Quality-Assuring Prices and Vertical Product Differentiation in Markets for Experience Goods*

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Abstract

We analyze vertical product differentiation in a model where a good's quality is unobservable to customers before purchase, a continuum of quality levels is technologically feasible, and minimum quality is supplied by a competitive fringe of firms. After purchase the true quality of the good is revealed with positive probability. To provide firms with incentives to actually deliver promised quality, prices must exceed unit variable costs. We derive sufficient conditions for these incentive constraints to determine equilibrium prices, and show that under certain conditions only one or both of the extreme levels of quality, minimum and maximum quality, are available in the market.

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1. Introduction

In a seminal paper, Klein and Leffler (1981) show that in order to provide incentives for a competitive firm to supply a high quality product, its price, the "quality-assuring price," has to be above average production cost and include an informational rent. The threat of losing the rent from future purchases of customers enforces high quality provision and the associated quality-assuring price.[1] In this paper, we apply Klein and Leffler's idea of quality-assuring prices to a market with price-setting firms and free but costly entry, and analyze which quality levels—out of a continuum of technologically feasible levels—are offered in equilibrium. This study is therefore primarily related to the literature on the incentive approach to high quality provision.[2] However, rather than focusing on the quality-assuring price of a given exogenous quality, as Klein and Leffler (1981), we study which quality levels are actually offered in equilibrium when firms compete simultaneously in prices and quality. That is, we examine the consequences of Klein and Leffler's insights about quality-assuring prices for the diversity of quality levels in the market.

Our results can explain the casual observation that in some markets with asymmetric information there is little variety in available levels of quality, obtainable quality is generally low, and high quality is either impossible to find or very expensive. In fact, we show that under certain conditions at most two different levels of quality are offered in equilibrium—one of them being the lowest and the other one the highest among the technologically feasible levels. Also, if high quality is available at all, then only for a price that is high relative to production cost.

The lack of high quality products in a particular market may simply be the consequence of customer preferences. Customers may just not be willing to pay the higher production cost associated with better quality. In such a case complaints about the lack of high quality are irrelevant from the point of view of market performance. However, our case is different. Our results are not based on any unwillingness of the customers to pay the higher production

[1] This repeat-purchase contract enforcing mechanism is an example of a self-enforcing contract mechanism that plays an important role in contract theory. See, for example, Crawford (1985), Levin (2003), MacLeod (2006), and the references cited therein.

[2] This literature was pioneered by Klein and Leffler 1981 and followed up by Shapiro 1983. See Bagwell and Riordan 1991, Bester 1998, Maksimovic and Titman 1991, Neeman and Orosel 2004, Riordan 1986, Rob and Sekiguchi 2004, Rogerson 1988, Wolinsky 1983, among others.

cost of better quality.

In our study, we consider a market for a good where (i) customers cannot observe its quality before deciding whether or not to buy, (ii) customers detect with positive probability the quality of the respective product after purchase, and (iii) contracts cannot be conditioned on quality. This type of good differs somewhat from a regular *experience good*, as introduced by Nelson (1970), in that even after purchase customers may find out the true quality not with certainty but only with positive probability.[3]

Examples of such goods can be found, e.g., among information services and products (Sarvary and Parker 1997), that is, services and products of legal, financial, medical or technical consultants or advisors, and the provision of financial and accounting audits. An illustration in case are retail financial services. Retail financial services firms collect and produce information and sell this information to investors, directly in the form of newsletters or reports, or indirectly through the set-up of a fund and the sale of shares on this fund that exploits this information.[4] Prospective customers of retail financial services firms "find it hard to assess the quality of individual products and suppliers" (Spencer 2000, p. 24).

With physical goods, unobservable quality is frequently associated with the good's genetic, organic, and chemical properties but may also relate to the respective production processes (e.g., as regards child labor, environmental implications or animal welfare). Most of the time consumers are not able to detect these attributes by their own experience. However, checks and tests by public agencies, consumer organizations or watchdogs can find out. Therefore, in our analysis we want to include goods where a customer's own experience will never reveal the true quality, but checks by certain agencies will, e.g., tests by health authorities will detect hazardous substances contained in a given product. Thus, checks and tests of such organizations are part of the contract enforcement mechanism. Since in any given period only few goods are tested, for a producer such tests are random events. Accordingly, we include in our model the case where customers cannot observe the true quality of the respective product even after consumption, but where after purchase the true quality of the good will become publicly known with positive probability.

We are not interested in the situation where insufficient competition, e.g., a monopoly,

[3]The good differs from a *credence good*, introduced by Darby and Karni (1973), in that customers have no ex ante uncertainty about the desired quality. For an account of "the economics of credence goods" see Dulleck and Kerschbamer (2006).

[4]See Admati and Pfleiderer 1986, 1990, Mahajan and Sweeney 2001; see also Fishman and Hagerty 1995.

causes the market to provide only meagre quality diversity. Rather, we want to investigate the effects of asymmetric information on quality diversity in equilibrium without limiting competition a priori. Therefore, we employ a model where market entry (and thus the intensity of competition) is endogenously determined by the interplay of a positive entry cost and market demand.

Our model is one of moral hazard and incentives rather than one of adverse selection and signaling. Since in the recent literature the term "reputation" is used predominantly, though not exclusively, in connection with adverse selection, we avoid that term in this paper.[5] In our analysis the problem is not firms' reputation with respect to (given) types but customers' trust regarding firms' *behavior*. Specifically, we assume that customers trust firms whenever conditional on this trust a firm has no incentive to "cheat," i.e., to provide lower quality than announced.[6] This assumption concerns off-equilibrium beliefs, and it excludes equilibria where firms never produce higher than minimal quality because customers stubbornly believe that quality is always at its lowest possible level. Whenever customers understand firms' incentives sufficiently well this assumption is justified.

The results of our model seem to be, by and large, consistent with empirical work on product differentiation. For example, Bresnahan (1981, p. 217) provides evidence for the US automobile market that "products are much more tightly bunched at the low-quality end of the product spectrum," which seems to suggest that close to the minimum a relatively large number of basically identical qualities is available in the market whereas only a few high qualities are offered. In addition, Bresnahan (1981, 1987) demonstrates that price-cost margins increase with higher quality. This observation, which is also supported by Kwoka (1992) and Feenstra and Levinsohn (1995), concurs with quality-assuring prices.

The rest of the paper is organized as follows. First, we discuss some related work in Section 2. In Section 3, we present the model. Then, in Section 4, we derive the quality-

[5] Examples of papers on reputation which include adverse selection as an essential element are, among others, Holmström (1999), Tadelis (1999), Mailath and Samuelson (1998), (2001), Hörner (2002), Cripps et al. (2004). For our analysis of the moral hazard problem we need not be concerned with the questions of how reputation can be acquired, how it is lost, and how it can be used strategically.

[6] Of course, firms' exact incentives to "cheat" depend on the details of how customers' trust in a firm is influenced when the firm provides lower quality than promised, specifically on how fast and to what extent a cheating firm loses its customers. However, our results depend on the *existence*, not on the details of the resulting incentive compatibility constraint.

assuring prices from the incentive compatibility constraints for firms to provide high quality. In Section 5, we show that due to these incentive compatibility constraints only low quality may be available in the market even though preferences for high quality may be "strong" relative to production costs. In Section 6, we demonstrate that under certain conditions equilibrium prices are determined by incentive compatibility constraints, whereas customer preferences and the distribution of customer types only determine the quantities demanded, given equilibrium prices. In Section 7, we analyze the case where customers' willingness to pay for quality is convex with respect to quality, and in Section 8 the case where it is concave. Finally, we conclude in Section 9. All mathematical proofs are relegated to the Appendix.

2. Related Work

As pointed out above, our analysis is based on Klein and Leffler (1981). In addition, it is also related to the literature on the analysis of Bertrand equilibria in markets with vertical product differentiation (see, e.g., Gabszewicz and Thisse 1979, 1980, and Shaked and Sutton 1982, 1983).[7] However, it differs from this literature (and the related analysis of endogenous sunk cost and the associated "finiteness result;" see, e.g., Sutton 1986, 1991; Shaked and Sutton 1983, 1987; Anderson et al. 1992, 305–313, Berry and Waldfogel 2006) in three important respects.[8] First, quality cannot be observed by customers before purchase and therefore, due to incentive reasons, high qualities must have high ("quality-assuring") prices relative to production costs. That is, costs must also include an incentive component.[9] Second,

[7] For a Cournot analysis of vertical product differentiation see, e.g., Gal-Or (1983). Gabszewicz and Grilo (1992) analyze Bertrand equilibria of a vertically differentiated duopoly where consumers have exogenous beliefs about which of the two firms sells high quality and which sells low quality. Another strain of literature, based on Mussa and Rosen (1978), deals with quality provision by a *monopolist* (see, e.g., Gabszewicz and Wauthy 2002 and the references therein). For an analysis of vertical product differentiation with high and low qualities under monopolistic or competitive conditions see Carlton and Dana (2004). However, due to differences in focus the conclusions of this literature are not readily comparable to our results.

[8] A fourth respect in which our model differs is our assumption that minimum quality is offered at marginal cost by a competitive fringe. However, although such an assumption would affect the equilibrium prices, the main results of the papers quoted above would remain essentially unchanged, since under the assumptions of these models the competitive fringe would have no customers.

[9] This may have significant effects on the equilibrium outcome. For example, for the duopoly case Bester (1998) has shown that if in a standard Hotelling model of spatial competition the horizontally differentiated good is an experience rather than an inspection good, the equilibrium outcome may be "minimum

the quoted literature on Bertrand equilibria in markets with vertical product differentiation assumes basically that the cost increase associated with an increase in quality is negligible in the sense that the cost increase is always below customers' willingness to pay for that increase in quality.[10] In contrast, we do not use the corresponding assumption in our analysis, which would be that customers' willingness to pay for higher quality is always larger than the increase in production-*cum-incentive* cost. Third, while the respective literature concentrates on the case where firms decide first on quality and then, given the quality choices, on prices, we consider the case where firms decide *simultaneously* about quality and price (i.e., knowing neither price *nor quality* chosen by competitors). This approach is natural in our context where quality is unobservable and firms have an incentive to "cheat" customers by producing and selling unobservably low quality for a high price.

Within the literature on vertical product differentiation there is a branch that investigates how customers' informational differences with respect to different brands may constitute a barrier to entry (see, e.g., Schmalensee 1982, Bagwell 1990). The informational differences are in fact differences with respect to supplier reputation. Whereas the incumbent has reputation (the quality of his product is "known"), a new entrant has no reputation (the quality of her product is not "known"). In this literature, firms are of different "types," e.g., high and low quality producers. Thus, customers face an adverse selection problem, and reputation concerns firms' (unalterable) types. In contrast, in our paper firms are not distinguished by types, and consequently, there is no adverse selection. Rather, we analyze the moral hazard problem associated with firms that can choose the quality they provide.

Our paper shares some aspects with repeated moral hazard models which assume that, in each repetition, players cannot observe the opponents' actions perfectly but only an imperfect stochastic signal thereof (see, e.g., Fudenberg et al. 1994, Green and Porter 1984, Rob and Sekiguchi 2004, among others). In contrast to these models, the unobservable actions (i.e., the *actual* quality choices) of opponents do not, in itself, influence a firm's stage-game profits in our model. However, quality monitoring at the end of each stage provides incentives, in our case, against a firm's repeated moral hazard problem of not providing the announced quality.

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differentiation" rather than "maximum differentiation."

[10] Shaked and Sutton (1983, p. 1472) characterize the respective assumption as follows: "Where that condition is satisfied, all consumers will be agreed in ranking the products in the same strict order, at unit variable cost."

3. The Model

We consider a market for a good that is homogeneous except for quality. That is, in our model there is (potentially) vertical product differentiation but no horizontal product differentiation. Time is measured in discrete periods $t \in \{1, 2, ...\}$. There is a pool of N (N possibly infinite) firms that are capable to produce each quality $v \in [0, 1]$ of the good considered at constant marginal cost $c(v) > 0$, where the function $c : [0, 1] \to \mathbb{R}_{++}$ is strictly increasing. That is, marginal cost is independent of quantity but increasing with respect to quality. Moreover, since $c(v)$ is strictly increasing, quality can be measured without loss of generality in such a way that cost is linear in quality, i.e.,

$$c(v) = c(0) + \gamma v, \tag{3.1}$$

where γ is some positive constant.[11] Put differently, we use marginal cost to measure quality. In addition to the N firms that are capable to produce each of the technologically feasible quality levels, there are infinitely many firms that are capable of producing minimum quality $v = 0$ at cost $c(0)$. Although a firm may be capable of producing multiple quality levels, we assume, in agreement with most of the literature,[12] that in each period it can produce and offer only one particular level of quality. The number N of potential producers of all quality levels $v \in [0, 1]$ should be interpreted to be large, implying that, in principle, the good could be supplied in many different quality levels.

In equilibrium not all firms will be active in the market and produce. Firms that are active in the market are distinguished between "brand names" and "no names." No names produce only minimum quality $v = 0$, whereas brand names may produce any quality level $v \in [0, 1]$. Any firm can be active as a no name, and each of the N of potential producers of all quality levels can be active as a brand name. Even if a no name is capable of producing positive quality, it will not do so because customers believe that all no names provide only minimum quality (Assumption 3 below). A brand name has to announce publicly the quality of its

[11] Let $V \in [V_{\min}, V_{\max}]$ be any measure of quality with associated marginal cost $C(V) > 0$ that increases in V but is constant in quantity. Defining $v \equiv [C(V) - C(V_{\min})] / [C(V_{\max}) - C(V_{\min})] \in [0, 1]$ gives the required normalized measurement of quality. For any quality level $\hat{V}$ with normalized equivalent $\hat{v}$ we get the marginal cost $c(\hat{v}) = C\left(\hat{V}\right) = C(V_{\min}) + [C(V_{\max}) - C(V_{\min})]\,\hat{v} = c(0) + \gamma\hat{v}$, where $\gamma \equiv C(V_{\max}) - C(V_{\min})$.

[12] See, for example, Shaked and Sutton 1982, 1983; Gabszewicz and Thisse 1979, 1980; Klein and Leffler 1981.

product in each period and we call this the "announced quality."[13] Actual quality is private information of the firm and may or may not coincide with the announced quality. Between periods brand names can change their announced and actual quality levels, respectively, at no cost. We assume that no names have no entry cost and thus form a competitive fringe that sells minimum quality at marginal cost and has zero payoff. In contrast, each brand name has to incur a positive entry cost $\eta > 0$, identical for all firms, in order to establish the respective brand together with an associated distribution channel. A brand name that voluntarily leaves the market may enter again, either simultaneously or later, but it has to pay the entry cost for each entry. Brand names choose their respective prices, together with their respective quality levels, simultaneously at the beginning of each period. That is, we assume Bertrand competition.

In each period there is an atomless continuum of customers of (Lebesgue) measure 1. Customers' total expenditures are non-negative and uniformly bounded in every period. Customers live one period and are distinguished by "types" $s \in S = [s_{\min}, s_{\max}] \subset \mathbb{R}$ according to their willingness to pay for quality. Customers' preferences are specified by the following assumption.

Assumption 1 (Customer Preferences). *Each customer buys at most one unit of the good. The payoff from not buying the good is normalized to zero. For customers of type* $s \in S$ *the payoff from buying one unit of the good of quality* v *for the price* p *is given by* $U(v, p, s) = R(v, s) - p$. *For all* $v \in (0, 1]$ *and* $s \in S$, *the function* $R(\cdot, \cdot)$ *is strictly increasing in quality* v.

The function $R(\cdot, \cdot)$ gives the willingness to pay of type s for quality v. Notice that the shape of this function, in particular, whether it is convex or concave, depends on the way

[13] Whenever public quality reports are manipulable they may play the role of the quality announcement. Examples in case are "report card" programs in medical-care markets in the US (see Cutler et al. 2004) and performance indicators introduced by the National Health Service in the U.K. to report publicly on the quality of health care provided by hospitals and physicians. A variety of measures, such as limiting access to high-risk patients, allow health care providers to manipulate quality reports and thus misrepresent quality (for references see Cutler et al. 2004, e.g., Dranove et al. 2003, 556-557). Similarly, in financial markets, public disclosure regulation is aiming at reporting publicly on the quality of retail financial products (see, for example, Financial Services Authority 2003). Again, quality may be misrepresented by the respective retail financial firms. Of course, rational customers will understand that these reports are manipulable.

quality is measured. The normalization of (3.1) implies that we use marginal cost to measure quality, and the function $R(\cdot,\cdot)$ is defined using this measurement.

If quality were immediately observable to customers and firms had no entry cost, competition would drive the price for quality v, denoted by $p(v)$, down to marginal cost $c(v)$, provided N is sufficiently "large." In the extreme case of no entry cost and a continuum of firms, each customer type $s \in S$ could find a firm that offers the level of quality that this type prefers most at prices $p(v) = c(v)$.[14] In the particular situation where at prices $p(v) = c(v)$ all types prefer maximum quality $v = 1$, all firms would offer the highest feasible quality for the price $c(1)$. However, we consider a good where neither customers nor other firms can observe the quality of a particular product, and contracts cannot be conditioned on quality. In such a situation, a firm could save on cost by producing lower quality than announced, and in the following we refer to such behavior as "cheating."

There are many different aspects of quality. Some of these a customer can observe immediately (like whether a fruit is rotten), some she can detect after a relatively short period of use (like the mileage of a car), some she can discern only after a long time (like the durability of a good), and some she will never know with certainty (like some features of product reliability, such as the probability of a breakdown). Sometimes, a customer can learn the quality of a product, for example, the quality of insurance or health care, only after random events, for example, the occurrence of an accident or illness (see Israel 2005). Moreover, some aspects, like the content of certain hazardous elements in food or toys, can be detected by an appropriate monitoring agency but not by the ordinary customer herself. We model the customers' uncertainty about the quality of the respective good in a way that is consistent with the latter cases above (but not restricted to them). Specifically, Assumption 2 below covers the case where in each period the true quality is revealed to the public with some probability $\varphi \in (0, 1]$, and where with the complementary probability $1-\varphi$ there is no information about a product's quality.

In some cases, the information that a certain product does not have the announced quality may spread slowly and the firm which sells it in the market may be able to exploit the trust among customers it has acquired in the past for a relatively long period after it has started

[14] The most preferred quality is either a corner solution ($v = 0$ or $v = 1$) or, assuming differentiability, satisfies the condition $\partial R(v, s)/\partial v = \gamma$, i.e., at an interior optimum the marginal willingness to pay for quality must equal the "marginal cost of quality."

to cheat. In other cases, this information may become public almost instantaneously and force the firm to close down. For example, a watchdog agency may randomly test brand names' products, and if the test shows that the true quality is below the announced quality the respective firm may have to close down, e.g., because it loses its customers or is forced by a legal authority to exit the market. The relevant point is not how fast or to what extent the respective firm's business is reduced, but that a cheating firm risks that it will be punished either by the public or by a legal authority. The expected punishment gives rise to an incentive compatibility constraint that, if satisfied, induces the firm to provide the announced quality. Regardless of the details of the model, this constraint necessarily implies that the price of a good above minimum quality must be sufficiently above its marginal cost. Otherwise the firm would cheat and produce only minimum quality. Since customers cannot immediately observe quality and contracts cannot be conditioned on quality, a firm that produces high quality must earn an informational rent. The threat of losing this rent if it cheats provides the incentive for the firm to actually produce the announced quality (Klein and Leffler 1981).

The previous discussion motivates the following assumption, which—among other cases—captures the situation where watchdog agencies perform random tests of the brand names' products with some probability $\varphi \in (0,1]$, and firms caught cheating have to exit the market and receive a payoff of zero from that moment onwards.[15, 16]

Assumption 2 (Quality Monitoring). *If in any period $t \in \{1, 2, ...\}$ the true quality of a product is below the quality announced by its producer, the respective brand name has to exit the market with positive probability $\varphi \in (0,1]$ at the end of this period and receives a payoff*

[15] The assumption that the payoff drops to zero is a simplification. For incentives a significant reduction of the payoff (to a still positive value) is sufficient. Empirically, such reductions can be observed. For example, Coyle (2002, pp. 29-30) points out (with respect to food quality): "A company involved in the spread of a foodborne pathogen can .... face costs imposed by courts or government agencies, including fines, product recalls, and temporary or permanent plant closures as well as large liability settlements and associated legal costs. Potential market and liability losses are strong incentives for food firms to ensure the food supply is as safe as possible."

[16] This implies that a firm that has to close down has no physical capital or other assets to sell. If the firm has some assets or a "scrap value," this would change the specific form of the incentive constraint below, but not the general argument. It would make a difference, though, if cheating firms could be forced to pay arbitrary large fines. However, for fines below a certain threshold the general argument remains valid.

of zero from the next period onwards.[17] *With the complementary probability $1-\varphi$ providing lower than announced quality in any period has no effect for the respective brand name.*

The assumption that the punishment for cheating is market exit need not be taken literally. The model covers all cases where a brand name that is detected cheating loses φ percent of its future payoff in expectation. For example, rather than having to leave the market with probability φ, a brand name that cheats may lose φ percent of its customers for sure. Or it may lose φ/π percent of its customers with probability $\pi \in (0,1)$.

This is not a paper on how firms achieve customers' trust. Rather, we want to characterize the equilibrium outcome when it is as easy as possible for a firm to convince rational customers to believe its quality announcement. Specifically, we assume that customers trust firms whenever conditional on this trust the respective firm has no incentive to cheat.

Assumption 3 (Customer Beliefs). *Customers cannot observe the true quality of a brand name. They believe that the true quality is the announced quality unless given these beliefs it is optimal for a firm to provide lower quality. Otherwise, they believe that the true quality is the minimum quality $v = 0$. No names are believed always to provide minimum quality.*

We assume that all firms have the same discount rate $\rho > 0$. The associated discount factor is denoted by $\delta \equiv \frac{1}{1+\rho} \in (0,1)$. A firm's payoff is the sum of its discounted profits (net of entry costs) if it has entered the market, and zero otherwise. If more than one brand name offers the same quality $v \in [0,1]$ and there is a unique firm that is the cheapest one, it gets all the demand for this quality. If the cheapest brand name among those offering the same quality is not unique, all brand names that sell for the lowest price share the respective demand equally.

Since no names have no entry cost, they always offer quality $v = 0$ for the price $p(0) = c(0)$ under perfectly competitive conditions. Therefore, we restrict the terms "entry" and "incumbent" to brand names. Moreover, because no names do not behave strategically, we do not treat them explicitly as players in the game. In contrast, potential and actual brand names act strategically. In their decisions they take the "competitive fringe" of no names into account, as well as the strategies of the other players.

Our analysis is based on the following game with imperfect information. The set of players consists of brand names, customers, and "nature." Brand names know the distribution of

[17] The respective firm may be allowed to enter as a no name and receive, as all no names, a payoff of zero.

customer preferences, but cannot observe the individual types. The cost function (3.1) and the rest of the model are common knowledge. In each period $t \in \{1, 2, ...\}$, the game proceeds in four phases. In the first phase, at the beginning of the period, brand names decide simultaneously about exit and entry. That is, brand names that are in the market decide whether to exit, and and brand names that are not, or not any more, in the market decide whether to enter. In the second phase, still at the beginning of the period, all brand names in the market observe the moves made in the first phase and choose simultaneously *announced* quality, *actual* quality, and price (each for the respective period). Customers and firms observe the announced qualities and prices, but *actual* qualities are private information, unobservable to customers and rival firms. In the third phase, which takes place at the end of the period, customers decide whether and from which supplier to buy one unit of the good. These decisions are executed and the period's payoffs accrue. Finally, in phase four, "nature" moves and each brand name that had provided some quality below the announced quality has to leave the market forever with probability $\varphi > 0$. A brand name that has to leave the market receives no further payoff (but keeps the payoffs received so far).

Since we are not interested in collusion among firms, we want to rule out folk theorem type results. In standard models this can be done by considering only those equilibria of the dynamic game that consist of playing a particular equilibrium of the stage game in every period. In our model the situation is somewhat different because the only reason why firms do not cheat are their future rents, and therefore the game that incumbent firms play in each period is not the relevant "stage game." In any one-period "stage game" (as well as in any finite version of the dynamic game) incumbents would always cheat, and since customers would anticipate this and thus not buy the respective product, no brand name would enter the market and no positive quality would be available in equilibrium. In our model, the analogue of an equilibrium (in pure strategies) of the dynamic game that consists of playing an equilibrium of the stage game in every period, is what we call an *equilibrium in stationary strategies* or, for short, a *stationary equilibrium.* We define a brand name's strategy to be *stationary,* if it satisfies the following two conditions: (i) the firm either enters the market in the first period $t = 1$ or not at all; (ii) if the firm had entered, its announced quality, actual quality, and price are constant in time and independent of the history of actions. Accordingly, an equilibrium (in pure strategies) is *stationary*, if all equilibrium strategies are stationary. It is the requirement that equilibrium strategies are independent of the history of actions that prevents collusion. In a stationary equilibrium, equilibrium strategies are not only constant along the equilibrium path but also at off-equilibrium nodes. However, this holds only for *equilibrium* strategies. *Deviating* strategies are not constrained to be stationary.[18] With the important exception of Section 5, our equilibrium concept throughout the paper is the one of a stationary equilibrium in pure strategies. Notice that since firms cannot observe each other's actual qualities, there are no proper subgames except the subgames starting at the second phase of the first period, after market entry. It is easy to check that any equilibrium of the overall game induces an equilibrium in each such proper subgame, and, therefore, any equilibrium is subgame perfect. For the results of Section 5 collusion plays no role and therefore we consider *all* Nash equilibria in pure strategies.

4. Incentive Compatibility Constraints

In this section, we derive the incentive (compatibility) constraint for a firm to provide a given quality $\bar{v} \in (0,1]$ in a stationary equilibrium. Consider a stationary equilibrium, where in some period t a firm offers a product of some announced and actual quality $\bar{v} \in (0,1]$ for some price $\bar{p}$, sells $\bar{x}$ units in this period, and considers to do the same in every future period. An alternative strategy is to cheat and provide only minimum quality for $K \geq 1$ periods (where K may be infinite), risking involuntary exit; and to produce again quality $\bar{v}$ thereafter (if K is finite), conditional on still being in the market. For the simplest case, $K = 1$, the incentive constraint for the firm to actually provide quality $\bar{v}$ is

$$\frac{\delta}{1-\delta}\left[\bar{p} - c\left(\bar{v}\right)\right]\bar{x} \quad \geq \quad \delta\left\{\left[\bar{p} - c\left(0\right)\right]\bar{x} + \left(1-\varphi\right)\frac{\delta}{1-\delta}\left[\bar{p} - c\left(\bar{v}\right)\right]\bar{x}\right\},$$

where the left hand side is the payoff from always producing quality $\bar{v}$ and the right hand side is the expected payoff from producing quality $v = 0$ in the next period and, if undetected, quality $\bar{v}$ thereafter. Equivalently, the benefit from cheating, which is $\left[c\left(\bar{v}\right) - c\left(0\right)\right]\bar{x} = \gamma\bar{v}\bar{x}$, must not exceed the cost of cheating, which is $\varphi\frac{\delta}{1-\delta}\left[\bar{p} - c\left(\bar{v}\right)\right]\bar{x} = \frac{\varphi}{\rho}\left[\bar{p} - c\left(\bar{v}\right)\right]\bar{x}$. Both ways of reasoning give the incentive constraint $\frac{\varphi}{\rho}\left[\bar{p} - c\left(\bar{v}\right)\right]\bar{x} \geq \gamma\bar{v}\bar{x}$. Re-arranging, we get

$$\bar{p} \geq c\left(\bar{v}\right) + \frac{\rho}{\varphi}\gamma\bar{v}. \tag{4.1}$$

[18] Below, we show in Lemma 1 of Section 6 that for stationary equilibria it is nevertheless sufficient to consider only stationary strategies.

It is intuitive, and easy to check, that if it is not profitable to cheat for one period, it is also not profitable to cheat for $K \in \{2, 3, ...\}$ periods and vice versa.[19] Thus, inequality (4.1) is the incentive compatibility constraint for quality level $\bar{v}$. In a stationary equilibrium, this inequality has to hold for all quality levels $\bar{v} \in (0, 1]$ that are actually produced. It cannot be that some quality $\bar{v}$ is sold at a lower price because at a lower price the respective firm would cheat and this would be anticipated by its customers.

Let $\hat{p}(v)$ denote the minimum price for quality $v \in [0, 1]$ in a stationary equilibrium, i.e.,

$$\hat{p}(v) \equiv c(v) + \frac{\rho}{\varphi}\gamma v, \quad v \in [0, 1]. \tag{4.2}$$

These prices include the incentive cost $\frac{\rho}{\varphi}\gamma v$ in addition to the production cost $c(v)$.[20] In any stationary equilibrium the price $p(v)$ must satisfy

$$p(v) \geq \hat{p}(v) \tag{4.3}$$

for each quality $v \in [0, 1]$ that is sold in the market.[21]

[19] If the firm provides only minimum quality for $K \geq 1$ periods, the sum of expected discounted profits is

$$\begin{aligned} & \delta \sum_{\tau=t}^{t+K-1} \delta^{\tau-t} (1-\varphi)^{\tau-t} [\bar{p} - c(0)] \bar{x} + (1-\varphi)^K \delta \sum_{\tau=t+K}^{\infty} \delta^{\tau-t} [\bar{p} - c(\bar{v})] \bar{x} \\ = \; & \delta \frac{1 - \delta^K (1-\varphi)^K}{1 - \delta(1-\varphi)} [\bar{p} - c(0)] \bar{x} + \delta^K (1-\varphi)^K \frac{\delta}{1-\delta} [\bar{p} - c(\bar{v})] \bar{x}, \end{aligned}$$

whereas it is $\frac{\delta}{1-\delta}[\bar{p} - c(\bar{v})]\bar{x}$, if the firm always provides quality $\bar{v}$. Consequently, the resulting incentive constraint is $\frac{\delta}{1-\delta}[\bar{p} - c(\bar{v})]\bar{x} \geq \delta\frac{1-\delta^K(1-\varphi)^K}{1-\delta(1-\varphi)}[\bar{p} - c(0)]\bar{x} + \delta^K(1-\varphi)^K \frac{\delta}{1-\delta}[\bar{p} - c(\bar{v})]\bar{x}$. Rearranging gives $\frac{1-\delta^K(1-\varphi)^K}{1-\delta}[\bar{p} - c(\bar{v})] \geq \frac{1-\delta^K(1-\varphi)^K}{1-\delta(1-\varphi)}[\bar{p} - c(0)]$, or $\bar{p} \geq c(\bar{v}) + \frac{1-\delta}{\delta\varphi}[c(\bar{v}) - c(0)] = c(\bar{v}) + \frac{1-\delta}{\delta\varphi}\gamma\bar{v}$. Since $\frac{1-\delta}{\delta} = \rho$, we get (4.1).

[20] If the producer does not sell the product directly to customers but via dealers, the respective dealers may have an opportunity to cheat by selling a fake product instead of the real one. In this case, similar to the producer, a dealer must earn a rent in order not to cheat. Consequently, incentive costs accrue at the dealer stage as well. In the price faced by the consumer a dealer's incentive cost will show up as a higher markup on the production cost. If the distribution channel for the good consists of several stages, the cumulative incentive costs may be very high. In order to avoid too high a markup on the producer's price, the producer may establish her own network of stores or company outlets, even if this would be inefficient if no incentive costs accrued in the distribution channel. In our model, we do not take such additional costs in selling the experience good into account. However, the presence of such costs is a reason to regard ρ/φ as large.

[21] There is a related problem in the regulation of banks: deposit-rate ceilings can be used as incentives for banks to invest in safe rather than in inefficiently risky assets ("gambling assets") because deposit-rate ceilings increase banks' profits per period and thus their franchise (or charter) values (Hellmann et al. 2000, Repullo 2004).

5. Minimum Quality Only

Although the incentive constraints (4.3) have been derived for *stationary* equilibria, this section shows that they have some relevance for *all* equilibria. We say that some given quality $v \in [0,1]$ is *available* in the market in some period t, if in period t some firm produces this quality v and offers its product at a price which at least some customers are willing to pay, i.e., at a price that results in sales. Recall that by normalization every customer's payoff is zero when she does not buy the good at all, and that minimum quality $v = 0$ is always offered for the price $p(0) = c(0)$. Assume that some given quality $\bar{v} > 0$ is offered for some price $p(\bar{v}) \geq \hat{p}(\bar{v})$. If $R(\bar{v}, s) - \hat{p}(\bar{v}) < \max[R(0,s) - c(0), 0]$ for all $s \in S$, customers will not buy quality $\bar{v} > 0$ for the price $p(\bar{v})$. Each customer prefers either to buy minimum quality for the price $c(0)$ or not to buy quality $\bar{v}$ at all. Consequently, quality $\bar{v} > 0$ will not be available in a stationary equilibrium. If this holds for all positive levels of quality, i.e., if $R(v,s) - \hat{p}(v) < \max[R(0,s) - c(0), 0]$ for all $s \in S$ and all $v \in (0,1]$, then in any stationary equilibrium either only minimum quality is available or the good is not available at all. Proposition 1 generalizes this observation to all equilibria.

Proposition 1. *If for no positive level of quality $v \in (0,1]$ customers are willing to pay the price $\hat{p}(v)$, i.e., if $R(v,s) - \hat{p}(v) < \max[R(0,s) - c(0), 0]$ for all $s \in S$ and all $v \in (0,1]$, then every Nash equilibrium in pure strategies has the property that positive quality is never available in the market. Depending on preferences, the respective good is either available only in minimum quality or, if $R(0,s) < c(0)$ for all $s \in S$, is not available at all.*

Under the assumption of Proposition 1, minimum quality $v = 0$ is available in the market in each period if $R(0,s) > c(0)$ for some $s \in S$, whereas if $R(0,s) < c(0)$ for all $s \in S$ the respective good is never available (due to lack of demand). In the borderline case $R(0, s_{\max}) = c(0)$, in any period t minimum quality of the good may or may not be sold.

The intuition of Proposition 1 is as follows. By the proposition's assumption customers are not willing to pay the price $\hat{p}(v)$ that is necessary to provide the incentive to actually produce quality $v > 0$ when profits per period are constant in time. Since positive quality can be sold only for some price below $\hat{p}(v)$, an equilibrium requires that along the equilibrium path profits of a firm that sells $v > 0$ in some period t increase in the future. Since profits can be positive in the future only if the respective firm provides again positive quality, the argument feeds on itself and profits have to increase ever more. In fact, it can be shown that with $t \to \infty$ profits must diverge. Since customers' expenses are uniformly bounded, this is not feasible and the proposition follows.

Even when customers are not willing to pay the prices $\hat{p}(v)$ for $v \in (0,1]$, they may be willing to pay the production cost $c(v) < \hat{p}(v)$. In particular, it may be the case that every single customer prefers the highest feasible quality to all other alternatives when prices equal marginal cost, i.e., $\max[R(v,s) - c(v), 0] < R(1,s) - c(1)$ may hold for all $v < 1$ and all $s \in S$. If this is the case, only the highest feasible quality would be consumed in the first-best, whereas due to asymmetric information either only minimum quality is available in equilibrium or the good is not available at all.

Proposition 1 holds analogously for the more general case where the payoff function $U(v,p,s)$ is any function $U : [0,1] \times \mathbb{R}_+ \times S \to \mathbb{R}$ that is strictly decreasing in the price p.[22] The proof of Proposition 1 shows that if $U[v, \hat{p}(v), s] < \max\{U[0, c(0), s], 0\}$ for all $v \in (0,1]$ and all $s \in S$, then in every Nash equilibrium in pure strategies positive quality $v > 0$ is never available in the market. Moreover, Proposition 1 does not depend on the details of the game, such as simultaneous entry of firms, and extends to equilibria in mixed strategies.

6. Equilibrium Prices and Incentive Constraints

For the reasons explained in Section 3, we examine stationary equilibria (in pure strategies) in the rest of the paper. Along any stationary equilibrium path, all incumbents' actions are constant. However, deviating strategies are not constrained to be stationary. If we restrict for each firm the strategy set to stationary strategies, we get a new game, which we call the *restricted game*. Although even for a *stationary* equilibrium of the original game *all* strategies have to be considered as possible deviations, the following lemma shows that it is actually sufficient to look only at stationary strategies.

Lemma 1. *A strategy profile that constitutes a Nash equilibrium in pure strategies of the restricted game is also a Nash equilibrium in pure strategies of the original game.*[23]

[22] If the parameter s is interpreted as "income" thought of as a Hicksian "composite commodity," the payoff becomes $U(v,p,s) = u(v, s-p)$, which frequently is further simplified to $U(v,p,s) = w(v)(s-p)$, as, e.g., in Gabszewicz and Thisse (1979, 1980) and Shaked and Sutton (1982, 1983).

[23] Obviously, the converse also holds: if a strategy profile of *stationary* strategies constitutes a Nash equilibrium of the original game, it also constitutes a Nash equilibrium of the restricted game.

In a stationary equilibrium of our game nothing can be learned from the past and the strategic situation is exactly the same in every period. From this it follows immediately that equilibrium strategies are sequentially rational and that a stationary equilibrium is a perfect Bayesian equilibrium.

Obviously, in a stationary equilibrium incumbents have positive profits, do not cheat and do not exit. For each quality $v \in [0, 1]$ that is available in the market there can be only one price $p(v)$ and it must hold that $p(v) \geq \hat{p}(v)$, where $\hat{p}(v)$ is given by (4.2). Otherwise the respective firm(s) would cheat. Below we will derive a condition that implies the *equality* $p(v) = \hat{p}(v)$ for all intermediate quality levels $v \in (0, 1)$. Consequently, under this (sufficient but non necessary) condition equilibrium prices of intermediate quality levels are completely determined by the incentive constraints (4.3), whereas customer preferences and the distribution of customer types only determine the quantities demanded, given (predetermined) equilibrium prices. In addition, the price for minimum quality $v = 0$ is given by its cost $c(0)$. For maximum quality $v = 1$ the incentive constraint determines the price $p(1) = \hat{p}(1)$ in two different circumstances. One case occurs when the incentive cost $\frac{\rho}{\varphi}\gamma$ is sufficiently large to make the price $\hat{p}(1) = c(1) + \frac{\rho}{\varphi}\gamma$ optimal (in the set $\{p \mid p \geq \hat{p}(1)\}$ of incentive compatible prices) even for a firm that is the sole brand name in the market. The other case, explained in the following paragraph, consists of the situation where two or more brand names offer quality $v = 1$.

If at least two brand names offer the same quality $\bar{v} > 0$, Bertrand competition will drive the price $p(\bar{v})$ to "the lowest possible value." In the case of observable quality this lowest possible value is the marginal cost $c(\bar{v})$. In contrast, when quality is unobservable, the lowest possible value is $\hat{p}(\bar{v})$ in any stationary equilibrium, since a brand name that had announced quality $\bar{v} > 0$ and charges a price $p(\bar{v}) < \hat{p}(\bar{v})$ will cheat. Because of Assumption 3 (Customer Beliefs), customers will buy its product for a price below $\hat{p}(\bar{v})$ only if the price is $c(0)$. Thus, if the firm charges a price $p(\bar{v}) < \hat{p}(\bar{v})$, its profit is zero, whereas it is positive if $p(\bar{v}) = \hat{p}(\bar{v})$. Consequently, whenever two or more brand names offer the same quality $\bar{v}$ it must hold that $p(\bar{v}) = \hat{p}(\bar{v})$ in any stationary equilibrium.[24, 25] Although it

[24] As Klein and Leffler (1981, p. 625) put it "... the quality-assuring price is, in effect, a minimum price constraint 'enforced' by rational consumers."

[25] Since brand names have no fixed costs of production, every brand name that has incurred the entry cost can stay in the market and guarantee itself a non-negative profit per period. Consequently, it is impossible that a brand name drives another brand name out of the market by undercutting $\hat{p}(\bar{v})$. In the case where

is implausible that two or more brand names offer the same *intermediate* quality $v \in (0,1)$, our analysis will show that for *maximum* quality $v = 1$ this may well be the case.

For the rest of the paper we assume that $R(\cdot,\cdot)$ is twice continuously differentiable and that $R_{vs}(\cdot,\cdot) > 0$ for $v > 0$, where subscripts of $R(\cdot,\cdot)$ always denote the respective partial derivatives. The assumption $R_{vs}(\cdot,\cdot) > 0$ is a single crossing condition and means that customer types can be defined in such a way that "higher" types have a higher willingness to pay for additional quality, i.e., have a higher *marginal* willingness to pay for quality.

Assumption 4 (Increasing Differences). *Higher customer types have a higher willingness to pay for additional quality, i.e., $R_{vs}(\cdot,\cdot) > 0$ for $v > 0$.*

Because of Assumption 4, the difference in willingness to pay between any two quality levels v' and $v'' > v'$, $R(v'',s) - R(v',s) = \int_{v'}^{v''} R_v(v,s)\,dv$, is strictly increasing in s for any such pair $v', v'' \in [0,1]$.[26] Specifically, the additional amount of money that a customer of type s is willing to pay when the good is of highest rather than of lowest possible quality, $r(s) \equiv R(1,s) - R(0,s)$, is strictly increasing. For convenience we identify customer types by this difference $r(s)$, i.e., without loss of generality we assume $s \equiv R(1,s) - R(0,s)$.

We provide a sufficient condition for the incentive constraint to determine the price charged by a firm that produces some quality $v \in (0,1)$ below the maximum quality. Specifically, we show that whenever it holds for all types $s \in S$ that $R_v(\bar{v},s) > \gamma$ for some $\bar{v} \in (0,1)$, then $p(\bar{v}) = \hat{p}(\bar{v})$. Consequently, if $R_v(v,s) > \gamma$ for all $v \in (0,1)$ and all $s \in S$, then $p(v) = \hat{p}(v)$ for all $v \in [0,1)$ since $p(0) = \hat{p}(0) = c(0)$ holds trivially. Substituting for $\hat{p}(v)$ gives $p(v) = c(v) + \frac{\rho}{\varphi}\gamma v = c(0) + \left(1 + \frac{\rho}{\varphi}\right)\gamma v = c(0) + \alpha\gamma v$, where α is defined as $\alpha \equiv 1 + \frac{\rho}{\varphi} = \frac{\varphi+\rho}{\varphi} > 1$. Because of $R_{vs}(\cdot,\cdot) > 0$, the condition $R_v(v,s) > \gamma$ for all $v \in (0,1)$ and all $s \in S$ is equivalent to $R_v(v,s_{\min}) > \gamma$ for all $v \in (0,1)$. Moreover, it is also equivalent to the condition that $R(v,s) - c(v) = R(v,s) - c(0) - \gamma v$ increases in v for all $s \in S$. As a

(contrary to our model) brand names have fixed costs of production, a different argument gives the same result. With fixed costs, undercutting will trigger a war of attrition, and in wars of attrition the most plausible equilibria are those in mixed strategies. Since this implies expected payoffs of zero, whereas cheating gives strictly positive payoffs, the respective brand names will cheat. Because customers will recognize this, undercutting is not profitable.

[26] Except for points of inflection the converse also holds. Whenever for all $v'', v' \in [0,1]$, $v'' > v'$, the difference $R(v'',s) - R(v',s) = \int_{v'}^{v''} R_v(v,s)\,dv$, is strictly increasing in s for all $s \in S$, then $R_{vs}(\cdot,\cdot) > 0$ except for isolated points $\tilde{v} \in [0,1]$. This follows because if $R_{vs}(v,s) \leq 0$ for all $v \in [v',v'']$ for some pair $v', v'' \in [0,1]$, $v'' > v'$, then $\frac{\partial[R(v'',s) - R(v',s)]}{\partial s} = \int_{v'}^{v''} R_{vs}(v,s)\,dv \leq 0$.

consequence, all agents prefer the highest feasible quality $v = 1$ to any other quality (though not necessarily to abstention), if for all $v \in [0,1]$ prices $p(v)$ equal marginal costs $c(v)$.[27]

Proposition 2. *If for some intermediate level of quality $\bar{v} \in (0,1)$ all customers' marginal willingness to pay for quality exceeds the "marginal cost of quality," i.e., $R_v(\bar{v},\cdot) > \gamma$, then in any stationary equilibrium the price $p(\bar{v})$ for this level of quality is given by $p(\bar{v}) = \hat{p}(\bar{v})$; that is, the price for quality $\bar{v}$ is determined by the incentive constraint. If for all intermediate levels of quality all customers' marginal willingness to pay for quality exceeds the "marginal cost of quality," i.e., $R_v(\cdot,\cdot) > \gamma$ for all $v \in (0,1)$, then in any stationary equilibrium the price $p(v)$ is given by $p(v) = \hat{p}(v) = c(0) + \alpha\gamma v$ for each level of quality $v \in [0,1)$ below the highest possible level. Thus, for each positive level of quality $v \in [0,1)$ below the highest possible level the price is determined by the incentive constraint.*[28]

Thus, if customers have a sufficiently strong preference, relative to production cost, for higher quality, then prices for all quality levels $v \in [0,1)$ below 1 are determined by the sum of production and incentive cost. Customer preferences, as long as they satisfy $R_v(\cdot,\cdot) > \gamma$ for $v > 0$, and the distribution of customer types only determine the quantities demanded, given the equilibrium prices (which are determined by the incentive constraints only). In addition, whenever two or more firms offer the highest possible quality $v = 1$, it follows from Bertrand competition that the price for maximum quality $v = 1$ is also determined by the

[27] If $R(\cdot,\cdot)$ is strictly concave in v (i.e., if $R_{vv}(\cdot,\cdot) < 0$), such a preference for the highest feasible quality is even equivalent to $R_v(\cdot,\cdot) > \gamma$ because then $R_v(v,\cdot) \leq \gamma$ for some $\bar{v} \in (0,1)$ implies $R_v(v,\cdot) < \gamma$ for all $v \in (\bar{v},1)$ and thus $R(\bar{v},\cdot) - c(\bar{v}) > R(v,\cdot) - c(v)$ for all $v \in (\bar{v},1)$. Notice that when $R(\cdot,\cdot)$ is strictly concave in v, $R(v,s) - c(v) = R(v,s) - c(0) - \gamma v$ achieves an interior maximum where $R_v(v,s) = \gamma$, provided such an interior maximum exists. Consequently, if $R(\cdot,\cdot)$ is strictly concave in v and all customers prefer the highest feasible quality $v = 1$ to any other quality at prices $p(v) = c(v)$, then it must hold that $R_v(v,\cdot) > \gamma$ for all $v \in (0,1)$.

[28] For any given $\bar{v} \in (0,1)$ it is sufficient that $R_v(\bar{v},s) > \gamma$ holds for the (unique) type $s(\bar{v})$ that solves $R(\bar{v},s) - R(0,s) = \alpha\gamma\bar{v}$ (thus, $s(\bar{v})$ is the "indifferent type" defined by $R(\bar{v},s) - \hat{p}(\bar{v}) = R(0,s) - c(0)$), whenever such a type exists. Because of $R_{vs}(\bar{v},s) > 0$, this implies $R_v(\bar{v},s) > \gamma$ for all higher types $s > s(\bar{v})$, but for lower types $s < s(\bar{v})$ it may hold that $R_v(\bar{v},s) \leq \gamma$. If a type $s(\bar{v})$ does not exist, either $R(\bar{v},s) - R(0,s) < \alpha\gamma\bar{v}$ for all $s \in S$ and no type will demand quality $\bar{v}$ at the price $\hat{p}(\bar{v})$, or $R(\bar{v},s) - R(0,s) > \alpha\gamma\bar{v}$ for all $s \in S$ and only then quality $\bar{v}$ can be available in the market and the assumption $R_v(\bar{v},\cdot) > \gamma$ is needed. If for each $\bar{v} \in (0,1)$ the type $s(\bar{v})$ exists and satisfies $R_v(\bar{v},s) > \gamma$, then $p(\bar{v}) = \hat{p}(\bar{v})$ for all $\bar{v} \in (0,1)$. Moreover, the proposition can be extended to the case where the strict inequality in $R_v(\bar{v},s) > \gamma$ is replaced by the weak inequality.

incentive constraint, i.e., $p(1) = \hat{p}(1)$.

It is useful to compare Proposition 2 with the corresponding results of standard models of vertical product differentiation, where quality is observable. In these models, it is crucial whether there is *simultaneous price-quality competition* (firms observe neither their competitors' quality choices nor their prices before making their own decisions) or *quality-then-price competition* (firms decide first about quality and then, after having observed their competitors' quality choices, about the price). Simultaneous price-quality competition has two consequences. First, prices are given by marginal production costs (see, e.g., Anderson et al. 1992, Section 8.3.2). Second, if marginal willingness to pay (for quality) exceeds marginal production cost (of quality), only maximum quality is available in equilibrium (Anderson et al. 1992, Section 8.3.2). These two results do not follow under asymmetric information about quality, but if marginal willingness to pay is larger than marginal production cost, the first result holds analogously in the sense that equilibrium prices are at their competitive levels (Proposition 2). In contrast, as long as marginal willingness to pay is not larger than marginal production *plus incentive* costs, the second result need not hold. More than one quality level may be available in equilibrium and maximum quality may not be available.

In the alternative case of quality-then-price competition (see, e.g., Gabszewicz and Thisse 1979, 1980, and Shaked and Sutton 1982, 1983), equilibrium prices will typically be above their competitive levels when quality is observable, and in our case they will be above marginal production plus incentive costs, provided these costs are not too high. However, in our context simultaneous price-quality competition, not quality-then-price competition is the appropriate assumption since the latter is not compatible with *unobservable* quality.

7. Equilibrium With Convex Willingness to Pay

The shape of customers' willingness to pay functions $R(v, s)$, specifically whether they are convex or concave with respect to quality v, has a significant effect on the equilibrium. We analyze the two cases where these functions are either convex or concave with respect to quality v for all $s \in S$.[29] First, we investigate the case where customers' willingness to pay

[29] Some customers will never demand positive quality at prices $p(v) \geq \hat{p}(v)$ and, thus, the shape of their willingness to pay functions—whether they are convex, concave or neither—is irrelevant. Specifically, if for a customer type $\tilde{s}$ it holds that $R(v, \tilde{s}) - \hat{p}(v) < \max[R(0, \tilde{s}) - c(0), 0]$ for all $v \in (0, 1]$, it follows from the incentive constraint (4.3) that this type will never demand positive quality. Thus we could limit any

$R(v,s)$ is convex in v, i.e., $R_{vv}(\cdot,\cdot) \geq 0$ for all $s \in S$.[30] That includes, in particular, the subcase where $R(\cdot,\cdot)$ is linear in v. We show that in this convex case at most the two extreme quality levels, $v = 0$ and $v = 1$, are available in the market. If two or more firms can cover their entry cost, all will offer quality $v = 1$ for the price $\hat{p}(1)$.[31] A plausible assumption is that higher types are willing to pay more for maximum quality, that is, $R(1,\cdot)$ is increasing in s. In this case, there exists a critical type $\bar{s} \in S = [s_{\min}, s_{\max}]$ such that all types $s > \bar{s}$ consume quality $v = 1$, whereas all types $s < \bar{s}$ either abstain from consuming the good or consume quality $v = 0$. Whenever $\bar{s} \in (s_{\min}, s_{\max})$, customer type $\bar{s}$ is indifferent between consuming quality $v = 1$ and the preferred one of the two alternatives abstention and consumption of quality $v = 0$.

Proposition 3. *Assume that customers' willingness to pay $R(\cdot,s)$ is convex in v, i.e., $R_{vv}(\cdot,s) \geq 0$, for all $s \in S$. Then in every stationary equilibrium at most the quality levels $v = 0$ and $v = 1$ are available in the market. If the market can accommodate two or more firms that offer positive quality, the only equilibrium in stationary strategies is that all those firms offer $v = 1$ for the price $p(1) = \hat{p}(1) = c(0) + \alpha\gamma$. If customers' willingness to pay for maximum quality $R(1,\cdot)$ is strictly increasing in type s, there exists a customer type $\bar{s} \in S$ such that all types $s < \bar{s}$ either abstain from consuming the good or consume quality $v = 0$, and all types $s > \bar{s}$ consume quality $v = 1$. Provided two or more firms offer positive quality, the (indifferent) customer type $\bar{s}$ decreases strictly with α for $\bar{s} \in (s_{\min}, s_{\max})$.*

Interestingly, more demand may reduce the price $p(1)$. With weak demand only one brand name is able to cover the entry cost, and in general a (monopoly) price $p^M(1) > \hat{p}(1)$ will be optimal for the single brand name. In contrast, with stronger demand two or more

assumption on the shape of $R(\cdot,\cdot)$ to "relevant" types.

[30] Recall that the shape of the function $R(\cdot,\cdot)$ depends on the way quality is measured. However, if there is some measurement of quality such that, given that measurement, cost is concave and willingness to pay is convex in that measurement, then the normalization of (3.1) implies that $R(\cdot,\cdot)$ is convex in v. To see this, let $V \in [V_{\min}, V_{\max}]$ be such a measurement of quality. As footnote 11 shows, $v \equiv \mu + \beta C(V)$, where $\beta > 0$ and μ are constants, gives (3.1). Consequently, $V = C^{-1}\left(\frac{v-\mu}{\beta}\right)$. Defining $B(v) \equiv C^{-1}\left(\frac{v-\mu}{\beta}\right)$, the willingness to pay of type s for quality V, denoted by $\hat{R}(V,s)$, gives $R(v,s) = \hat{R}(V,s) = \hat{R}[B(v), s]$. Therefore, $\frac{\partial^2 R(v,s)}{\partial v^2} = \frac{\partial^2 \hat{R}(V,s)}{\partial V^2}[B'(v)]^2 + \frac{\partial \hat{R}(V,s)}{\partial V} B''(v)$, where $\frac{\partial \hat{R}(V,s)}{\partial V} > 0$. Thus, $C''(v) \leq 0$ (which implies $B''(v) \geq 0$) and $\frac{\partial^2 \hat{R}(V,s)}{\partial V^2} \geq 0$ gives $\frac{\partial^2 R(v,s)}{\partial v^2} \geq 0$.

[31] Since we have normalized the population to have measure 1, the entry cost has to be measured relative to the size of the population.

brand names are able to cover the entry cost and thus the equilibrium price is $p^B(1) = \hat{p}(1) < p'(1)$ because of Bertrand competition. This gives the following corollary.

Corollary 1. *If customers' willingness to pay $R(\cdot, s)$ is convex in v for all $s \in S$ (i.e., $R_{vv}(\cdot,\cdot) \geq 0$), then an increase in some or all customers' willingness to pay for quality $v = 1$ may reduce the equilibrium price of quality $v = 1$.*

Linearity of marginal cost $c(v)$ and convexity of willingness to pay $R(v, s)$ in quality v imply that consumer welfare and total profits can be maximized with just the two extreme quality levels $v = 0$ and $v = 1$. Thus, the lack of intermediate quality levels is efficient. It does not follow, however, that the equilibrium outcome is constrained efficient in the sense that a planner who does not have more information than the customers and cannot influence the parameters of the model, could not improve the outcome.[32] Given the inefficiency of intermediate quality levels, the planner would allow at most one brand name to enter, because of the positive entry cost. When setting prices, the planner has to provide incentives for the firms not to cheat and hence is constrained by the incentive constraints (4.3). Consequently, in the constrained efficient outcome either one brand name offers maximum quality $v = 1$ for the price $\hat{p}(1)$, or—if demand is insufficient—there is no brand name. The market outcome will not be constrained efficient, in general, because it needs at least two brand names, and thus too many, to make sure that the price $\hat{p}(1)$ is charged. Only if there is no brand name in the market or if there is just one brand name and $p(1) = \hat{p}(1)$ is nevertheless the payoff-maximizing price, the equilibrium outcome is constrained efficient.

Does it make a difference for the customers whether a given level of unit variable cost consists only of production cost or of production and incentive cost? To answer this question we alter our model in two respects: we assume for the moment that quality is observable (so no incentive costs accrue) and that constant marginal production costs for quality v are given by $\tilde{c}(v) = c(0) + \alpha\gamma v$, $v \in [0, 1]$, which is identical to the sum of production and incentive costs considered so far. In all other respects, in particular regarding preferences and entry cost, the model remains the same. At least when customers' willingness to pay $R(\cdot, s)$ is *strictly* convex in v (i.e., $R_{vv}(\cdot, s) > 0$), the proof of Proposition 3 can be applied to the modified model as well, although with a slight variation.[33] It follows that only one

[32] Specifically, the hypothetical planner cannot change the detection parameter φ.

[33] The only difference is that when quality $\bar{v} < 1$ and price $p(\bar{v})$ are increased to $v = 1$ and $p(1) =$

brand name can be in the market, because with two or more brand names all will offer $v = 1$ for the price $\hat{p}(1) = c(0) + \alpha\gamma$ that equals marginal cost and gives zero profits. Since the brand names could not cover the positive entry cost, only one brand name can be in the market in a stationary equilibrium. The single brand name will offer quality $v = 1$ for a price $p(1) > \hat{p}(1)$. Therefore, customers will be worse off than in the case where the same unit variable costs consist of production costs and incentive costs, provided that the entry cost is sufficiently low for two or more brand names to be active and thus $p(1) = \hat{p}(1)$.

8. Equilibrium With Concave Willingness to Pay

In this section, we consider the case where each customer's willingness to pay $R(\cdot,\cdot)$ is strictly concave in v, i.e., $R_{vv}(\cdot,\cdot) < 0$.[34] Under this assumption the model becomes similar to a location model, as will be explained below. Depending on customer preferences, the distribution of customer types, and other parameters, many outcomes are possible. Moreover, because of the discontinuity associated with Bertrand competition an equilibrium need not exist.

We provide sufficient conditions for a lack of variety in the sense that no or at best relatively few firms offer an intermediate quality level. The intuition for this result is that it may be more profitable to share the high price market for maximum quality with competitors than to offer an intermediate quality level. Since the price for maximum quality cannot fall below $\hat{p}(1) > c(1)$, the effect of competition is limited. Provided two or more firms already offer maximum quality, entry into this market does not decrease the price although it exceeds marginal cost. Thus, the incentive to differentiate is significantly reduced. If, given the entry cost, the overall market for positive quality levels can accommodate only few firms, all (or most) incumbents may offer maximum quality.

Consider the situation where for each quality $v \in [0,1]$ the price is given by $p(v) = \hat{p}(v) = c(0) + \alpha\gamma v$. For a customer of type s buying quality v generates the payoff $R(v,s) - p(v)$, which has an interior maximum where $R_v(v,s) = \alpha\gamma$, provided an interior maximum exists. If an interior maximum does not exist, either $R_v(0,s) \leq \alpha\gamma$ and $\arg\max_v [R(v,s) - p(v)] = 0$, or $R_v(1,s) \geq \alpha\gamma$ and $\arg\max_v [R(v,s) - p(v)] = 1$. Let $v^*(s)$ be the solution of $R_v(v,s) =$

$p(\bar{v}) + (1-\bar{v})\alpha\gamma$, respectively, profit per customer $p(1) - c(0) - \alpha\gamma = p(\bar{v}) + (1-\bar{v})\alpha\gamma - c(0) - \alpha\gamma = p(\bar{v}) - c(0) - \alpha\gamma\bar{v}$ does not increase but remains constant.

[34] An analogue to footnote 30 for the convex case holds for the concave case.

$\alpha\gamma$ if such a solution exists for $v \in (0,1)$, and define $v^*(s) = 0$ if $R_v(0,s) \leq \alpha\gamma$, and $v^*(s) = 1$ if $R_v(1,s) \geq \alpha\gamma$. If $R(v^*(s), s) \geq \hat{p}(v^*(s))$, we can interpret $v^*(s)$ as the "location" of customer s.[35] If $R(v^*(s), s) < \hat{p}(v^*(s))$, type s prefers not to buy the product at all and thus is located "outside" of the space $[0,1]$ of quality levels. Types that have the location $v^*(s) = 0$ or satisfy $R(v^*(s), s) < \hat{p}(v^*(s))$ will never buy positive quality and are irrelevant for the following analysis. Let μ_1 denote the measure (or "quantity") of types with location $v^*(s) = 1$, and $\mu_{(0,1)}$ the measure (or "quantity") of types with an interior location $v^*(s) \in (0,1)$. If maximum quality $v = 1$ is available at the price $\hat{p}(1)$, μ_1 is a lower bound of the demand for it, since $p(v) \geq \hat{p}(v)$ for $v \in (0,1)$, whereas $\mu_{(0,1)}$ is an upper bound of total demand for all intermediate quality levels $v \in (0,1)$. The ratio $\lambda \equiv \frac{\mu_1}{\mu_{(0,1)}+\mu_1}$ is the share of types with location $v^*(s) = 1$ in the set of types with location $v^*(s) > 0$. For reasons that will become clear below, we are interested in the situation where λ is "large," say 2/3 or higher. This will be the case if (but not only if) the willingness to pay is not too concave in v.

Consider now a stationary equilibrium where prices are given by $p(v) = \hat{p}(v)$ for all available intermediate quality levels $v \in (0,1)$, N firms offer positive quality, and n of those offer maximum quality $v = 1$ for the price $\hat{p}(1)$, where $1 \leq n \leq N-1$. Among the firms that offer some intermediate quality $v \in (0,1)$ the one with the fewest customers has at most $\frac{\mu_{(0,1)}}{N-n}$ customers. If it deviates and offers maximum quality $v = 1$ for the price $\hat{p}(1)$, it gets at least $\frac{\mu_1}{n+1}$ customers and, in addition, makes a larger profit per customer, since $\hat{p}(v) - c(v) = \alpha\gamma v$ achieves a maximum at $v = 1$. Because in an equilibrium such a deviation must not be profitable, $\frac{\mu_1}{n+1} < \frac{\mu_{(0,1)}}{N-n}$, which is equivalent to $n > \lambda(N+1) - 1$.[36] The following proposition generalizes this observation.

Proposition 4. *Consider the case where each customer's willingness to pay is strictly concave in v, i.e., $R_{vv}(\cdot,\cdot) < 0$. Assume that for all intermediate levels of quality all customers' marginal willingness to pay for quality exceeds the "marginal cost of quality," i.e., $R_v(\cdot,\cdot) > \gamma$ for all $v \in (0,1)$. Then in every stationary equilibrium with $N \geq \frac{1}{\lambda} + 1$ brand names in the market (which implies $N \geq 2$) the price for maximum quality $v = 1$ is $p(1) = \hat{p}(1)$, and*

[35] This assumes the (innocuous) tie breaking rule that a type with $R(v^*(s), s) = \hat{p}(v^*(s))$, who is indifferent between buying quality $v^*(s)$ and not buying the good at all, buys quality $v^*(s)$.

[36] The inequality $n > \lambda(N+1) - 1$ is equivalent to $\frac{n+1}{N+1} > \lambda$, which gives a lower bound to the "modified share" of firms that offer maximum quality.

the number of brand names that offer maximum quality is at least $n > \lambda(N+1) - 1$ *for* $\lambda < 1$ *and is* N *for* $\lambda = 1$. *Moreover, in every stationary equilibrium with* $N \in \left[3, \frac{\lambda}{1-\lambda}\right]$ *brand names in the market (which implies* $\lambda \geq \frac{3}{4}$*) all* N *brand names in the market offer maximum quality* $v = 1$ *for the price* $p(1) = \hat{p}(1)$.[37]

The number of brand names that offer maximum quality will frequently exceed its lower bound $\lambda(N+1) - 1$ considerably. However, without more specific assumptions on the willingness to pay functions and the distribution of customers it is impossible to derive tight bounds on demand for maximum and intermediate quality, respectively.[38] In most cases the term μ_1 will significantly underestimate demand for maximum quality $v = 1$ at prices $p(v) = \hat{p}(v)$, $v \in [0, 1]$, while simultaneously the term $\mu_{(0,1)}$ will significantly overestimate the aggregate demand for intermediate qualities at these prices. Moreover, since intermediate quality $v < 1$ can be arbitrarily close to 1, the fact that profits per customer increase with quality v cannot be exploited. Therefore, the proposition's result that brand names will provide exclusively or at least predominantly maximum quality can be expected to hold for considerably lower values of λ than suggested by Proposition 4.

Under the conditions of Proposition 4, brand names tend to provide maximum quality $v = 1$ rather than an intermediate quality $v \in (0, 1)$ whenever λ is clearly above $1/2$. For example, consider the case $\lambda = \frac{2}{3}$. If $N = 2$, no intermediate quality will be available in the market. Even when $N = 8$ at most 2 brand names offer intermediate quality levels, whereas at least 6 brand names offer maximum quality. In fact, in most cases $N = 8$ will imply that 7 (or even all 8) brand names offer maximum quality and at most one offers some intermediate level.[39] Thus, although some intermediate quality levels may be offered when customer's

[37] All stationary equilibria where $N \geq 2$ brand names are in the market and offer maximum quality, are equivalent from the customers' point of view. They differ only with respect to the identity of the firms. All stationary equilibria where $N = 1$ brand name is in the market and offers maximum quality, are also equivalent from the customers' point of view, but if the sole brand name charges a price $p(1) > \hat{p}(1)$ customers are worse off than in stationary equilibria with $N \geq 2$ brand names offering maximum quality.

[38] Also, without more specific assumptions on the willingness to pay functions and the distribution of customers we cannot determine what quality a monopolist would offer.

[39] This follows from the fact that the bounds used for the proof of Proposition 4 are not tight, as explained in the previous paragraph. If N is sufficiently large, such that more than 1 brand name *may* offer intermediate quality (as with $N = 8$), there is an additional loss of tightness: the proof does not exploit the fact that the profit of the *least profitable* firm among several firms that all offer intermediate qualities will be below the *average* profit of those firms.

willingness to pay is concave in v, frequently they will be inefficiently few. In addition, since profit per customer $\hat{p}(v) - c(v) = (\alpha - 1)\gamma v$ increases in quality v, the intermediate quality levels that are actually available will be biased towards maximum quality $v = 1$. In contrast, in the second-best allocation every brand name would offer a different level of quality and the different quality levels would be spread out over the interval $(0, 1]$. Moreover, in any stationary equilibrium with more than two firms offering maximum quality, a reduction to only two such firms would increase welfare in the sense that entry costs would decrease whereas prices and total profits would be the same in each period and customers would be unaffected. In other words, at the upper quality end (as, in fact, at any given quality level) two firms are "sufficient;" every additional firm offering maximum quality has only a "business-stealing" effect (Mankiw and Whinston 1986) on the allocation after entry and thus from the social point of view the additional entry cost is a waste of resources.

The assumption that for all positive levels of quality all customers' marginal willingness to pay for quality exceeds the "marginal cost of quality" is used only to make sure that profits per customer are maximal at $v = 1$, which follows from the assumption's implication that $p(v) = \hat{p}(v)$ for all $[0, 1)$. Even when customers' marginal willingness to pay for quality is lower than required by the assumption, profit per customer may well be maximal at $v = 1$ because optimal prices for intermediate quality levels may not exceed $\hat{p}(v)$ significantly, if at all. And if the optimal price for an intermediate quality level v does exceed $\hat{p}(v)$ significantly, this may lead to low demand and the difference in profit per customer may be compensated by a higher number of customers, and thus a higher profit, at $v = 1$.

As pointed out above, because of the discontinuity associated with Bertrand competition an equilibrium need not exist. In the rest of this section we address this issue and provide a sufficient condition for the existence of a stationary equilibrium where all brand names in the market offer maximum quality $v = 1$.

If only minimum and maximum quality are available and the prices are $p(0) = c(0)$ and $p(1) = \hat{p}(1)$, respectively, types who satisfy $R(1, s) - \hat{p}(1) \geq \max[R(0, s) - c(0), 0]$ will buy maximum quality (this assumes the innocuous tie breaking rule that indifferent types buy maximum quality), whereas all others prefer either to buy quality $v = 0$ for the price $c(0)$ or not to buy at all. Let $\bar{\mu}$ denote the measure (or "quantity") of customers who satisfy $R(1, s) - \hat{p}(1) \geq \max[R(0, s) - c(0), 0]$. Thus, $\bar{\mu}$ is the demand for quality $v = 1$ in the situation specified in this paragraph. If $N \geq 2$ brand names are in the market and all offer

$v = 1$, the price is $p(1) = \hat{p}(1)$ and the profit per period of each brand name is $\frac{1}{N}\bar{\mu}(\alpha - 1)\gamma$. Under the assumptions of Proposition 4, if one of the brand names deviates and offers some intermediate quality $v \in (0, 1)$, its profit per period is less than $\mu_{(0,1)}(\alpha - 1)\gamma$. Thus, if $\mu_{(0,1)}(\alpha - 1)\gamma \leq \frac{1}{N}\bar{\mu}(\alpha - 1)\gamma$, no firm will deviate.

In any stationary equilibrium each brand name in the market must be able to cover the entry cost η, which means that its profit per period must be at least $\eta\rho$ (where ρ denotes the discount rate). The constellation where $N \geq 2$ brand names are in the market and all offer maximum quality $v = 1$ for the price $p(1) = \hat{p}(1)$, is a stationary equilibrium if three conditions hold: (i) each brand name in the market is at least able to cover the entry cost, (ii) an additional brand name, if it entered, could at best just cover the entry cost and thus cannot get a positive payoff, and (iii) no brand name in the market can increase its profit per period by deviating from maximum quality $v = 1$ to some lower quality level. The following proposition provides sufficient conditions (a) for the existence of a stationary equilibrium where all brand names in the market offer maximum quality, and (b) for the conclusion that in *every* stationary equilibrium all brand names in the market offer maximum quality.

Proposition 5. *Consider the case where each customer's willingness to pay is strictly concave in v, i.e., $R_{vv}(\cdot, \cdot) < 0$. Assume that for all intermediate levels of quality all customers' marginal willingness to pay for quality exceeds the "marginal cost of quality," i.e., $R_v(\cdot, \cdot) > \gamma$ for all $v \in (0, 1)$. If the two conditions $\frac{\bar{\mu}}{N+1}(\alpha - 1)\gamma \leq \eta\rho \leq \frac{\bar{\mu}}{N}(\alpha - 1)\gamma$ and $\mu_{(0,1)}(\alpha - 1)\gamma \leq \eta\rho$ are both satisfied for some $N \geq 2$, there exists a stationary equilibrium where N brand names are in the market and all offer $v = 1$ for the price $p(1) = \hat{p}(1)$. Whenever, in addition, the first condition implies $N \geq 3$, all N brand names in the market offer $v = 1$ for the price $p(1) = \hat{p}(1)$ in every stationary equilibrium.*

The first condition makes sure that all brand names cover the entry cost and no additional firm wants to enter and offer maximum quality. The second condition implies that an additional entrant that offers intermediate quality cannot cover the entry cost, provided maximum quality is sold for the price $\hat{p}(1)$. It follows immediately that no intermediate quality is available if two or more brand names offer maximum quality, since that implies $p(1) = \hat{p}(1)$. However, Proposition 5 does not exclude a stationary equilibrium with $N = 2$ brand names where one brand name offers maximum quality $v = 1$ at a price $p(1) > \hat{p}(1)$ and the other brand name offers some intermediate quality $v \in (0, 1)$ for the price $\hat{p}(v)$.

Since $p(1) > \hat{p}(1)$, the second brand name's demand can be greater than $\mu_{(0,1)}$ and thus its profit per period can exceed $\eta\rho$, in which case that brand name can cover its entry cost. In fact, in such an equilibrium the second brand name's profit per period must not be less than the first brand name's profit, because otherwise it could increase its payoff by slightly undercutting the first brand name's price $p(1)$. The stationary equilibrium with only one out of $N = 2$ brand names offering maximum quality does not generalize to $N > 2$ brand names. The proof of Proposition 5 shows that with more than two brand names there does not exist a stationary equilibrium where only one brand name offers maximum quality $v = 1$ at a price $p(1) > \hat{p}(1)$ and all other brand names offer some intermediate qualities.

If Proposition 5 holds, the market for each specific intermediate quality level $\bar{v} \in (0, 1)$ is too "thin" or has too low a markup (or both) to justify entry, given the availability of minimum and maximum quality at the prices $c(0)$ and $\hat{p}(1)$, respectively. Although many customers may prefer *some* intermediate quality, every *particular* level of intermediate quality would attract only a limited number of customers or generate only an insignificant profit per customer, and thus would result in an insufficient profit relative to the entry cost. Since for all $v \in (0, 1)$ the price is $\hat{p}(v)$, profit per customer is proportional to v, and consequently for small v the associated profit is low even when demand is high.

9. Conclusions

In this paper, we investigated the effects of "quality-assuring" prices (Klein and Leffler 1981) on quality diversity in a model with an endogenous number of oligopolistic firms that simultaneously compete in price and quality. We considered a model of vertical product differentiation where before purchase the respective good's quality is unobservable to customers, a continuum of quality levels is technologically feasible, and minimum quality is supplied under competitive conditions. After purchase the true quality of the good is revealed with positive probability. For each feasible quality level, except for the minimum, the price must exceed marginal cost in order to provide incentives for the respective firm not to produce lower quality than announced. Our analysis shows that if, relative to production cost, customers have a preference for high quality, equilibrium prices are completely determined by incentive constraints for all quality levels, except for the maximum. Consequently, for those quality levels customer preferences and the distribution of customer types only affect the quantities demanded, given equilibrium prices, but not the prices themselves. Moreover, if the information problem is sufficiently severe, no positive quality is available in the market even though all customers may prefer maximal quality to any other quality when each quality level is sold for a price that is equal to its marginal cost.

The shape of customers' willingness to pay functions—whether they are convex or concave with respect to quality—has a significant effect on the equilibrium. When each customer's willingness to pay for quality is convex with respect to quality, at most two levels of quality, the lowest and the highest one among the technologically feasible levels, are available in the market, and this is constrained efficient. When customers' willingness to pay for quality is strictly concave, the model becomes similar to a location model and an equilibrium may not even exist. Existence of an equilibrium and its properties depend on details of customers' willingness to pay functions and the distribution of types. We derived conditions under which firms that offer above minimum quality tend to concentrate at maximum quality $v = 1$ rather than to spread out over all above minimum quality levels, as would be efficient. Only few firms, if any, will offer intermediate quality levels. Indeed, we provided sufficient conditions for the existence of an equilibrium where *all* brand names in the market offer maximum quality, and for the uniqueness of this equilibrium outcome. Intuitively, quality-assuring prices (Klein and Leffler 1981) limit price competition and the need of firms to differentiate themselves from competitors. This, in turn, implies a lack of equilibrium quality diversity and, provided demand for maximum quality is sufficiently high, a bias of equilibrium quality levels towards maximum quality.

Appendix

Proof of Proposition 1: The proof is by contradiction. Assume that there exists a Nash equilibrium in pure strategies where in some period t some firm offers some quality $v_t > 0$ at a price p_t such that sales x_t are positive. The firm will not cheat along the equilibrium path (and thus we need not distinguish between announced and actual quality levels). Therefore, and because we consider only pure strategies, the respective firm's sales are deterministic along the equilibrium path. Denote equilibrium sales by $x_\tau \geq 0$, equilibrium prices by $p_\tau \geq 0$, and equilibrium quality levels by $v_\tau \in [0,1]$, $\tau \in \{t, t+1, ...\}$. Along the equilibrium path the firm's payoff from any period $\tau \in \{t, t+1, ...\}$ onwards is $V_\tau \equiv \sum_{s=\tau}^{\infty} \delta^{s+1-\tau} [p_s - c(v_s)] x_s = \delta [p_\tau - c(v_\tau)] x_\tau + \delta V_{\tau+1}$. First, we derive the incentive constraint that prevents the firm from cheating (only) in period t. If the firm follows its equilibrium strategy, $V_t = \delta [p_t - c(v_t)] x_t + \delta V_{t+1}$. If it cheats in period t, but not thereafter its expected payoff from period t onwards is $\delta [p_t - c(0)] x_t + (1-\varphi) \delta V_{t+1}$. For the firm not to cheat in period t, it must hold that $\delta [p_t - c(v_t)] x_t + \delta V_{t+1} \geq \delta [p_t - c(0)] x_t + (1-\varphi) \delta V_{t+1}$, and thus $\varphi \delta V_{t+1} \geq \delta [c(v_t) - c(0)] x_t = \delta \gamma v_t x_t$ or $V_{t+1} \geq \frac{1}{\varphi} \gamma v_t x_t > 0$, where positivity follows from $v_t > 0$ and $x_t > 0$. By assumption, $R(v,s) - \hat{p}(v) < \max [R(0,s) - c(0), 0]$ for all $v \in (0,1]$ and all $s \in S$. Hence there exists a positive function $\kappa : (0,1] \to \mathbb{R}_{++}$ such that $\max [R(0,s) - c(0), 0] - [R(v,s) - \hat{p}(v)] > \kappa(v) > 0$ for all $v \in (0,1]$ and all $s \in S$. Therefore, since $v_t > 0$ and $x_t > 0$, $p_t < \hat{p}(v_t) - \kappa(v_t) = c(v_t) + \frac{\rho}{\varphi} \gamma v_t - \kappa(v_t)$ because for $p_t \geq \hat{p}(v_t) - \kappa(v_t)$ and $v_t > 0$ sales x_t are zero. Thus, $p_t - c(v_t) < \frac{\rho}{\varphi} \gamma v_t - \kappa(v_t)$. Due to this and $\frac{1}{\varphi} \gamma v_t x_t \leq V_{t+1}$, we have $V_t = \delta [p_t - c(v_t)] x_t + \delta V_{t+1} < \delta \frac{\rho}{\varphi} \gamma v_t x_t - \delta \kappa(v_t) x_t + \delta V_{t+1} \leq \rho \delta V_{t+1} + \delta V_{t+1} - \delta \kappa(v_t) x_t = V_{t+1} - \delta \kappa(v_t) x_t$, i.e., $V_{t+1} - V_t > \delta \kappa(v_t) x_t > 0$. The preceding argument can be applied to any period τ. Consequently, whenever $v_\tau x_\tau > 0$ (which is equivalent to $v_\tau > 0$ and $x_\tau > 0$) for some τ, $V_{\tau+1} - V_\tau > \delta \kappa(v_\tau) x_\tau > 0$. Consider next the case where $v_\tau x_\tau = 0$, i.e., where $v_\tau = 0$ or $x_\tau = 0$, or both. Notice that if $v_\tau = 0$ and $x_\tau > 0$, then $p_\tau = c(0)$ and thus $p_\tau - c(v_\tau) = 0$. From $V_\tau = \delta [p_\tau - c(v_\tau)] x_\tau + \delta V_{\tau+1}$ and $\frac{1}{\delta} = 1 + \rho$ we get $V_{\tau+1} = (1+\rho) V_\tau - [p_\tau - c(v_\tau)] x_\tau$. Therefore, $v_\tau x_\tau = 0$ implies $V_{\tau+1} - V_\tau = \rho V_\tau$. We know that $V_{t+1} > 0$. If $v_{t+1} x_{t+1} = 0$, the previous argument gives $V_{t+2} > V_{t+1}$. Moreover, we have shown that $v_\tau x_\tau > 0$ for any τ implies $V_{\tau+1} - V_\tau > \delta \kappa(v_\tau) x_\tau > 0$, hence $v_{t+1} x_{t+1} > 0$ implies $V_{t+2} - V_{t+1} > \delta \kappa(v_{t+1}) x_{t+1} > 0$. It follows that $V_{t+2} > V_{t+1}$ regardless of v_{t+1} and x_{t+1}. Repeating this argument shows that $V_{\tau+1} > V_\tau$ for all $\tau \in \{t, t+1, ...\}$.

By assumption, total expenditures $p_\tau x_\tau$ are uniformly bounded. Consequently, per period profits $[p_\tau - c(v_\tau)]\, x_\tau \leq p_\tau x_\tau$ are bounded by some number M and thus V_τ, $\tau \in \{t, t+1, ...\}$, is bounded by M/ρ. Therefore, the (increasing) sequence $\{V_\tau\}_t^\infty$ must converge to some $\bar{V}$. Together with $V_{t+1} > 0$ this implies that there exists a sufficiently large $T > t$ such that for all $\tau \in \{T, T+1, ...\}$ it holds that $\frac{\rho}{2} V_{t+1} > V_{\tau+1} - V_\tau$ and thus $\frac{\rho}{2} V_{t+1} > V_{\tau+1} - V_\tau = \rho V_\tau - [p_\tau - c(v_\tau)]\, x_\tau \geq \rho V_{t+1} - [p_\tau - c(v_\tau)]\, x_\tau$, which gives $[p_\tau - c(v_\tau)]\, x_\tau > \frac{\rho}{2} V_{t+1} > 0$ (in addition, $p_\tau - c(v_\tau) > 0$ follows). Since $p_\tau - c(v_\tau) < \hat{p}(v_\tau) - c(v_\tau) = \frac{\rho}{\varphi} \gamma v_\tau \leq \frac{\rho}{\varphi} \gamma$, $x_\tau > \frac{\varphi}{2\gamma} V_{t+1} > 0$. Moreover, $[p_\tau - c(v_\tau)]\, x_\tau > 0$ implies $p_\tau > c(0) > 0$ and thus, because expenditures $p_\tau x_\tau$ are uniformly bounded, $x_\tau < \bar{x}$ for some $\bar{x}$. Together with $[p_\tau - c(v_\tau)]\, x_\tau > \frac{\rho}{2} V_{t+1}$ this gives $p_\tau - c(v_\tau) > \frac{\rho}{2x_\tau} V_{t+1} > \frac{\rho}{2\bar{x}} V_{t+1} > 0$. In addition, $0 < p_\tau - c(v_\tau) < \hat{p}(v_\tau) - c(v_\tau) = \frac{\rho}{\varphi} \gamma v_\tau$ implies $[p_\tau - c(v_\tau)] \to 0$ for $v_\tau \to 0$. Therefore, there exists a $\bar{v} > 0$ such that $v_\tau > \bar{v}$ for all $\tau \in \{T, T+1, ...\}$. Since $v_\tau > \bar{v} > 0$ for all $\tau \in \{T, T+1, ...\}$, $\kappa(v_\tau) > \bar{\kappa}$ for some $\bar{\kappa} > 0$ for all $\tau \in \{T, T+1, ...\}$. Finally, $v_\tau x_\tau > 0$ implies $V_{\tau+1} - V_\tau > \delta \kappa(v_\tau)\, x_\tau > \delta \bar{\kappa} \frac{\varphi}{2\gamma} V_{t+1} > 0$ for all $\tau \in \{T, T+1, ...\}$, and thus the sequence $\{V_\tau\}_t^\infty$ cannot converge to some $\bar{V}$. This contradiction proves the proposition. ■

Proof of Lemma 1: Since the respective strategy profile is a Nash equilibrium of the restricted game, the incentive constraints for always actually producing the announced quality are satisfied. Otherwise customers would not buy the respective firm's product for a price above $c(0)$, the firm's profit would be zero and it would not have entered the market (since it has a positive entry cost). In addition, since all competitors' actions are constant in time, each firm's (constant) choice of quality and price maximizes *in each individual period* its per period profit, given all other incumbents' qualities and prices. Consequently, no incumbent can improve her payoff by deviating to a different (stationary or non-stationary) strategy with respect to quality and price. Moreover, each incumbent makes in each period a constant positive profit that must exceed the entry cost times the discount rate ρ (otherwise the respective firm would not have entered the market). Therefore, it is optimal for each incumbent to enter in the first period and never to exit. ■

Proof of Proposition 2: The proof is by contradiction. Consider some firm that in a stationary equilibrium offers quality $\bar{v} \in (0, 1)$ for the price $p(\bar{v}) > \hat{p}(\bar{v})$. Assume the firm increases quality $\bar{v}$ to $\bar{v} + \varepsilon > \bar{v}$. Since $R_v(\bar{v}, s_{\min}) > \gamma$, there exists a price $p(\bar{v} + \varepsilon)$ such that $p(\bar{v}) + \varepsilon R_v(\bar{v}, s_{\min}) > p(\bar{v} + \varepsilon) > p(\bar{v}) + \varepsilon\gamma$. In addition, $p(\bar{v}) > \hat{p}(\bar{v})$ implies that for a sufficiently small $\varepsilon > 0$ the inequality $p(\bar{v}+\varepsilon) \geq \hat{p}(\bar{v}) + \frac{\rho\gamma}{\varphi}\varepsilon = \hat{p}(\bar{v}+\varepsilon)$ is satisfied. Moreover, if $\varepsilon > 0$ is sufficiently small, it holds for each customer of the respective firm that $R(\bar{v}+\varepsilon, s) - R(\bar{v}, s) \geq \varepsilon R_v(\bar{v}, s_{\min}) \geq p(\bar{v}+\varepsilon) - p(\bar{v})$ because $R_{vs}(\cdot,\cdot) > 0$ for all $v > 0$, and thus $R(\bar{v}+\varepsilon, s) - p(\bar{v}+\varepsilon) \geq R(\bar{v}, s) - p(\bar{v})$. Therefore, under the alternative strategy of announcing and producing quality $\bar{v}+\varepsilon$ and charging the price $p(\bar{v}+\varepsilon) < p(\bar{v}) + \varepsilon R_v(\bar{v}, s_{\min})$ the firm has at least as many customers as under the original strategy of selling quality $\bar{v}$ for the price $p(\bar{v})$. In addition, since $p(\bar{v}+\varepsilon) > p(\bar{v}) + \varepsilon\gamma$ and thus $p(\bar{v}+\varepsilon) - c(\bar{v}+\varepsilon) > p(\bar{v}) + \varepsilon\gamma - c(\bar{v}) - \gamma\varepsilon = p(\bar{v}) - c(\bar{v})$, the firm's profit per customer will increase. Consequently, the firm's payoff will increase, and hence the strategy to offer quality $\bar{v} \in (0,1)$ for the price $p(\bar{v}) > \hat{p}(\bar{v})$ cannot optimal. This contradicts the assumption that it is an equilibrium strategy. If $R_v(v,s) > \gamma$ for all $v \in (0,1)$ and all $s \in S$, then the same argument shows that it cannot be optimal to offer any quality $v \in (0,1)$ for a price $p(v) > \hat{p}(v)$. Finally, $p(0) = c(0) = \hat{p}(0)$ holds trivially. ■

Proof of Proposition 3: First, we show by contradiction that if there is only one brand name, it will offer maximum quality $v = 1$. Clearly, in a stationary equilibrium the respective single brand name will not offer minimum quality $v = 0$ since profits would be zero and the entry cost could not be covered. Assume the respective brand name offers some quality level $\bar{v} \in (0,1)$ for some price $p(\bar{v}) \geq \hat{p}(\bar{v}) = c(0) + \alpha\gamma\bar{v}$. For any customer s of the brand name it must hold that she prefers $v = \bar{v}$ for the price $p(\bar{v})$ to (i) $v = 0$ for the price $p(0) = c(0)$ and (ii) to not buying at all, i.e., it must hold that $R(\bar{v}, s) - p(\bar{v}) \geq \max[R(0,s) - c(0), 0]$. If $R(\bar{v}, s) - p(\bar{v}) \geq R(0,s) - c(0) \geq 0$, then $R(\bar{v}, s) - R(0,s) \geq p(\bar{v}) - c(0) \geq \hat{p}(\bar{v}) - c(0) = \alpha\gamma\bar{v}$. Moreover, if for some $s \in S$, $R(\bar{v}, s) - R(0,s) \geq \alpha\gamma\bar{v}$ for some $\bar{v} \in (0,1)$, then $R(1,s) - R(\bar{v}, s) \geq (1-\bar{v})\alpha\gamma$, because from $R_{vv}(\cdot, s) \geq 0$ it follows that $\frac{R(1,s)-R(\bar{v},s)}{1-\bar{v}} \geq \frac{R(\bar{v},s)-R(0,s)}{\bar{v}}$. Thus, $R(\bar{v}, s) - p(\bar{v}) \geq R(0,s) - c(0)$ implies $R(1,s) - [p(\bar{v}) + (1-\bar{v})\alpha\gamma] \geq R(\bar{v}, s) - p(\bar{v})$. In addition, if $R(\bar{v}, s) - p(\bar{v}) \geq 0 > R(0,s) - c(0)$, then $R(\bar{v}, s) - R(0,s) > p(\bar{v}) - c(0) \geq \hat{p}(\bar{v}) - c(0) = \alpha\gamma\bar{v}$ and thus $R(1,s) - [p(\bar{v}) + (1-\bar{v})\alpha\gamma] > R(\bar{v}, s) - p(\bar{v})$ because $\frac{R(1,s)-R(\bar{v},s)}{1-\bar{v}} \geq \frac{R(\bar{v},s)-R(0,s)}{\bar{v}} > \alpha\gamma$ gives $R(1,s) > R(\bar{v}, s) + (1-\bar{v})\alpha\gamma$. Consequently, $R(\bar{v}, s) - p(\bar{v}) \geq \max[R(0,s) - c(0), 0]$ always implies $R(1,s) - [p(\bar{v}) + (1-\bar{v})\alpha\gamma] \geq R(\bar{v}, s) - p(\bar{v})$. Therefore, the respective brand name will not lose demand, if it offers quality $v = 1$ for the price $p(1) = p(\bar{v}) + (1-\bar{v})\alpha\gamma$ instead of quality $\bar{v} \in (0,1)$ for the price $p(\bar{v})$. In addition, its profit per customer will increase from $(\alpha-1)\gamma\bar{v}$ to $(\alpha-1)\gamma$. It follows that if there is only one brand name, it will offer maximum quality $v = 1$.

Consider now the case of at least two brand names. By the same argument as before, at least one firm offers quality $v = 1$. If two or more firms offer quality $v = 1$, $p(1) = \hat{p}(1)$ because of Bertrand competition. In this case, any third firm that offers some quality $\bar{v} \in (0,1)$ for some price $p(\bar{v}) \geq \hat{p}(\bar{v})$ will attract customer type s only if $R(\bar{v}, s) - p(\bar{v}) \geq \max[R(1,s) - \hat{p}(1), R(0,s) - c(0), 0]$. However, we have already shown that $R(\bar{v}, s) - p(\bar{v}) \geq \max[R(0,s) - c(0), 0]$ implies $R(\bar{v}, s) - p(\bar{v}) \leq R(1,s) - [p(\bar{v}) + (1-\bar{v})\alpha\gamma] \leq R(1,s) - [\hat{p}(\bar{v}) + (1-\bar{v})\alpha\gamma] = R(1,s) - \hat{p}(1)$. That is, whenever some type s prefers $v = \bar{v}$ for the price $p(\bar{v})$ to $v = 0$ for the price $p(0) = c(0)$ and to not buying at all, that type prefers, at least weakly, $v = 1$ for the price $\hat{p}(1)$ to $v = \bar{v}$ for the price $p(\bar{v})$. Consequently, any firm that offers some quality $\bar{v} \in (0,1)$ for some price $p(\bar{v}) \geq \hat{p}(\bar{v})$ has demand of measure zero, and thus zero profits, and cannot cover its entry cost. Hence whenever at least two firms offer $v = 1$ no intermediate quality $v \in (0,1)$ will be available in the market. The same conclusion follows if only one firm offers $v = 1$ for the price $p(1) = \hat{p}(1)$. The remaining case is the one where $v = 1$ is being offered for some price $p(1) > \hat{p}(1)$ by a single firm and at least one other firm offers $\bar{v} \in (0,1)$ for some price $p(\bar{v}) \geq \hat{p}(\bar{v})$. We have already shown that $R(\bar{v}, s) - p(\bar{v}) \geq \max[R(0,s) - c(0), 0]$ implies $R(1,s) - [p(\bar{v}) + (1-\bar{v})\alpha\gamma] \geq R(\bar{v}, s) - p(\bar{v})$. Therefore, the firm offering $v = \bar{v}$ can have customers only if $p(1) \geq p(\bar{v}) + (1-\bar{v})\alpha\gamma$ since otherwise $R(\bar{v}, s) - p(\bar{v}) \geq \max[R(0,s) - c(0), 0]$ implies $R(\bar{v}, s) - p(\bar{v}) \leq R(1,s) - [p(\bar{v}) + (1-\bar{v})\alpha\gamma] < R(1,s) - p(1)$, i.e., rather than buying $v = \bar{v}$ for the price $p(\bar{v})$ each type $s \in S$ prefers either not to buy at all, to buy $v = 0$ or to buy $v = 1$. If equality $p(1) = p(\bar{v}) + (1-\bar{v})\alpha\gamma$ holds, the set of customers is of measure zero, thus positive profits of the firm offering $v = \bar{v}$ imply $p(1) > p(\bar{v}) + (1-\bar{v})\alpha\gamma$. If the respective firm offers instead $v = 1$ for the price $\tilde{p}(1) = p(\bar{v}) + (1-\bar{v})\alpha\gamma < p(1)$, it will not lose any customers (because, as shown, $R(1,s) - \tilde{p}(1) \geq R(\bar{v}, s) - p(\bar{v})$ for each customer s) and take all customers from the rival firm that offers $v = 1$ for some price $p(1) > \tilde{p}(1)$. In addition, its profit per customer and hence its payoff increases. Thus, also in this last case it cannot be an equilibrium that some firm offers some intermediate quality $\bar{v} \in (0,1)$. Consequently, there is no case where a brand name offers an intermediate quality $v \in (0,1)$. In contrast, it is an equilibrium that all brand names offer quality $v = 1$ for the price $\hat{p}(1)$, provided the market can accommodate at least two firms (i.e., if two firms offer quality $v = 1$ for the price $\hat{p}(1)$, each firm's discounted stream of profits covers the entry cost). If the market accommodates at least one firm (i.e., if for a single firm that offers quality $v = 1$ for the profit-maximizing price $p(1) \geq \hat{p}(1)$ the discounted stream of profits covers the entry cost), quality $v = 1$ will be available in the market for some price $p(1) \geq \hat{p}(1)$. For the proof of the second part of the proposition, which assumes $R_s(1, \cdot) > 0$, consider first the case where quality $v = 1$ is available for some price $p(1) \geq \hat{p}(1)$ and is demanded by some but not all customers, i.e., $R(1, s_{\max}) - p(1) \geq \max[R(0, s_{\max}) - c(0), 0]$ and $R(1, s_{\min}) - p(1) \leq \max[R(0, s_{\min}) - c(0), 0]$. Then there exists a unique type $\bar{s} \in S$ such that $R(1, \bar{s}) - p(1) = \max[R(0, \bar{s}) - c(0), 0]$. Uniqueness follows because $R(1, \cdot)$ and $s = R(1, \cdot) - R(0, \cdot)$ are both strictly increasing in s and because of the following contradiction: if $R(1, \bar{s}) - p(1) = R(0, \bar{s}) - c(0) > 0$ and $R(1, \bar{s}') - p(1) = 0 > R(0, \bar{s}') - c(0)$ for some $\bar{s} \in S$ and $\bar{s}' \in S$, then $R(1, \bar{s}) - p(1) > 0 = R(1, \bar{s}') - p(1)$ implies $\bar{s} > \bar{s}'$ because $R_s(1, \cdot) > 0$, whereas $R(1, \bar{s}) - p(1) = R(0, \bar{s}) - c(0)$ and $R(1, \bar{s}') - p(1) > R(0, \bar{s}') - c(0)$ imply $\bar{s} = R(1, \bar{s}) - R(0, \bar{s}) = p(1) - c(0) < R(1, \bar{s}') - R(0, \bar{s}') = \bar{s}'$ and thus $\bar{s}' > \bar{s}$. Moreover, because $R(1, \cdot)$ and $s = R(1, \cdot) - R(0, \cdot)$ are both strictly increasing in s, all types $s \in [s_{\min}, \bar{s})$ either consume quality $v = 0$ or abstain from consuming the good, and all types $s \in (\bar{s}, s_{\max}]$ consume quality $v = 1$. Since $R(1, s) - \hat{p}(1) = R(1, s) - c(0) - \alpha\gamma$ decreases strictly with α, the solution $\bar{s}$ of $R(1, \bar{s}) - \hat{p}(1) = \max[R(0, \bar{s}) - c(0), 0]$ must also strictly decrease with α for $\bar{s} \in (s_{\min}, s_{\max})$. Finally, if all types $s \in [s_{\min}, s_{\max}]$ demand $v = 1$, $\bar{s} = s_{\min}$; and if no type $s \in [s_{\min}, s_{\max}]$ demands $v = 1$, $\bar{s} = s_{\max}$. ■

Proof of Proposition 4: By construction, demand for quality $v = 1$ at the price $p(1) = \hat{p}(1)$ is bounded from below by μ_1, regardless of the availability of other quality levels. If $n \geq 1$ brand names offer $v = 1$ for the price $\hat{p}(1)$, each has a profit per period of at least $\frac{\mu_1}{n}[\hat{p}(1) - c(1)] = \frac{\mu_1}{n}(\alpha - 1)\gamma$. If $n \geq 2$ brand names offer quality $v = 1$, Bertrand competition implies that all such firms charge the price $p(1) = \hat{p}(1)$. For all intermediate qualities $v \in (0, 1)$ the price is $p(v) = \hat{p}(v)$ because of Proposition 2 and $R_v(\cdot, \cdot) > \gamma$ for $v \in (0, 1)$. Therefore, the profit per customer that a firm offering $v < 1$ achieves is $(\alpha - 1)\gamma v < (\alpha - 1)\gamma$. Moreover, if some brand name offers quality $v = 1$ for the price $p(1) = \hat{p}(1)$, total demand for all intermediate quality levels $v \in (0, 1)$ is bounded from above by $\mu_{(0,1)}$. In this case, if $k \geq 1$ brand names offer (identical or different) intermediate quality levels $v \in (0, 1)$, the lowest profit per period must be less than $\frac{\mu_{(0,1)}}{k}(\alpha - 1)\gamma$, because $\frac{\mu_{(0,1)}}{k}(\alpha - 1)\gamma$ exceeds the average profit of those k firms. A necessary condition for a stationary equilibrium where $k \geq 1$ firms offer some intermediate quality levels and n firms offer quality $v = 1$, is that no firm can increase its payoff, if it switches to some other quality. This implies, in particular, that a brand name with the lowest profit per period among the k brand names that offer intermediate quality levels must not be able to increase its profit per period by switching to quality $v = 1$. Denote by π the lowest profit per period that a brand name that has chosen some intermediate quality $v \in (0, 1)$ achieves, and by ψ the one that this brand name achieves instead if it switches to quality $v = 1$. If $k \geq 1$ firms offer some intermediate quality levels $v \in (0, 1)$ and either $n \geq 2$ firms offer quality $v = 1$, which implies $p(1) = \hat{p}(1)$, or $n = 1$ firm offers quality $v = 1$ for the price $p(1) = \hat{p}(1)$, the equilibrium condition $\pi \geq \psi$, together with the previous arguments, implies $\frac{\mu_{(0,1)}}{k}(\alpha - 1)\gamma > \pi \geq \psi \geq \frac{\mu_1}{n+1}(\alpha - 1)\gamma$, and thus $\frac{\lambda}{1-\lambda} = \frac{\mu_1}{\mu_{(0,1)}} < \frac{n+1}{k}$. Because of $n + k = N$, this gives the proposition's inequality $n > \lambda(N + 1) - 1$ for the case where $k \geq 1$ (and thus $1 \leq n \leq N - 1$) and $p(1) = \hat{p}(1)$.

Consider next the case $n = 0$, i.e., each brand name in the market offers some intermediate quality level $v \in (0, 1)$. In this case the average (and thus the minimal) profit per period is strictly less than $\frac{\mu_{(0,1)}+\mu_1}{N}(\alpha - 1)\gamma$. Since a brand name can always offer quality $v = 1$ for the price $p(1) = \hat{p}(1)$ and receive a profit per period of at least $\mu_1(\alpha - 1)\gamma$, it must hold that $\mu_1 < \frac{\mu_{(0,1)}+\mu_1}{N}$ and thus $\frac{1}{N} > \frac{\mu_1}{\mu_{(0,1)}+\mu_1} = \lambda$, hence $\lambda N < 1$. Consequently, the proposition's assumption $N \geq \frac{1}{\lambda} + 1$ implies that at least one firm offers maximum quality $v = 1$. Consider now the case where out of N brand names in the market $N - 1$ offer some intermediate quality levels $v \in (0, 1)$ and one offers maximum quality $v = 1$. We have to distinguish the subcase where the firm that offers $v = 1$ charges a price $p(1) = \hat{p}(1)$ from the subcase where it charges a price $p(1) > \hat{p}(1)$. The subcase $p(1) = \hat{p}(1)$ has already been dealt with above and we have shown that in this subcase the proposition holds. Consider now the subcase $p(1) > \hat{p}(1)$. In this subcase the average (and thus the minimal) profit per period of the $N - 1$ firms is strictly less than $\frac{\mu_{(0,1)}+\mu_1}{N-1}(\alpha - 1)\gamma$. If a brand name switches from intermediate quality to maximum quality and charges the price $\hat{p}(1)$, it gets all the customers buying $v = 1$ and thus receives a profit per period of at least $\mu_1(\alpha - 1)\gamma$. Since this must not increase the respective firm's payoff, $\mu_1 < \frac{\mu_{(0,1)}+\mu_1}{N-1}$. Therefore, $(N - 1)\lambda < 1$, a contradiction to the proposition's assumption $N \geq \frac{1}{\lambda} + 1$. This contradiction eliminates the subcase $p(1) > \hat{p}(1)$ and implies $p(1) = \hat{p}(1)$. Summing up, it follows that $n > \lambda(N + 1) - 1$ whenever $n \leq N - 1$. For $n = N$ that inequality is trivially satisfied for $\lambda < 1$. If $\lambda = 1$ (and $N \geq 2$), $n = N$ follows because every $n \leq N - 1$ leads to the contradiction $n > \lambda(N + 1) - 1 = N \geq n$. Hence for every possible case that is compatible with $N \geq \frac{1}{\lambda} + 1$ it must hold (i) that $n > \lambda(N + 1) - 1$

if $\lambda < 1$ and $n = N$ if $\lambda = 1$, and (ii) that $p(1) = \hat{p}(1)$.

Finally, assume $N \in \left[3, \frac{\lambda}{1-\lambda}\right]$. This implies $\frac{\lambda}{1-\lambda} \geq 3$ and thus $\lambda \geq \frac{3}{4}$. First we show that at least one brand name offers quality $v = 1$. If no brand name offers quality $v = 1$, brand names' average profit per period is less than $\frac{\mu_{(0,1)}+\mu_1}{N}(\alpha - 1)\gamma \leq \frac{\mu_{(0,1)}+\mu_1}{3}(\alpha - 1)\gamma = \frac{1}{3}\frac{\mu_1}{\lambda}(\alpha - 1)\gamma \leq \frac{4}{9}\mu_1(\alpha - 1)\gamma < \mu_1(\alpha - 1)\gamma$ because $N \geq 3$ and $\lambda \geq \frac{3}{4}$. Thus, at least one brand name can do better by offering $v = 1$ for the price $p(1) = \hat{p}(1)$. Hence in a stationary equilibrium at least one brand names offer $v = 1$. If $p(1) = \hat{p}(1)$, total demand for intermediate quality levels is bounded by $\mu_{(0,1)}$. Thus, if $p(1) = \hat{p}(1)$ and $k \in \{1, ..., N-1\}$, firms offer intermediate quality levels, their average profit per period is less than $\frac{\mu_{(0,1)}}{k}(\alpha - 1)\gamma \leq \frac{\mu_1}{N}(\alpha - 1)\gamma \leq \frac{\mu_1}{N-k+1}(\alpha - 1)\gamma$, where the first inequality holds because $N \leq \frac{\lambda}{1-\lambda} = \frac{\mu_1}{\mu_{(0,1)}}$ gives $\mu_{(0,1)} \leq \frac{\mu_1}{N}$. Consequently, a firm with the minimum profit among the $k \in \{1, ..., N-1\}$, firms offering intermediate quality levels can increase its payoff, if it offers $v = 1$ for the price $p(1) = \hat{p}(1)$. Therefore, whenever a brand name offers $v = 1$ for the price $p(1) = \hat{p}(1)$, this must hold for all N brand names in the market. Proposition 4 follows, if we can exclude that $n = 1$ brand name offers $v = 1$ for some price $p(1) > \hat{p}(1)$ and the $k = N - 1$ remaining brand names all offer intermediate quality levels. Because $N \geq 3$ and $\mu_{(0,1)} + \mu_1 = \frac{\mu_1}{\lambda} \leq \frac{4}{3}\mu_1$ (since $\lambda \geq \frac{3}{4}$), the average profit per period of the latter $k = N - 1$ remaining brand names is less than $\frac{\mu_{(0,1)}+\mu_1}{N-1}(\alpha - 1)\gamma \leq \frac{1}{2}\frac{4}{3}\mu_1(\alpha - 1)\gamma < \mu_1(\alpha - 1)\gamma$. Since $p(1) > \hat{p}(1)$, a brand name that switches from $v \in (0,1)$ to $v = 1$ and charges the price $\hat{p}(1)$ gets all the demand for $v = 1$, which is at least μ_1, and thus achieves a profit per period of at least $\mu_1(\alpha - 1)\gamma$. The previous inequality shows that this is more than a brand name with the minimal profit per period among those offering intermediate quality levels achieves. Hence it cannot be a stationary equilibrium that $n = 1$ brand name offers $v = 1$ for the price $p(1) > \hat{p}(1)$ and the $k = N - 1$ remaining brand names all offer intermediate quality levels. Thus, for $N \in \left[3, \frac{\lambda}{1-\lambda}\right]$ we have excluded all cases where some brand name offers some intermediate quality level as an outcome of a stationary equilibrium. Since $N \geq 3$, $p(1) = \hat{p}(1)$ follows. ■

Proof of Proposition 5: First we show that it is an equilibrium that there are $N \geq 2$ brand names in the market, all of which offer quality $v = 1$. Recall that $R_v(\cdot,\cdot) > \gamma$ for all $v \in (0,1)$ implies $p(v) = \hat{p}(v)$ for all $v \in (0,1)$ and thus a profit per customer of $(\alpha - 1)\gamma v$. Since by assumption $N \geq 2$, Bertrand competition implies $p(1) = \hat{p}(1)$. Aggregate demand for quality $v = 1$ is $\bar{\mu}$ and each brand name in the market has a profit per period of $\frac{\bar{\mu}}{N}(\alpha - 1)\gamma$. Since $\eta\rho \leq \frac{\bar{\mu}}{N}(\alpha - 1)\gamma$, each brand name in the market has a non-negative payoff $\frac{\bar{\mu}}{\rho N}(\alpha-1)\gamma-\eta\geq 0$ and thus is at least not worse off than if it stayed out of the market. Consider a firm that deviates from the strategy not to enter the market and enters as a brand name. The profit per period it gets is $\frac{\bar{\mu}}{N+1}(\alpha-1)\gamma$, if it offers quality $v=1$, and is less than $\mu_{(0,1)}(\alpha-1)\gamma$, if it offers some intermediate quality $v\in(0,1)$. The inequalities $\frac{\bar{\mu}}{N+1}(\alpha-1)\gamma\leq\eta\rho$ and $\mu_{(0,1)}(\alpha-1)\gamma\leq\eta\rho$ imply that the resulting payoff is non-positive in each case. Thus, the deviation does not increase the payoff. Finally, if a brand name in the market deviates to some quality $v\in[0,1)$, its profit per period is less than $\mu_{(0,1)}(\alpha-1)\gamma$, and since $\mu_{(0,1)}(\alpha-1)\gamma\leq\eta\rho\leq\frac{\bar{\mu}}{N}(\alpha-1)\gamma$ the deviation decreases its payoff. This proves the first part of the proposition.

Consider now the case where $N\geq 3$. By construction, $\bar{\mu}\leq\mu_1+\mu_{(0,1)}$. From the propositions's two conditions follows $\mu_{(0,1)}(\alpha-1)\gamma\leq\eta\rho\leq\frac{\bar{\mu}}{N}(\alpha-1)\gamma$, which implies $\mu_{(0,1)}\leq\frac{\bar{\mu}}{N}$. Therefore, $\mu_1+\mu_{(0,1)}\leq\mu_1+\frac{\bar{\mu}}{N}\leq\mu_1+\frac{\mu_1+\mu_{(0,1)}}{N}$ and thus $\mu_1+\mu_{(0,1)}\leq\frac{N}{N-1}\mu_1$. If $k\in\{N-1,N\}$ brand names offer intermediate quality levels, their average profit per period is less than $\frac{\mu_1+\mu_{(0,1)}}{k}(\alpha-1)\gamma\leq\frac{N}{2(N-1)}\mu_1(\alpha-1)\gamma<\mu_1(\alpha-1)\gamma$ because $N\geq 3$ implies $k\geq 2$ (given $k\geq N-1$) and $\frac{N}{2(N-1)}<1$. Whenever either no firm offers maximum quality ($k=N$) or one firm offers maximum quality ($k=N-1$) at a price $p(1)>\hat{p}(1)$, a brand name that switches from intermediate to maximum quality and charges the price $\hat{p}(1)$ gets all the demand for $v=1$ and thus achieves a profit per period that is at least $\mu_1(\alpha-1)\gamma>\frac{\mu_{(0,1)}+\mu_1}{k}(\alpha-1)\gamma$. Consequently, it cannot be a stationary equilibrium that k brand names offer intermediate quality levels and either $k=N-1$ and $p(1)>\hat{p}(1)$ or $k=N$. To conclude the proof we have to consider two remaining cases: (i) $k\in\{1,...,N-2\}$ brand names offer intermediate quality levels, which implies that $n=N-k\geq 2$ brand names offer maximum quality and thus $p(1)=\hat{p}(1)$; (ii) $k=N-1$ brand names offer intermediate quality levels and the single brand name that offers maximum quality charges the price $p(1)=\hat{p}(1)$. In both cases it holds that $p(1)=\hat{p}(1)$, and thus that the average profit per period of the brand names that offer some intermediate quality $v\in(0,1)$ is less than $\frac{\mu_{(0,1)}}{k}(\alpha-1)\gamma\leq\mu_{(0,1)}(\alpha-1)\gamma\leq\eta\rho$. Therefore, at least one brand name that offers intermediate quality cannot cover its entry cost and thus this scenario cannot be the outcome of a a stationary equilibrium. ■

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