02 to -.06 with u_{0it} and un_t (obviously we drop the latter when year dummies are added) poorly determined and often incorrectly signed (Grant finds contemporaneous unemployment also to be significant).

We suggested above that, despite its significance and robustness, the minu variate may not be important in terms of explaining the variation in wages over time and we now examine this suggestion. If we add year dummies to a basic BDN regression to absorb macro effects (hence we regress log wages on characteristics, minu and year dummies) then we find, unsurprisingly, that in both the short (BDN) and long samples minu explains less than 0.5% of the within year variation in log wages and accounts for less than 1% of the explained variance of wages in the pooled regression. More important from the macroeconomic viewpoint is minu's contribution to the year by year/macro movements in log wages. The macro movements in log wages in our sample may be interpreted as the 26 coefficients on the year dummies. Forming a single time series of 26 data points from these coefficients and regressing it on the 26 yearly means of the minu variate plus a time trend we find that the yearly means of minu are wholly insignificant (see Table 3 below). More importantly a further calculatoin using output fro mthis regression tells us that trend deviations in mean minu explain only 3.8% of the variation in the trend deviations of log wages.

Finally we report one further robustness test for the BDN theory. When we apply a BDN regression to *new hires only* (over the full time period). the coefficient on minu (which in this case is just u_{0it}, the current unemployment rate) falls to below -.01 and is insignificant. In our data this is a robust finding.

Overall then it seems that minu is significant only in explaining differentials between workers within a year and does not explain year to year movements in real wages.On this evidence it is hard to argue that the minimum rate of unemployment during job tenure can help explain cyclical variations in real wages.

Macroeconomic Evidence from the PSID

We now extend the empirical analysis to compare the relative ability of macro movements in minu versus the model based simulated wage series in explaining movements in aggregate wages garnered from the PSID. One advantage of using the PSID for this purpose is that the aggregate annual wage we extract has been purged of the effects of changes from year to year in worker characteristics. By contrast the BLS aggregate wage series may move purely because of compositional changes of the working labour force over the business cycle. Given that our theory makes predictions for a representative individual the PSID panel is in many ways more appropriate benchmark "target" than is the BLS aggregate data. There is a problem with this however. Our theory explains macroeconomic movements in wages purely in terms of productivity shocks. A glance at Figure 1 shows that between 1968 and 1993 real wages (non farm private sector) and aggregate TFP (likewise private nonfarm) have opposite trends - real wages fall whilst TFP rises - a general feature of postwar US data. But as is well known, there is an increasing discrepancy between total (wage plus nonwage) compensation and wages, due largely to sharp rises in company medical and pension,etc. benefits. If we look at total worker compensation (Figure 1 again) we see this clearly. Interestingly total compensation has roughly the same trend as TFP. Any theory of wages based on TFP such as ours will thus be at odds with the real wage data in a major way. In what follows therefore we adjust annual real wage measures extracted from the PSID to allow for non wage compensation.This is consistent with our model in which wages are driven largely by the demand for labour which depends on total compensation.

To compute an aggregate simulated wage series for the period 1968 to 1993 we took a weighted average of that given for manufacturing above with that generated for the "residual" sector with weights which reflect the relative weights in private value added of the two aggregate sectors over the years in question, i.e. .6 for manufacturing and .4 for nonmanufacturing.

To use PSID data for our macro analysis there are two problems. First it reports wages not total compensation and some adjustment must be made when matching it with our simulated series. Second one has to take into account the fact that the characteristics of each yearly PSID cohort (e.g., the proportion of professionals) changes markedly from year to year and is rarely representative of the workforce as a whole. We deal with the second issue first. We may write the following empirical model for PSID observations, w_{it} :

$$w_{it} = \pi^{'} c_{it} + \alpha \widetilde{m}_{it} + \alpha \overline{m}_t + \theta^{'} x_t + \varepsilon_t + \epsilon_{it}, \qquad \#$$

where w_{it} are individual i's log of wages deflated by the annual CPI in year t ($i = 1, \ldots n_t$), c_{it} is a $k \times 1$ vector of individual i's characteristics (6 occupation dummies, sex, 3 race dummies, tenure, tenure squared, age, age squared, state of residence and 8 education dummies) at time t, x_t is a vector of variables that have direct common influence on PSID wages (trend, cyclical variates, etc., to be specified below) with $\pi = (\pi_1, \pi_2 \ldots)$ with $\theta = (\theta_1, \theta_2 \ldots)$ being a conformable vector of parameters, and $\widetilde{m}_{it} = m_{it} - \overline{m}_t$ with m_{it} being the BDN measure of individual $i's$ tightest labour market during his current job tenure at time t ("minu") and $\overline{m}_t$ being the sample mean of m_{it} in year t. The errors $\varepsilon_t = (\varepsilon_1, \varepsilon_2 \ldots \varepsilon_T)$ and $\epsilon_{it} = (\epsilon_{11} \ldots \epsilon_{n_1 1}, \epsilon_{12} \ldots \epsilon_{n_2 2} \ldots \epsilon_{1T} \ldots \epsilon_{n_T T})$ are for now and for simplicity assumed to be mean zero $i.i.d.$ Taking annual averages of (ref: emp1) gives the "macro" model for PSID wages as

$$\overline{w}_t = \pi^{'} \overline{c}_t + \alpha \overline{m}_t + \theta^{'} x_t + \varepsilon_t + O_p(n_t^{-1}), \qquad \#$$

where $\overline{c}_t = \sum_{i=1}^{n_t} \frac{c_{it}}{n_t}$ contains the annual means of c_{it}. We assume that characteristic means are not elements of x_t. In effect this means that the only role of characteristics in the macro model is to allow for year to year compositional changes in the panel. If for example the 1968 data had a preponderance of professionals but the 1969 data was dominated by unskilled workers, we would expect a drop in wages that reflects a combination of the change in composition and the wage differential between unskilled and professional workers. From the viewpoint of our macro theory we are really only interested in α and θ in (ref: emp2) because $\pi^{'} \overline{c}_t$ merely picks up aggregate wage movements associated with changes in the mix of characteristics in any particular year in the PSID. Explicitly we wish to analyse the relative importance of $\overline{m}_t$ and rival macro variables in x such as trend, the simulated wages from our model and TFP. One obvious and direct way to do this would be to treat (ref: emp2) as a simple regression (assuming that the $O_p(n_t^{-1})$ terms are negligible) but there are two problems with this. First we have over 60 characteristics plus at least two further (macro) regressors but only 26 annual data points. Second, the left hand side variable $\overline{w}_t$ excludes non wage benefits (pension, health insurance etc) which, from arguments given previously, are the relevant compensation measure in the theory. The answer to the first conundrum is to exploit the cross section of the panel to obtain estimates of π and to the second problem is to adjust wages for non wage benefits using aggregate data. We achieve these aims via two stage estimation. In the first stage we estimate (ref: emp1) but allowing $\alpha \overline{m}_t + \theta^{'} x_t + \varepsilon_t$ to be absorbed into year dummies. We therefore estimate the model

$$w_{it} = \pi^{'} c_{it} + \alpha \widetilde{m}_{it} + \sum_{t=1}^{T} \beta_t D_t + \epsilon_{it}, \qquad \#$$

where D_t takes the value 1 in year t but zero otherwise. All of the macro effects are now captured in the estimates $\widehat{\beta}_t$ so that we can write

$$\widehat{\beta}_t = \alpha \overline{m}_t + \theta' x_t + \varepsilon_t + O_p(n_t^{-1}). \qquad \#$$

To adjust for non wage benefits we add the BLS measure of the log of the ratio of aggregate total compensation to wages (r_t) to the left hand side of (ref: emp4).

We therefore estimate

$$\widehat{\tau}_t \equiv \widehat{\beta}_t + r_t = \alpha \overline{m}_t + \theta' x_t + \varepsilon_t + O_p(n_t^{-1}) \qquad \#$$

free of restrictions and in particular we do not impose the restriction that the coefficient on $\overline{m}_t$ be equal to α in (ref: emp3). This allows $\overline{m}_t$ the freedom to have a macroeconomic impact that is separate from and unconstrained by its cross sectional/within year effects on individual workers.

Table 3 gives the results for (ref: emp5) for a number of different x vectors. The regression of $\widehat{\tau}_t$ on $\overline{m}_t$ (first line) gives implausible results. Although $\overline{m}_t$ is significant and explains a substantial proportion (39%) of the year to year variation in compensation α has the wrong sign. The positive coefficient implies that an increase in average minu from one year to another will lead to higher worker compensation, not lower as the theory would suggest. As the second line in the table shows, adding a time trend to the regression renders $\overline{m}_t$ insignificant, suggesting possibly that its original importance was down to trend. By contrast and turning to lines 3 to 6 of the Table, we see that our simulated wage series, w_t^*, is highly significant, explains 58% of the variation in $\widehat{\tau}_t$ and has a coefficient close to unity. Furthermore its significance survives the addition of a trend term (line 4) and when both $\overline{m}_t$ and w_t^* are included with and without trend (lines 5 and 6) the latter remains highly significant but the former becomes insignificant. Finally adding aggregate TFP (y_t) to the equation (line 7) reduces the siginificance of w_t^* somewhat - to be expected given that w_t^* is correlated with y_t to some degree - but adds nothing to the explanatory power of the equation.

For completeness we report in lines 8 to 10 the regression of unadjusted PSID time effects $\widehat{\beta}_t$ on $w_t^*, trend$,(line 8), $\overline{m}_t, trend$ (line 9) and on $w_t^*, \overline{m}_t$ and *trend* (line 10). The reason for including a trend is, as noted above, TFP and therefore w_t^* have a positive trend whereas wages from the PSID have a negative trend. Overall the results are similar in that, once trend is controlled for, w_t^* is the only important and robustly significant explanatory variable and despite hving a different to trend it appears to pick up the dynamics of $\widehat{\beta}_t$ rather well.

Finally we comment on the stationarity of the errors. In the regressions of $\widehat{\tau}_t$ on w_t^* Engel Granger ADF(1) tests reject the null of no cointegration at the 5% level with a statistic value of 4.01. This is reassuring should it be the case that these series are I(1) rather than trend stationary.

How can we reconcile the robustness of the BDN minu variable in the original regressions with the fact that our w_t^* variable appears much better at explaining macro movements in the wage? Partly this is an unfair comparison; BDN's model is not formulated to explain macroeconomic phenomena, but to test alternative theories of contracting. In addition, the fact that BDN's model is driven by labor supply is important: they assume a stationary relationship between the unemployment rate and outside opportunities (i.e., outside of the labor market) for workers, so that for a given unemployment rate, the wage for new hires and for any incumbents who have binding outside opportunity constraints is determined by the unemployment rate. In our model, labor demand, which depends on TFP, is the primary driving force. It is conceivable that a reformulation of their model in which the outside opportunity rises in response to TFP changes would generate better macro predictions, although it is unclear how this might be justified. However our preferred interpretation is that cohort effects, as BDN model them, may *exist* but are restricted in size, and thus they are small relative to overall movements in wages which are better accounted for by our approach.

Table 1: Data Means and Standard Deviations

	Our Sample 1976-1984	BDN'S Sample
Log of Real Wages	1.01	1.12
	(.003)	(.004)
Minu	4.6%	4.2%
	(.009)	(.013)
Months in Post (Tenure)	81.7	83.0
	(.57)	(.66)
Percent White	37%	32%
	(.003)	(.003)

Table 2: BDN Pooled Regressions

	m_{it}	u_{0t}	u_t
BDN Sample(1976-1984)	-.059	.013	.000
	(.006)	(.004)	(.002)
	-.045	-	-
	(.003)	-	-
Our Sample(1976-1984)	-.024	.011	.026
	(.004)	(.002)	(.003)
	-.022	-	-
	(.003)	-	-
Our Sample(1968-1993)	-.052	.004	.019
	(.002)	(.001)	(.001)
	-.033	-	-
	(.001)	-	-

Table 3: Annual Time Series Regressions 1968-1993

	$\overline{m_t}$	*trend*	w_t^*	y_t	R^2
LHS Variable $\widehat{\tau_t}$					
	.03	-	-	-	.39
	(.008)	-	-	-	
	.021	.001	-	-	.40
	(.017)	(.002)	-	-	
	-	-	.654	-	.61
	-	-	(.106)	-	
	-	-.002	.851	-	.64
	-	(.002)	(.206)	-	
	-.015	-	.889	-	.64
	(.013)	-	(.226)	-	
	-.011	-.001	.932	-	.64
	(.016)	(.002)	(.243)	-	
	-	-	.528	.200	.63
	-	-	(.172)	(.214)	
LHS Variable $\widehat{\beta}_t$	.019	-.01	-	-	.60
	(.019)	(.003)	-	-	
	-	-.013	.864	-	.73
	-	(.002)	(.248)	-	
	-.013	-.012	.969	-	.74
	(.019)	(.002)	(.292)	-	

Thoughts

1. suppose not hiring; one would expect that constraint only binds when full employment, so there is hiring. otherwise we just follow the target wage growth but set employment to δN. ?problem is MP condition fails at full employment as hiring a workewr might imply that he cuts future profits; what about layoffs? maybe they can be included, and maybe teh firm has no incentive to offer insurnace.
2. Note that the strategy in the relaxed problem of paying w=∞ in one period and setting employment =0 won't work because under $\alpha\gamma > 1$ either utility is bounded above so the benefits in terms of particpation are small, or production fn goes to −∞ so profits would be −∞.

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The solution under commitment by workers

So far we have assumed that workers are not committed to contracts, and hence, as in BDN, it is the ex post mobility of workers which drives the wage dynamics. Here we drop the assumption that workers can costlessly quit the firm. What changes? Suppose there is a symmetric equilibrium in which firms hire every period. Then it must be identical to a symmetric hiring equilibrium with ex post mobility since the same participation constraint needs to be satisfied each period - if the continuation contract offers enough to prevent a worker from leaving, then it will also offer enough to entice a new worker. In terms of giving sufficient conditions for the equilibrium to exist, however, we have to be a little more careful. With commitment a firm has the option of choosing not to hire in some periods, and of violating the participation constraint (under no commitment the participation constarint must hold every period, independently of hiring, or else the entire workforce would leave, which is assumed to be harmful to the firm). So it is necessary to check that this is not a profitable deviation strategy. Intuitively, the participation constraint binds when productivity is high – and there is a high cost to not filling vacancies. So it might be expected that a strategy of not hiring in such a state may not be attractive. However there is a benefit in that the firm may avoid a large fluctuation in wages for its incumbents. Conjecture: for small enough variance in the process, this deviation strategy will not be profitable. The idea is that if there is no variability, then a constant wage contract is optimal and the exogenous loss of incumbents will always be replaced. Close to this situation, the loss incurred by not hiring will always be larger than any savings in terms of reduced wage variability.

In our model worker mobility/commitment *does not affect* the optimal contract under the additional assumption that the firm needs to hire in each period (provided a symmetric equilibrium exists). The intuition here is straightforward: if the firm desires to hire then it must offer outsiders at least their reservation utilities, and if it cannot discriminate in favour or against incumbents then it follows that the latter too must be offered continuation utilities which do not lie below the outside determined reservation utilities. This latter condition is precisely the usual one where there is no worker commitment. Thus we argue that the existing empirical research cannot be interpreted as evidence in favour of either worker mobility or of the existence of cohort effects.

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There is a close connection between the equilibria solved for here under the no commitment assumption, and equilibria under full or other degrees of worker commitment. Specifically, if there is a symmetric equilibrium under full commitment on the part of workers (with other assumptions maintained) in which firms always hire, then this will also be an equilibrium if there is less than full commitment. Intuitively, hiring requires that a participation constraint is satisfied which is at least as strong as the constraint which implies that workers will not leave, hence the latter can never affect the optimal contract.

Also, without the no discrimination assumption, this problem would be posed as in BDN, *at each point of hiring*, as choosing a wage contract subject to (ref: partic) which minimised costs of that worker (using now $\beta_f\delta$ to discount costs).

It does not follw that this is an equilibrium with commitment on the part of workers. It may pay firms to choose to not hire in some periods (to avoid increases in wages) and let $V_t^*(h_t)$ fall below $\chi(h_t)$. In the no commitment case a firm doing this will lose its incumbent workers too, something by assumption it wants to avoid. The two cases will coincide if however we additionally assumed that a firm must always hire some (even measure zero) workers to replace dying workers; this could be justified if there are 'key' workers who cannot be replaced by reallocating incumbents. The converse however does hold: if there is a symmetric hiring equilibrium with full commitment then it will also be an equilibrium with less or no commitment. This follows even though the participation constraint for hiring is different from that for preventing quits: in the full commitment equilibrium, only the hiring constraint needs to satisfied. In periods of unemployment this means that only $U(h_t)$ needs to be offered. However if $V_t^*(h_t) \geq U(h_t)$ then it also follows that $V_t^*(h_t) \geq \chi(h_t)$ (see (ref: outside)).