Consumption Risk and the Cross-Section of Government Bond Returns*

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February 2010

Abstract

We use a consumption-based asset pricing model with Epstein-Zin-Weil recursive preferences to explain the cross-section of excess returns on nominal US Treasury bond portfolios. Our model has two factors: innovations to current consumption growth and innovations to expected future consumption growth. We find that, over the period 1975-2006, nominal government bonds are risky assets that pay off in good times characterized by good prospects for future consumption growth. The model explains well the cross-sectional variation in mean excess bond returns and provides plausible estimates of the structural parameter. Our results are robust to using alternate test assets and sample period, different definitions of consumption and estimation methods.

JEL classification: G0, G10, G12
Keywords: Epstein-Zin preferences, consumption risk, asset pricing tests, government bonds, dynamic factor analysis

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*We would like to thank David Babbel, Laurence Copeland, Antonio Diez de los Rios, Kabir Dutta, Javier Gil-Bazo, Lynda Khalaf, Chung-Ming Kuan, Patrick Minford, Francisco Penaranda, Jesper Rangvid, seminar participants at the Universities of Aarhus, Autonoma de Barcelona, Cardiff, Carlos III de Madrid, Essex, National Central University, National Taiwan University, Pompeu Fabra, Reading and the participants at the 2009 Warsaw International Economic Meeting, 2009 Econometric Society European Meeting Barcelona, 2009 ASSET Istanbul, XVII Foro Finanzas Madrid, XXXIV SAEe Valencia, 5th PhD Meeting of RES London for helpful discussions and comments.

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1 Introduction

We investigate, using a consumption-based capital asset pricing model (C-CAPM) with Epstein-Zin-Weil recursive utility, the cross-section of excess returns on portfolios of US Treasury bonds with varying times to maturity. Specifically, we ask the following question: do investments in nominal government bonds pay off in good times and therefore are risky assets that investors need an inducement to hold or do they pay off in bad times and may help to hedge macroeconomic risk. More generally, we add to the literature on consumption-based models for pricing bonds that is "surprisingly small, given the vast amount of attention given to consumption-based models of equity pricing"[1].

The evaluation of risks in nominal government bonds has recently attracted considerable attention. Campbell, Sunderam and Viceira (2009) point out that this can be done in at least three ways. We could, for example, decompose realized excess bond returns into components arising from shocks due to movements in real rates, expected inflation and future expected excess bond returns. Measures of the covariances of these components with some proxy for the marginal utility of investors can then be used to determine the riskiness of nominal government bonds. Another approach would be to estimate a dynamic model of the term structure to understand the evolution of bond market risks. However, this requires a specification of particular stochastic processes for, at a minimum, the real interest rate, expected inflation and investor risk aversion. Finally a direct approach is to measure the covariance of bond returns with a proxy for the marginal utility of the consumers like the return on market portfolio (as in the classical CAPM) or the aggregate consumption growth (as in the C-CAPM). Indeed early attempts to evaluate the risks of nominal bonds followed this approach (see for example Gultekin and Rogalski, 1985). More recently, Viceira (2010) finds that the consumption beta for bonds is negative, over the 1980 and 1990s, suggesting that nominal bonds help investors hedge aggregate market risks. Our work is in this spirit and builds on prior but differs in two important respects. First, we use a consumption CAPM with Epstein-Zin-Weil utility rather than the standard power utility C-CAPM. This allows us to focus on the covariance between government bond returns and innovations to both current and expected future consumption growth. Second, we study a cross-section of ten government bond portfolios of varying times to maturity rather than a single index of government bonds.

Our C-CAPM has two factors: innovations to current consumption growth and innovations to expected future consumption growth. These factors are usually estimated using Vector Autoregressive (VAR) models[2] where specific state variables are selected that are known to forecast consumption growth well. Our implementation of this methodology is however novel. Instead of choosing specific predictor variables we use a set of dynamic factors obtained, following Stock and Watson (2002a,b and 2005), from a large panel of macroeconomic and financial time series. We then estimate a factor-augmented VAR, in the spirit of Bernanke, Boivin and Eliasz (2005), and extract shocks to current and expected future consumption growth. This approach has some advantages. First, we can be agnostic in our choice of state variables thus mitigating to

[1] Campbell (2007)

[2] Brunnermeier and Julliard (2007), Campbell and Vuolteenaho (2004), Lustig and Nieuverburgh (2008).

some extent concerns about the choice of specific state variables (see for example Chen and Zhao, 2008). Second, there is evidence (see for example Stock and Watson, 2006) that dynamic factors have good forecasting properties even in the presence of structural breaks. Third, the pre-estimation of the dynamic factors does not affect the consistency of OLS estimates in the VAR model, which is important in our application (see for example Bai and Ng, 2008). Our test assets are bond portfolios that are constructed using US Treasury bonds with times to maturity ranging from over a year to longer than 10 years. The sample period is 1975–2006. We use standard Fama-MacBeth cross-sectional regressions to study how well our two-factor C-CAPM explains the cross-section of average excess returns on government bonds. We also estimate, using GMM, the stochastic discount factor representation of the model to study the relative importance of the model factors.

Our main results can be summarized as follows. We find that a C-CAPM with current and expected future consumption growth shocks as factors explains well the cross-section of average excess returns on portfolios with US Treasury bonds of differing maturities (around 95% of the cross-sectional variation). Further, we find that the innovations in current consumption growth do not play a role in pricing the cross-section of mean excess nominal government bond returns reflecting the failure of the standard power utility C-CAPM. On the other hand, the risk premium related to news in expected future consumption growth is positive and significant. In other words, the risk related to prospects of future consumption matter in pricing government bonds and induce a risk premium. We also find that bond portfolios with long maturity bonds are riskier in terms of higher and positive consumption betas compared to the portfolios with short term bonds. Finally, we obtain realistic estimates of the structural parameter of our model (the coefficient of relative risk aversion) which provide strong evidence for the fit of the model over and above that seen in the statistical tests. Our results are robust to a battery of tests: use of an alternate test assets and sample period, alternate measures of consumption growth and estimation methods. Overall our results suggest that investors must be rewarded to hold government bond portfolios that are risky rather than safe assets whose price movements matter and which may not be useful in hedging against other risks.

The rest of the paper is organized as follows. Section 2 provides an overview of related research while Section 3 provides details of our model. Section 5 outlines key features of the methodology used and Section 4 describes the data. We discuss our empirical results and tests for robustness in Section 6. Section 7 concludes the paper. The Appendix provides details on additional tests including tests for robustness and the details of data sets used in the paper.

2 Related Literature

Expositions of the canonical consumption-based asset pricing model for equities are now standard textbook material but applications in the context of bonds are not common; Wolman (2006) is a recent example of a pedagogic guide to the consumption-based modelling of bonds. He derives expressions for the yield differential as well as the holding period return from buying a bond and selling it at a future date. We note here however that while the literature on empirical tests of the C-CAPM for equities is vast, there is surprisingly little empirical research on the cross-section of nominal government bond returns.

Gultekin and Rogalski (1985) are possibly the first to use constant maturity bond portfolios, over the 1960-1979 period, from the CRSP data base as test assets. They study how well Ross's APT model and the CAPM can price the cross-section of twelve US government bond portfolios. They find that, in the case of the APT, mean returns on bond portfolios are explained by at least two "priced" factors obtained using factor analysis on the data. Further, using tests for the CAPM available at that time, they find that estimates of the factor risk premia on the market portfolio are all negative but not significantly different from zero. They conclude that "... [their] tests should be viewed simply as the first empirical attempt to properly measure interest-rate risk for bonds using factor-generating models. Our results in terms of the existence of priced risk premia are more favorable to multifactor models than to single-factor models or the CAPM". In a related study using corporate bond portfolios, Chang and Huang (1990) observe that the focus, in the literature[3], on stocks rather than bonds may be due " [to] the lack of convincing empirical evidence…show[ing] that covariance risks are priced in bond markets". In recent work, in a similar vein, Viceira (2010) finds considerable time variation, persistence and mean reversion in bond market betas over the 1962-2003 period using a CAPM. He also finds that the consumption beta for a 5-year maturity bond index is negative over this period, suggesting that nominal bonds help investors hedge aggregate consumption risk. Our work builds on this prior work but differs from it in two important respects. First, we use a C-CAPM with Epstein-Zin-Weil utility rather than the standard power utility C-CAPM. We are able to study the covariance between government bond returns and innovations to both current and expected future consumption growth and returns. In addition we use ten portfolios of government bonds with varying times to maturity, rather than a single bond index, to examine the role of uncertainty about future consumption prospects on bond portfolios with times to maturity varying from a year to more than ten years.

We note here that there is a huge related literature on modelling the term structure of interest rates (see Piazzesi, 2009 for a recent survey). This literature assumes that: the price of bonds is driven by a continuous time stochastic process, there are continuous trading opportunities and the principle of "no arbitrage" holds. One can then obtain (as in Vasicek, 1977 for example) equilibrium relationships between prices of bonds of different maturities. Early models like Vasicek assume that there is only one source of uncertainty - the current level of the short rate. However, more recent work assumes that bond prices are driven by multiple state variables. In these multifactor models the factors are assumed to be latent variables - identified by data from yields of different maturity bonds. The factors are further assumed to be "affine" functions of the state variables which have the convenient property that yields are maturity-dependent linear functions of state variables (see for example Duffie and Kan, 1996). There have been attempts however to link these factors to observed macroeconomic variables (see Ang and Piazzesi, 2003). Piazzesi and Schneider (2006), for example, consider a representative agent model with recursive utility and solve it for average yields. Gallmeyer, Hollifield,

[3]A more recent example is Gebhardt, Hvidkjaer, Swaminathan (2005).

Palomino, and Zin (2007) also demonstrate how the literature on affine models can be linked with a structural equilibrium model of investors' preferences and opportunities using the Epstein-Zin-Weil utility framework. In contrast to this work, our paper, in the spirit of Gultekin and Rogalski (1985), studies the covariance risk or correlation of consumption growth with government bond returns using a linear factor pricing framework. This has been done in the case of corporate bonds (see for example Gebhardt, Hvidkjaer, Swaminathan, 2005). The role of linear factor models has been studied in a similar manner in the case of futures markets (see for example Khan, Khoker and Simin, 2007) and for options markets (Coval and Shumway, 2001). Estimation of linear factor models for bond portfolios is of practical relevance. For example, in 2008 out of US $10,349 billion invested in all mutual funds about US $1,552 billion (15%) was in funds that invested exclusively in US government and related securities. The performance appraisal of these funds is closely linked to specifications of appropriate asset pricing models that will enable the identification of differential performance for investors[4].

Our work is also related to the literature that examines whether stock returns are priced by their exposure to consumption risk measured over different horizons. For example, Daniel and Marshall (1997) find that the performance of a standard power utility C-CAPM improves if they use covariances with consumption growth at the two-year horizon. In recent work, Parker and Juilliard (2005) and Jagannathan and Wang (2007) also find that the power utility C-CAPM explains the cross-section of expected stock returns better when risk is measured by the covariance of an asset's return and consumption growth at longer horizons. In this paper, in contrast, we estimate a measure of the innovation in expectations about the present value of future consumption growth rates. The "long-run" measures of consumption growth used, for example, in Parker and Julliard (2005) can be regarded as the truncation of such an infinite series at economically sensible horizons. This point is also made by Malloy, Moskowitz and Vissing-Jorgensen (2009).

Finally, our work intersects with the burgeoning literature on long-run consumption risks in the C-CAPM using the framework of Epstein-Zin-Weil utility[5]. Malloy, Moskowitz and Vissing-Jorgensen (2009) also study the role of long-run consumption risks for stockholders and non-stockholders using the Consumer Expenditure Survey (CEX) data. They find that the C-CAPM with covariance of excess equity returns with long-run consumption growth rates in the case of households who own stocks provides a better fit and plausible estimates of the coefficient of risk aversion. The model and approach used in this paper is similar to Malloy, Moskowitz and Vissing-Jorgensen (2009). Finally, Tedongap (2007) also uses equity market data to test a model with two factors: expected changes in consumption growth and the volatility of consumption growth. He finds that for equity portfolios the consumption growth factor is positive while that for the consumption volatility factor is negative. In contrast, we find that the variance of future expected consumption growth is not an important factor in pricing the cross-section of nominal returns on portfolios of US government bonds of varying times to maturity.

[4]Data from the Investment Company Institute (2009 Investment Company Institute Factbook, Investment Company Institute).

[5]See Bansal (2007) for an accessible review.

3 Consumption CAPM with Epstein-Zin-Weil Preferences

We consider a representative agent in an endowment economy which has Epstein-Zin-Weil preferences over the consumption stream. In equilibrium asset prices are such that it is optimal for the agent to consume the endowment. The preferences are represented by the recursive utility function of Epstein and Zin (1989,1991) and Weil (1989) of the form:

$$V_t = \{(1-\beta)C_t^{1-\rho} + \beta[E_t(V_{t+1}^{1-\gamma})]^{\frac{1-\rho}{1-\gamma}}\}^{\frac{1}{1-\rho}} \tag{1}$$

where V_{t+1} is the continuation value, γ is the coefficient of relative risk aversion, $\frac{1}{\rho}$ represents the intertemporal elasticity of substitution (IES) and β is the time discount rate. When $\gamma = \frac{1}{\rho}$, the expression in Eq.(1) collapses to the familiar power utility function. With Epstein-Zin-Weil recursive utility the stochastic discount factor M_{t+1} is given by:

$$M_{t+1} = \beta(\frac{C_{t+1}}{C_t})^{-\gamma}(\frac{V_{t+1}}{E_t(V_{t+1}^{1-\gamma})^{\frac{1}{1-\gamma}}})^{\rho-\gamma} \tag{2}$$

There are two contributions in the stochastic discount factor. One is a simple consumption growth $\frac{C_{t+1}}{C_t}$. The other one is related to the continuation value V_{t+1} which represents future utility. This is a forward looking term linked to the future consumption via the recursion (1) and it is present only when the coefficients of intertemporal elasticity of substitution and risk aversion differ. In order to implement empirically this recursive preference framework we follow the structural approach of Hansen, Heaton and Li (2005). They solve exactly for the continuation value V_{t+1} assuming the following stochastic process for consumption growth:

$$\Delta c_{t+1} = \mu_c + \alpha(L)w_{t+1} = \mu_c + (\sum_{s=0}^{\infty} \alpha_s L^s)w_{t+1} = \mu_c + \sum_{s=0}^{\infty} \alpha_s w_{t+1-s} \tag{3}$$

where $\Delta c_{t+1} \equiv \log(\frac{C_{t+1}}{C_t})$ and $\{w_{t+1}\}$ is an *iid* standard normal process. They fix as well the intertemporal elasticity of substitution to one since it allows a closed form solution for the model. In our case, as also in Malloy, Moskowitz and Vissing-Jorgensen (2009), we are interested in studying the cross-section of expected returns, where the value of the IES has little effect.

Hansen, Heaton and Li (2005) then show that the expression for the log of the stochastic discount factor (SDF) is given by:

$$\begin{aligned} m_{t+1} &= -\ln\beta - \Delta c_{t+1} - (\gamma-1)(\sum_{s=0}^{\infty}\alpha_s\beta^s)w_{t+1} - \frac{1}{2}(\gamma-1)^2(\sum_{s=0}^{\infty}\alpha_s\beta^s)^2 \\ &= -\ln\beta - \Delta c_{t+1} - (\gamma-1)\sum_{j=0}^{\infty}\beta^j(E_{t+1}-E_t)\Delta c_{t+1+j} \\ &\quad - \frac{1}{2}(\gamma-1)^2 var_t[\sum_{j=0}^{\infty}\beta^j(E_{t+1}-E_t)\Delta c_{t+1+j}] \end{aligned} \tag{4}$$

Ignoring the constant, there are two components in the above expression that are important in an asset pricing context. The first is the current consumption growth Δc_{t+1} and this is what is captured in the classical power utility C-CAPM. For conditional pricing, what matters is innovation to current consumption growth, i.e., $\Delta c_{t+1} - E_t(\Delta c_{t+1})$, and we refer to this pricing factor as the current consumption growth shock. The second term $(\sum_{s=0}^{\infty} \alpha_s \beta^s) w_{t+1}$ equals:

$$\sum_{j=0}^{\infty} \beta^j (E_{t+1} - E_t) \Delta c_{t+1+j} \tag{5}$$

and represents the innovation to expectations about the present value of consumption growth in all future periods. In other words, this term reflects the change in the expectations about future consumption growth. We refer to this forward-looking term as the expected future consumption growth shock. The last term in Eq.(4) is the variance of this innovation. In the Hansen, Heaton and Li (2005) specification this variance is time-invariant so it does not influence the asset's excess returns. Some research (see Tedongap, 2007, for example) investigates the importance of this volatility of innovations to expected future consumption growth and find that it is a priced factor. In our data however, we find, as by predicted by the theory, that this factor does not contribute to explaining the cross-section of excess bond returns. The results for this estimation are available in the Appendix.

Given the form of the SDF we can obtain the innovation to the log SDF:

$$m_{t+1} - E_t(m_{t+1}) = -\eta_{c,t+1} - (\gamma - 1)\varepsilon_{c,t+1} \tag{6}$$

where

$$\eta_{c,t+1} \equiv \Delta c_{t+1} - E_t(\Delta c_{t+1})$$
$$\varepsilon_{c,t+1} \equiv \sum_{j=0}^{\infty} \beta^j (E_{t+1} - E_t) \Delta c_{t+1+j}$$

The Euler equation implies that the expected excess return on any asset i is given by:

$$E_t(r_{i,t+1} - r_{f,t+1}) + \frac{1}{2}\sigma_i^2 = -Cov_t(m_{t+1} - E_t(m_{t+1}), r_{i,t+1}) \tag{7}$$

We can now write our two-factor model for the risk premium on an asset i as follows:

$$E_t(r_{i,t+1} - r_{f,t+1}) + \frac{1}{2}\sigma_i^2 = Cov_t(\eta_{c,t+1}, r_{i,t+1}) + (\gamma - 1)Cov_t(\varepsilon_{c,t+1}, r_{i,t+1})^6 \tag{8}$$

The unconditional expected return-beta form is then the following:

[6] The model to test empirically when volatility of innovations to expected future consumption growth matter is: $E_t(r_{i,t+1} - r_{f,t+1}) + \frac{1}{2}\sigma_i^2 = Cov_t(\eta_{c,t+1}, r_{i,t+1}) + (\gamma - 1)Cov_t(\varepsilon_{c,t+1}, r_{i,t+1}) + \frac{1}{2}(\gamma - 1)^2 Cov_t(Var(\varepsilon_{c,t+1}), r_{i,t+1})$. The results for this model are available in the Appendix 6.

$$E_t(r_{i,t+1} - r_{f,t+1}) + \frac{1}{2}\sigma_i^2 = \lambda_\eta \beta_{i,\eta} + \lambda_\varepsilon \beta_{i,\varepsilon} \tag{9}$$

where betas reflect different types of risk and are defined in a usual way:

$$\begin{aligned} \beta_{i,\eta} &= \frac{Cov(\eta_{c,t+1}, r_{i,t+1})}{\sigma_\eta^2} \\ \beta_{i,\varepsilon} &= \frac{Cov(\varepsilon_{c,t+1}, r_{i,t+1})}{\sigma_\varepsilon^2} \end{aligned} \tag{10}$$

and the loadings on betas are:

$$\begin{aligned} \lambda_\eta &= \sigma_\eta^2 \\ \lambda_\varepsilon &= (\gamma - 1)\sigma_\varepsilon^2 \end{aligned} \tag{11}$$

These loadings represent the factor risk premia and are linked to the structural parameter γ of the model. We use this relation to back-out the value of γ from the estimated parameters. The magnitude of the implied risk aversion coefficient γ, obtained from the above relation can be used as a important metric to assess the success of the model apart from the purely statistical tests.

4 Methodology

We proceed with our empirical analysis in two steps. We first estimate the factors[7] in our linearized C-CAPM; innovations to current and future expected consumption growth using a factor-augmented VAR. The dynamic factors used in the VAR are obtained from a large panel of macroeconomic and financial time series using principal component analysis. Next, we estimate the expected return-beta representation of our model using a standard Fama-MacBeth procedure and OLS cross-sectional regressions using GMM. We also assess the relative importance of the model factors by estimating, using GMM, the stochastic discount factor representation of the model. Additionally we evaluate the goodness of fit of our model using the test for the model R^2 if Kan, Robotti and Schanken (2009).

4.1 Estimation of the Factor-Augmented VAR

We use a VAR methodology to estimate the model factors: innovations to current and expected future consumption growth. This approach allows us to incorporate estimates of conditional expectations and compute the expressions in Eq. (5) such as the present value of expected future consumption growth. A cost to this approach is that there is an element of estimation error in the VAR parameter estimates due to misspecification of

[7]The term factors here refers to the "factors" in the linearized CCAPM – unfortunately the term factor is also used in the literature (see, for example, Bai and Ng, 2008) for the "dynamic factors" extracted using principal components from a large panel of macroeconomic and financial time series. However the context should make clear which "factors" we are referring to.

the predictive or state variables used in the VAR. However, as we argue later, the issue of misspecification and estimation errors is mitigated with our use of dynamic factors as state variables in the VAR. A benefit of using a VAR, however is that we can extract a forward looking measure of "expected future consumption growth" as emphasized by Malloy, Moskowitz and Vissing-Jorgensen (2009). This differs from the approach in for example, Parker and Julliard (2005) and Jagannathan and Wang (2007), where the covariance between excess returns and consumption growth is truncated at specific lag lengths. Finally, the VAR methodology allows us to investigate empirically if the variance of the innovations to expected future consumption growth has a role in pricing the cross-section of government bond returns.

We now provide, in brief, the method used to extract the model factors using a factor-augmented VAR. Let Z_t denote a vector which has log consumption growth $g_{c,t}$ as its first element and the other elements x_t are a set of K state variables. Let this vector follows the process:

$$Z_{t+1} = AZ_t + v_{t+1} \tag{12}$$

where $v_{t+1} = \begin{bmatrix} \eta_{c,t+1} & \varepsilon_{c,t+1} \end{bmatrix}'$. Specifically, we assume the following dynamics for Δc_t and x_t:

$$\begin{bmatrix} \Delta c_{t+1} \\ x_{t+1} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \Delta c_t \\ x_t \end{bmatrix} + \begin{bmatrix} \eta_{c,t+1} \\ \varepsilon_{c,t+1} \end{bmatrix} \tag{13}$$

Let $e1$ be a vector with the first element equal to 1 and all others equal to zero. Using $e1$ we can now write consumption growth in terms of the elements of the VAR, i.e., $\Delta c_t = e1'Z_t$. We earlier defined two quantities: the current consumption growth shock,

$$\eta_{c,t+1} = \Delta c_{t+1} - E_t(\Delta c_{t+1}) \tag{14}$$

and expected future consumption growth shock,

$$\varepsilon_{c,t+1} = \sum_{j=0}^{\infty} \beta^j (E_{t+1} - E_t)\Delta c_{t+1+j} \tag{15}$$

where β is the time discount parameter. Once we estimate the VAR with a suitable set of predictor variables x_t, we can easily compute the shocks using the following expressions:

$$\begin{aligned} \eta_{c,t+1} &= e1'v_{t+1} \\ \varepsilon_{c,t+1} &= e1'\beta A(I - \beta A)^{-1} v_{t+1} \end{aligned} \tag{16}$$

In most applications the VAR is estimated using a specific set of state (or predictive) variables. For example, Campbell and Vuolteenaho (2004) use the term yield spread, the PE ratio and the small-stock value-spread to extract shocks or news about changes expected cash flows and discount rates. These series are then used as pricing factors in a two-factor I-CAPM to explain the difference between value and growth portfolios. Campbell and Vuolteenaho (2004) note that their results are sensitive to the inclusion of certain specific state variables and this point is further elaborated by Chen and Zhao (2008).

In contrast, in this paper we use, instead of specific predictive variables, dynamic factors in a VAR in the spirit of Bernanke, Boivin, and Eliasz (2005). These dynamic factors obtained from a large panel of macroeconomic and financial series are, as Ludvigson and Ng (2009) point out, likely to contribute to the forming of investor's expectations since they reflect a common set of underlying fundamentals. Briefly put, in Eq.(13) x_t is now our subset of dynamic factors, i.e., $x_t = \bar{F}_t$.

We now provide a brief background to the estimation of these dynamic factors and refer the reader to the cited papers for fuller details. Let us suppose that we have a panel of macroeconomic and financial data of dimension $(T \times N)$ where the elements are denoted as x_{it}, $i = 1, ..., N$, $t = 1, ..., T$. We assume that the cross-sectional dimension, N, is large, and could possibly be larger than the number of time periods, T. We also assume that x_{it} has a factor structure of the form $x_{it} = \lambda_i' f_t + e_{it}$, where f_t is a $(r \times 1)$ vector of latent common factors, λ_i is a corresponding $(r \times 1)$ vector of latent factor loadings, and e_{it} is a vector of idiosyncratic errors. These common factors are not observed and are estimated by the method of asymptotic principal components. Let Λ be an $(N \times r)$ matrix defined as $\Lambda \equiv \begin{bmatrix} \lambda_1' & ... & \lambda_N' \end{bmatrix}$. The estimated time t factors $\hat{f}_t$ are linear combinations of each element of the $(N \times r)$ vector $x_t = \begin{bmatrix} x_{1t} & ... & x_{Nt} \end{bmatrix}'$, where the linear combination is chosen optimally to minimize the sum of squared residuals $(x_{it} - \Lambda \hat{f}_t)^2$. The number of dynamic factors chosen is based on the information criteria in Bai and Ng (2002).

Further, in our application we focus on a subset of the dynamic factors that have predictive power for future consumption growth. We do this following the procedure detailed in Ludvigson and Ng (2009). Briefly, let F_{kt} be subsets of estimated dynamic factors, where k indicates the number of factors included in a subset, with $k = 1, ..., r$. For example, F_{1t} can have one of r possible factors, $\hat{f}_{1t}, ..., \hat{f}_{rt}$. The composition F_{kt} of is determined by evaluating the Bayesian information criterion (BIC) and the Akaike information criterion (AIC) from the following regression of on F_{kt}:

$$\Delta c_{t+1} = \gamma' F_{kt} + e_{t+1} \tag{17}$$

where γ is the parameter vector on the subset of factors and e_{t+1} is the disturbance at $t+1$. Once the composition of factors that minimizes the selection criteria is identified, we add another factor from a pool of $(r - k)$ remaining factors F_{kt} to in order to expand the subset to $F_{(k+1)t}$. This procedure is continued until the subset includes all estimated factors, i.e.,F_{rt}. Having identified the subsets, we select from them the one $\bar{F}_t$ that minimizes either or both the BIC and AIC from regression specified in Eq.(17).

Our use of dynamic factors[8] in this context is novel and has a number of advantages. First, we can avoid having to choose specific individual predictive variables and instead use the dynamic factors as state variable in the VAR. Second, as Bai and Ng (2007) show, under the assumption that both $N, T \longrightarrow \infty$ while $\frac{\sqrt{T}}{N} \longrightarrow \infty$, the coefficients obtained

[8]There is a growing literature using dynamic factor analysis in a VAR to study the macroeconomic effects of policy interventions or patterns of co-movements in economic activity and as inputs into dynamic stochastic general equilibrium models (see, for example, Bernanke, Boivin and Eliasz, 2005 and Stock and Watson, 2005).

from ordinary least squares estimation of the regressions in Eq.(13) are $\sqrt{T}$-consistent and asymptotically normal. They also show that the asymptotic variance is such that inference can proceed as if $\hat{f}_t$'s are observed rather than estimated. In other words, the pre-estimation of the factors using principal components analysis does not affect the consistency of the OLS estimates or the standard errors in the VAR system. This is of particular relevance in our case since the VAR in Eq.(13) is estimated equation-by-equation using OLS. We also note that N needs to be large; otherwise the factor space cannot be consistently estimated even if T is very large. In our case, $N = 126$ and this is of a size similar to that used in earlier work. Third, dynamic factors are found to be robust to structural instability that plagues low-dimensional forecasting regressions (Stock and Watson, 2006). The intuition for this result is that such instabilities average out in the construction of common factors if the instability is sufficiently dissimilar from one series to the next.

We now turn to a brief description of the standard methodologies used to estimate the model parameters and test its performance.

4.2 Estimation of the Model

4.2.1 Cross-Sectional Regression

We estimate our model using the standard Fama-MacBeth (FMB hereafter) procedure. As it is well-known, this procedure has two stages. In the first stage we run the time-series regressions and estimate betas:

$$R^e_{it} = \alpha_0 + \beta_{i,\eta}\eta_{c,t} + \beta_{i,\varepsilon}\varepsilon_{c,t} + e_{it} \tag{18}$$

for $i = 1, ..., N$ and $t = 1, ..., T$. In the second stage, we run the cross-sectional regressions of excess returns on the estimated betas at each time period in the sample:

$$R^e_{it} = \hat{\beta}'\lambda_t + \alpha_{it} \tag{19}$$

The second stage of the FMB procedure results in a time series of lambda estimates, $\{\hat{\lambda}_t\}^T_{t=1}$, and time series of pricing errors, $\{\hat{\alpha}_{it} = R^e_{it} - \hat{\beta}'\hat{\lambda}_t\}^T_{t=1}$. The parameter estimates $\hat{\lambda}$ and pricing errors $\hat{\alpha}_i$ $(i = 1, ..., N)$ are then the averages of the appropriate time-series estimates: $\hat{\lambda} = E_T(\hat{\lambda}_t)$ and $\hat{\alpha}_i = E_T(\hat{\alpha}_{it})$. The coefficients of interest are λ's which represent the factor risk prices[9]. Fama and MacBeth (1973) suggest using the standard deviations of the cross-sectional regression estimates to generate the sampling errors for the parameter estimates: $cov(\hat{\lambda}) = \frac{1}{T}E_T[(\hat{\lambda}_t - \hat{\lambda})(\hat{\lambda}_t - \hat{\lambda})']$ and $cov(\hat{\alpha}) = \frac{1}{T}E_T[(\hat{\alpha}_t - \hat{\alpha})(\hat{\alpha}_t - \hat{\alpha})']$.

It is well-known that cross-sectional regressions suffer from an errors-in-variable problem since the betas used in the second-pass regression are estimates of the true unknown betas. One way to deal with this problem is to use Shanken (1992) asymptotic standard errors with a correction factor given by $Sh = (1 + \hat{\lambda}'\hat{\Sigma}^{-1}_f\hat{\lambda})$, where $\hat{\Sigma}_f = E_T\{[f_t - E_T(f_t)][f_t - E_T(f_t)]'\}$ is the sample variance-covariance matrix of the

[9] Alternatively we estimate as well the betas using univariate regressions of the form: $R^e_{it} = \beta^0_{i,\eta} + \beta^u_{i,\eta}\eta_{c,t} + e^{\eta}_{it}$ and $R^e_{it} = \beta^0_{i,\varepsilon} + \beta^u_{i,\varepsilon}\varepsilon_{c,t} + e^{\varepsilon}_{it}$. These betas, in contrast to those estimated using Eq.(18), represent the riskiness of the assets as they are directly proportional to the covariances of the pricing factors with assets' returns. The results related to univariate betas are available in the Appendix 4.

factors. The Shanken correction assumes that the returns are stationary and conditionally homoskedastic[10]. Further, we note, as shown in Jagannathan and Wang (1998), that the Fama-MacBeth procedure does not necessarily overstate the precision of the standard errors in the presence of conditional heteroskedasticity.

An elegant way to deal with the problem of generated regressors is to use a GMM framework (see, for example, Cochrane, 2005). In this approach, both time-series and cross-sectional moments are minimized simultaneously. The moments are the following:

$$g_T(\theta) = \begin{bmatrix} E(R_t^e - a - \beta f_t) \\ E[(R_t^e - a - \beta f_t) \otimes f_t] \\ E(R^e - \beta\lambda) \end{bmatrix} = \begin{bmatrix} 0_{(Nx1)} \\ 0_{(NLx1)} \\ 0_{(Nx1)} \end{bmatrix} \tag{20}$$

where $a_{(N\times 1)}$ is a vector of constants for the time-series regressions; $\beta_{(N\times L)}$is a matrix of L factor loadings for the N test assets; $\lambda_{(L\times 1)}$ is a vector of beta risk prices; $\otimes$ denotes the Kronecker product and 0 denotes conformable vectors of zeros. The parameter vector in this GMM system is $\theta' = \begin{bmatrix} a' & \beta' & \lambda \end{bmatrix}'$: a and β are identified by the first two groups of moment conditions and the cross-sectional estimates of λ are identified by the third group of moments weighted by the time-series β.

The GMM estimation of this system in Eq.(20) with the identity weighting matrix is equivalent to simple OLS cross-sectional regression or the FMB procedure in the sense that it produces the same estimates of the parameters . An advantage of using the GMM framework is in the estimation of the standard errors for the lambda coefficients. Since GMM minimizes time-series and cross-sectional moments simultaneously, the standard errors of the cross-sectional estimates are directly affected by the time-series properties of the input data. In this way the GMM standard errors account for the fact that the betas are estimated and also correct for heteroskedasticity and serial correlation in the data.

Following most of the literature, we report tests of the null hypothesis that all pricing errors $\hat{\alpha}$ are jointly zero which asymptotically follows the χ^2_{N-L} distribution, where N is the number of test assets and L is the number of parameters in the cross-sectional regression that needs to be estimated. We note that the null of zero pricing errors may not be rejected, not because of small pricing errors, but because of their high sampling error (see for example Lettau and Ludvigson, 2001b). We also report some commonly used informal criteria that help assess the goodness-of-fit of the model: the root mean square pricing error $RMSE = \sqrt{\frac{1}{N}\hat{\alpha}'\hat{\alpha}}$, the mean absolute error $MAE = \frac{1}{N}||\hat{\alpha}||$, the classical R^2, its adjusted value $R^2 - adjusted$ and plot of the actual versus the model predicted excess returns. However, these results need to be interpreted with some caution as pointed out by Lewellen, Nagel and Shanken (2008). In response to their critique of empirical methods used is asset pricing tests we also test whether the true R^2 of the model is significantly different from zero and from one: $H_0 : R^2 = 1$ and $H_0 : R^2 = 0$ vs appropriate two-sided alternatives. To do this we rely on the asymptotic distribution of the sample cross-sectional uncentered R^2 derived by Kan, Robotti and

[10]The correction is directly related to the magnitude of each coefficient and inversely related to the variability of the pricing factors. Lettau and Ludvigson (2001b) point out that macro factors are not very volatile and as a result this tends to "blow up" the Shanken correction factor so that the corresponding t-statistics are not significant.

Shanken (2009). The uncentered R^2 is an alternative measure of goodness of fit of the model and has the same interpretation as the commonly used R^2.

4.2.2 Linear Stochastic Discount Factor approach using GMM estimation

The expected return-beta representation of asset pricing models can be written in the form of a linear stochastic discount factor model. Let m denote a stochastic discount factor (SDF), represented by $m = a - bf'$, where a is chosen based on the normalization of the mean of the SDF and b is a parameter vector that indicates whether a particular factor (f) in a proposed asset pricing model is marginally useful in pricing test assets in the presence of other factors. If the portfolios are correctly priced by the proposed SDF, the pricing errors will be zero when the test assets are excess returns, i.e. $E(mR^e) = 0$. Since the mean of m cannot be identified from the zero pricing errors, we normalize it to one, i.e., $E(m) = 1$, and as a result m is specified as $m = 1 - b'[f - E(f)]$. This specification implies that the SDF is a linear function of the demeaned factors and is advocated by Kan and Robotti (2008). The pricing errors, the difference between the actual and predicted excess returns, are now denoted as:

$$g_T = E_T(mR^e) = E_T(R^e) - E_T[R^e(f - E(f))']b \tag{21}$$

and will be used as the moment conditions while implementing GMM. Since $E(mR^e) = 0$,

$$E_T(R^e) = E_T[R^e(f - E(f))']b = Cov(R^e, f')b \tag{22}$$

which shows that the GMM estimation is an equivalent to a cross-sectional regression of average excess returns on the covariances between excess returns and factors. Recalling the expected return-beta representation, $E_T(R^e) = \beta'\lambda = Var(f)^{-1}Cov(R^e, f')\lambda$, the factor risk prices λ and b are related in the following way:

$$\lambda = Var(f)b \tag{23}$$

Since the mean of factors, $E(f) \equiv \mu_f$, is also unknown parameter, the GMM estimate is formed from:

$$Min_{\{b,\mu_f\}} g_T(b, \mu_f)'Wg_T(b, \mu_f) \tag{24}$$

where W is a weighting matrix that determines on which moment conditions or linear combinations of moments to put more emphasis over others. We first use the identity weighting matrix, $W = I$. In this case all assets are treated symmetrically and parameters are estimated by minimizing the sum of squared pricing errors. We also choose as another weighting matrix the inverse of the covariance matrix of the excess returns, i.e., $\{[R^e - E(R^e)]'[R^e - E(R^e)]/T\}^{-1}$, as suggested in Kan and Robotti (2008). This is a modified version of Hansen-Jagannathan (HJ) distance which uses the inverse of the second moment of the excess returns, i.e., $[R^{e\prime}R^e/T]^{-1}$. Lettau and Ludvigson (2001b) point out however that parameters can be poorly estimated using HJ weighting matrix in Eq.(24) if the size of the available sample T is small compared to the number of test assets N. As such, if the modified HJ-distance based GMM estimates differ greatly from those estimated using identity weighting matrix, this may be due to the poor finite sample estimate of the asymptotic covariance matrix of the pricing errors of the excess returns.

Once the parameter vector is estimated and g_T is identified through Eq.(24), we can test the model using the J_T test with the following test statistic:

$$J_T = g_T'(b, \mu_f)[Var(g_T(b, \mu_f))]^+ g_T(b, \mu_f) \sim \chi^2_{N-L} \tag{25}$$

where $[]^+$ denotes the pseudo-inverse since the variance-covariance matrix of the g_T is singular. The χ^2 distribution has degrees of freedom equal to the difference between the number of moments and the number of estimated parameters.

We also report estimate of the coefficient of relative risk aversion γ obtained from the estimated value of the b_ε using the relation $\hat{b}_\varepsilon = (\hat{\gamma} - 1)$. The estimate of the structural parameter of the model based on the data provides an alternate and important way to validate the model. We use this as as a metric to asses the success of the model apart from the econometric results. This additional way of model evaluation is stressed and recommended by Malloy, Moskowitz and Vising-Jorgensen (2009).

5 Data

Our test assets are 10 US Treasury bond portfolios from the CRSP Fama Maturity Portfolios Returns Files. The bonds in these portfolios include callable, non-callable and non-flower U.S. government notes and bonds but exclude partially or fully tax-exempt issues. Quarterly returns are calculated using monthly holding period returns for each Fama portfolio. These holding period returns in the CRSP database are equal weighted averages of the unadjusted ex-post one-month holding period returns of each bond in the portfolio. In this paper, we use Fama portfolios with the following maturities: (1) from 13 to 18 months, (2) from 19 to 24 months, (3) from 25 to 30 months, (4) from 31 to 36 months, (5) from 37 to 42 months, (6) from 43 to 48 months, (7) from 49 to 54 months, (8) from 55 to 60 months, (9) from 61 to 120 months and (10) greater than 120 months from the quote date. Our sample starts from the first quarter of 1975 (the first date from which a complete set of returns for all the Fama portfolios is available) through to the fourth quarter of 2006. This results in 128 quarterly returns on each Treasury bond portfolio. We compute quarterly simple excess returns on these ten portfolios as the difference between the quarterly portfolio returns and quarterly returns on 30-day Treasury bills obtained from the CRSP.

We also use a second set of test assets to check for the robustness of our results. These are quarterly returns on 7 CRSP Fixed Term Indices with target maturities of 1, 2, 5, 7, 10, 20 and 30-years. They are taken in excess over quarterly 30-day Treasury bill rate.

Finally, we obtain the dynamic factors from a balanced panel of 126 macroeconomic and financial time series (described individually in the Appendix 7) from the Global Insights Basic Economics and the Conference Board's Indicators Databases. We started with the 132 series used and described in Ludvigson and Ng (2009); however, in updating their data to December 2006, six series were dropped (details in the Appendix) due to missing data or discontinuance of the series[11]. The 126 series represent broad categories of economic and financial time series such as Real Output & Income, Employment and Hours, Housing Starts and Sales, Real Inventories, Orders and Unfilled Orders, Money and Credit, Stock Prices, Interest Rates, Exchange Rates, Price Indexes and Consumer Expectations Survey data. We use quarterly data and following Stock and Watson (2002a,b) standardize and transform the data where necessary to ensure stationarity prior to the estimation of the dynamic factors using principal component analysis. Details of specific transformations used in each series are described in the Appendix.

We construct our real consumption series using personal consumption expenditure on nondurables and services deflated by a weighted average of price index for nondurables and price index for services (base: 2000 = 100), following Hansen, Heaton and Li (2005). This series is then divided by population for the corresponding time period to obtain a per capita real consumption measure. Our consumption growth data are log changes in real per capita quarterly consumption of nondurables and services[12]. We also use, as a robustness check, data on quarterly consumption growth obtained following an alternate procedure. We follow Piazzesi and Schneider (2006) who measure per capita consumption growth as equal to the growth rate of the raw consumption NIPA data minus a constant and assume that population growth is constant. This allows them to avoid taking a stand on which population series to use since they find large differences in the standard population series available from various data sources and the presence of very large spikes at points where the census data is collected every decade. Further, the Piazzesi and Schneider (2006) methodology avoids the use of population series that suffer from interpolation issues between each census. The source for all data used to construct the consumption growth series is the U.S. Department of Commerce, Bureau of Economic Analysis.

6 Empirical Results

We now present our empirical results beginning with some summary statistics that describe our tests assets and the consumption data. Next, we provide estimates for the factor-augmented VAR and the extracted series of innovations to current and expected future consumption growth. Finally, we describe the results of the tests of the expected beta-return and the stochastic discount factor representation of the model.

6.1 Summary Statistics

In contrast to the stylized facts for equity portfolios the features of the Fama Maturity bond portfolios that we use are less well-known. We report, in Table 1, some summary statistics for excess returns on our 10 test assets over the full sample period. The government bond portfolio which consists of bonds with maturities from 12 to 18 months is denoted as BP1. Each subsequent portfolio (BP2,. . . etc.) has bonds with maturity

[11] Ludvigson and Ng (2009) makes some of their data available on their webpage. We find that the dynamic factors obtained using their data are similar to ours with sample correlations exceeding 0.98.

[12] We consider expenditure on nondurables and services, following a large literature on consumption-based models (see, for example, Lettau and Ludvigson, 2001a).

increasing in increments of 6 months. The portfolio BP10 consists of all bonds with maturities greater than 10 years. The excess returns in Table 1 are quarterly simple returns in excess of quarterly 30-day T-Bill rate (multiplied by 100 for clarity in the Table). The full sample period begins from the first quarter of 1975 and ends in the fourth quarter of 2006 with a total of 128 observations. Over the 30-year sample period, average excess returns on the bond portfolios increase with the maturity of the constituent bonds; the mean quarterly excess return for the shortest maturity (12-18 months) portfolio BP1 is 0.338%, increasing in a monotonic fashion to 0.886% for BP10, the portfolio that contains the longest maturity bonds (more than 120 months). The volatility of average excess returns also increases from 1.318% (Portfolio BP1) to more than four times that for the longest maturity portfolio BP10 (5.638%). The Sharpe ratio is highest for the portfolio with bonds with shortest maturity (0.256) and decreases with the increase of the maturity of the constituent bonds to 0.157 for the longest-maturity portfolio. These patterns can be clearly seen in Figure 1. Our results are similar to those reported by Pilotte and Sterbenz (2006). We can also see from Table 1 that there is little correlation in quarterly excess returns for all the Treasury bond portfolios.

Our data for real consumption growth is log-differenced per capita consumption, obtained as indicated earlier following the methodology in Hansen, Heaton and Li (2005) and an alternate measure following Piazzesi and Schneider (2006). Table 2 reports summary statistics for the quarterly real consumption growth series using both these methods. We note that, while their means are different, the autocorrelations are similar, showing positive and significant correlation up to three lags.

6.2 Dynamic Factors

The number of factors is determined based on the information criteria in Bai and Ng (2002). We note that factors are zero mean (by construction) but the standard deviation decreases from factor $\hat{f}_{1t}$ to factor $\hat{f}_{8t}$ as might be expected using principal components. The factors are also persistent and serially correlated.

Next, Table 3 reports results of the procedure, following Ludvigson and Ng (2007), we use to select the subset of factors with strong predictive power for consumption growth. We estimate Eq.(17) and find that the BIC and AIC are minimized in the model with the 3-factor subset $\bar{F}_{3t} = \left[\begin{array}{ccc} \hat{f}_{1t} & \hat{f}_{2t} & \hat{f}_{8t} \end{array} \right]$ and the 6-factor subset $\bar{F}_{6t} = \left[\begin{array}{cccccc} \hat{f}_{1t} & \hat{f}_{2t} & \hat{f}_{3t} & \hat{f}_{4t} & \hat{f}_{5t} & \hat{f}_{8t} \end{array} \right]$ respectively. Since and have low BIC and AIC very close $\bar{F}_{3t}$ to $\bar{F}_{6t}$ and respectively, we test for model refinement using the log-likelihood ratio test on models with $\bar{F}_{kt}$ and $\bar{F}_{(k+1)t}$, where $k = 3, 4, 5$. We find that $\hat{f}_{5t}$, not included in $\bar{F}_{3t}$ but in $\bar{F}_{4t}$, matters significantly in predicting future consumption growth as the computed χ^2 between $\bar{F}_{3t}$ and $\bar{F}_{4t}$ is greater than the critical value with 1 degree of freedom, while those between $\bar{F}_{4t}$ and $\bar{F}_{5t}$ and between $\bar{F}_{5t}$ and $\bar{F}_{6t}$ are not statistically significant. Therefore, we use a subset of the following factors $\bar{F}_{4t} = \left[\begin{array}{cccc} \hat{f}_{1t} & \hat{f}_{2t} & \hat{f}_{5t} & \hat{f}_{8t} \end{array} \right]$ as our state or predictive variables in the factor-augmented VAR.

A point of interest in including these factors in the VAR is whether they have any economic interpretation. In general, interpretation of the factors as representing specific types of macroeconomic or financial series is inappropriate since the construction of each one is affected to some degree by all the variables in our large dataset. In addition, the orthogonalization process means that none of them will correspond exactly to a precise economic concept like output or unemployment especially when such series are naturally correlated. With this caveat, but with a view to get some intuition of what the factors might represent, we follow Stock and Watson (2002b) and Ludvigson and Ng (2009) in characterizing[13] the factors as they relate to the 126 variables in our panel dataset. We depict in Figure 2 the marginal R-squares for our estimates of these four factors $\hat{f}_{1t}$, $\hat{f}_{2t}$, $\hat{f}_{5t}$ and $\hat{f}_{8t}$. The marginal R-square is the R^2 statistic from regressions of each of the 126 individual series from our panel data onto each estimated factor, one at a time, using the full sample of data. Each plot displays the R^2 statistics as bars in the chart separately for each of the four factors we use. The individual series that make up the panel dataset are grouped by broad category and labelled using the numbered ordering given in the Appendix.

As Figure 2.A shows, the first factor ($\hat{f}_{1t}$) loads heavily on measures related to industrial production, employment, new manufacturing orders and housing, while displaying little correlation with prices or financial variables. The second factor ($\hat{f}_{2t}$), on the other hand, appears to load most heavily on financial variables, especially several interest rate spreads, but displays little correlation with macroeconomic measures (Figure 2.B). The third and fourth factors ($\hat{f}_{5t}$ and $\hat{f}_{8t}$) are correlated with nominal variables related to housing (Figures 2.C and 2.D).

6.3 VAR Estimation and Consumption Growth Factors

We report in Table 4 the estimation results of our factor-augmented VAR model with per capita consumption growth and the 4 dynamic factors along with OLS and bootstrapped standard errors for the coefficient estimates. The equation for Δc_{t+1}presented in the first column of the Table suggests that, while lagged three factors $\hat{f}_2$, $\hat{f}_5$ and $\hat{f}_8$ are statistically significant, lagged Δc_{t+1} and $\hat{f}_1$ are not. Given that both Δc_t and $\hat{f}_{1t}$ have strong partial effects on Δc_{t+1}, this may be due to strong contemporaneous correlation between Δc_{t+1} and $\hat{f}_1$: the correlation coefficient between them is -0.483. The R^2 for the equation is high (0.356), which is as expected since VAR models with highly persistent predictive variables tend to have high R^2 (for example, when variables like the dividend yield are used as regressors as in Campbell, 1991).

Table 5 reports some summary statistics for the current consumption growth shock η_c and the expected future consumption growth shock ε_c for the period from 1975 to 2006, using real per capita consumption growth. The future growth shock series is more volatile than the current shock series: the standard deviations for the future growth shock and the current growth shock are 0.454 and 0.332 respectively using the per capita consumption measure.

6.4 Results of Asset Pricing Tests

We now turn to results of the formal asset pricing tests.

[13]While interesting, this analysis takes us away from the main theme of our paper and we refer the reader to Ludvigson and Ng (2009) who have a detailed and interesting analysis of this issue when using dynamic factors in empirical analysis.

6.4.1 Fama-MacBeth Cross-Sectional Regression Results

Table 7 reports estimates for the factor risk prices or λ's based on the FMB two-pass procedure. We calculate three types of standard errors: with *iid* assumption, corrected for generated regressors using Shanken's (1992) correction and GMM-based standard errors.

Over the full sample period (1975-2006), the point estimate of λ_η or the price of risk related to innovations to current consumption growth is small and positive (0.032) but not statistically different from zero. This is consistent with the well-documented poor performance of the standard consumption CAPM (Hansen and Singleton, 1982). On the other hand, the point estimate of λ_ε, the price of risk related to innovations to expected future consumption growth, is positive (0.193) and significant. We note here that this factor is a proxy for the present value of innovation to expected future consumption growth. It thus encompasses the "ultimate consumption risk" factor in Parker and Julliard (2005) who use the covariance of an asset's return with consumption growth cumulated over a finite number of quarters. A positive and significant value of λ_ε has an important interpretation: it means that that the risk related to expected future consumption growth is priced and the factor risk premium equals 0.193.

We also report in the Table 7 some measures of the fit of the model. We find that the root mean square pricing error of the model is small 0.03%, the mean absolute errors is 0.025% and the R^2 and adjusted R^2 are 96% and 95% respectively. These values mean that the co-movements of bond returns with news to current consumption growth and news to expected future consumption growth are able to explain around 95% of the variation in mean excess bond returns. We present a plot of the mean excess returns predicted from the model against the actual ones in Figure 5. If the observed mean excess returns are consistent with risks measured from the models, the predicted and actual mean excess returns should line up along a 45 degree line from the origin. It is clear therefore from the plot that our two-factor models provide a close fit to the data. We note that this explanation needs to be interpreted with caution as highlighted by Lewellen, Nagel and Shanken (2008).

To allay these types of concerns we also report in Table 7 results of tests of whether the true uncentered R^2 of the model is significantly different from zero and from one: $H_0 : R^2 = 1$ and $H_0 : R^2 = 0$ vs appropriate two-sided alternatives. In this test, high p-values for the null: $H_0 : R^2 = 1$ imply a strong evidence in favour of this null hypothesis. We find that for our model the p-values for this null are 0.606 and 0.539 implying that there is strong evidence that the model is correct and explains well the cross-sectional differences in excess returns on government bond portfolios. On the other hand, low p-values for the null: $H_0 : R^2 = 0$ imply weak evidence in favour of this null hypothesis. We find that for our model the p-values for this null are 0.047 and 0.106 implying only weak evidence that this cannot explain at all the cross-section of the excess returns on our test assets.

6.4.2 SDF- GMM Approach

Table 8 reports the results for the stochastic discount factor (SDF) representation of the model, estimated with GMM. We specify the SDF as a linear function of the demeaned factors, i.e., $m = 1 - [f - E(f)]'b$, so that the in the cross-sectional we regress mean excess bond returns on covariances between returns and factors. As Cochrane (2005) explains, b_j captures whether factor f_j is marginally useful in pricing assets, given the presence of other factors. Thus, if $b_j = 0$, we can price assets just as well without factor f_j as with it. We estimate the SDF using GMM with the identity matrix and the HJ-type matrix as weighting matrices. Given that our SDF formulation is based on demeaned factors we use the inverse of the covariance matrix, instead of the second moment matrix of excess returns following Kan and Robotti (2006).

We can see, in Table 8, that the factor loading b_ε related to the future growth shock factor is positive and significantly different from zero (1.015) when estimated using the identity weighting matrix. This implies that innovation to expected future consumption growth plays an important role in pricing the cross-section of government bonds with different maturities. The factor loading b_η related to the two current growth shock is not significantly different from zero signalling that the innovations to current consumption growth do not help to price the cross-section of test assets and they can dropped off. This supports the finding from the FMB cross-sectional regression. The estimation results using the HJ-type weighting matrix suggest that both the innovations to current and expected future consumption growth help pricing the test assets since the point estimate of b_η is also statistically significant.

Finally, following Malloy, Moskowitz and Vissing-Jorgensen (2009) we also take theoretical restrictions of the model seriously and report the magnitude of the implied risk aversion coefficient γ, obtained from the relation: $\hat{b}_\varepsilon = \hat{\gamma} - 1$. We find that the estimate of γ has a value of 2.015 and 1.821 for identity matrix and Hansen Jagannathan weighting matrix respectively, which suggest a plausible economic magnitude for this parameter that could be used as an important metric to asses the success of the model apart from the results of statistical test related to Fama MacBeth regression and SDF-GMM estimation.

6.5 Discussion of the Results

What lies behind the findings in the previous section? We show below that it is not by accident that both the risk price λ_ε and the factor loading b_ε related to the news to expected future consumption growth are positive and significant.

In Table 6 we report the estimates of betas related to the two pricing factors: innovations to current consumption growth and innovations to expected future consumption growth. Betas measure the riskiness of test assets related to each pricing factor and they are directly proportional to the covariance between bond returns and given pricing factor. We can see, from the Table 6, that only betas related to news in expected future consumption growth are positive and significant for all 10 Treasury bond portfolios. This implies that only the risk related to expected future consumption matters in pricing these portfolios. Positive values of betas are implied by positive covariances between bond returns and news to expected future consumption growth. These covariances indicate that governments bonds have low payoffs when times are bad or payoff well in good times. Thus investors require an inducement to hold such assets which do not smooth consumption. This explains why the government bonds were, on average, risky assets in the period 1975-2006.

Additionally we can observe, from the Table 6, that among our 10 test assets the portfolios of bonds with longer maturity have greater betas implying that they are riskier than portfolios of bond with shorter maturity. Because of positive and significant estimates of λ_ε and b_ε we can conclude that there is enough variation in the riskiness of bond portfolios with different maturities, that is in their betas, to explain the variation in observed mean excess returns on these bond portfolios. This can be seen clearly in the Figure 3 where we plot betas related to each pricing factor against observed mean excess bond returns. We can see that betas related to innovations to expected future consumption growth increase along with the average excess returns of bond portfolios and further that these increase with the maturity of the constituent bonds.

Summing up then, bonds with longer maturities command a higher risk premium than bonds with shorter maturities because they are more risky. The main source or risk is related to the prospects in future consumption (movements in expected future consumption growth). In the Figure 4 we show that indeed the risk premium related to the prospects in future consumption constitutes the main proportion of the risk premium of each bond portfolio.

6.6 Tests for Robustness

We also perform a battery of tests to assess the robustness of our results. Here we describe only the conclusions of these tests and provide full details in the Appendix.

First, we use an alternate set of test assets. These are quarterly holding period returns on the CRSP Fixed Term Indices in excess of quarterly 30-day Treasury bill rates. The Fixed Term Indices are categorized into 7 groups, each with the following target maturity: 1, 2, 5, 7, 10, 20 and 30 years. We calculate quarterly returns using the unadjusted monthly returns (computed as the price change plus interest divided by last month's price) available from the CRSP Monthly US Treasury database. The summary statistics and the estimation results of cross-sectional regressions are reported in the Appendix 2. Our results (Tables A2.2 and A2.3) are qualitatively similar to those obtained using the Fama Maturity Portfolio returns.

Next, we estimate the model for the same two groups of test assets but using a different consumption measure in extracting consumption growth shocks following Piazzesi and Schneider (2006). These results are detailed in Appendix 3 and again support the results obtained using the consumption measure of Hansen, Heaton and Li (2005).

We also report results for the Fama MacBeth cross-sectional regressions using univariate betas. Univariate betas, in contrast to multivariate, measure the riskiness of an individual test asset. The parameters estimated in the second stage of Fama MacBeth regression capture whether each pricing factor is marginally useful in pricing assets, given the presence of other factors. The results of the regressions with univariate betas are reported in the Appendix 4.

Finally, we also stress test our model using an alternate sub-sample of our data. The results for the sub-sample are reported in detail in Appendix 5. Our full sample period, 1975-2006, has 128 data points and is dictated by the availability of non-missing data on the CRSP Fama and Fixed Index portfolios. We therefore test the model using data over the sub-period 1982-2006 - a sub-period marked by a large decline in volatility of major macroeconomic variables like GDP growth or inflation rate, known as the Great

Moderation. As a result, we can test the model during a period characterized by a common macroeconomic environment while at the same time avoid exacerbating the problem of small sample size given the total number of data points available to us. We find again that there is strong support for the main conclusions obtained using the full sample of data.

Further, in Appendix 6 we report the Fama MacBeth cross-sectional regressions and SDF-GMM estimation results for the model in which we include the third pricing factor: the variance of expected future consumption growth shock, $Var(\varepsilon_{c,t})$ calculated by squaring the demeaned $\varepsilon_{c,t}$ as justified in Section 3 of the paper. The main conclusion is that the variance of expected future consumption growth is not important in pricing the cross-section of government bonds.

7 Summary and Conclusion

In this paper we explain the cross-sectional differences in mean excess returns on US Treasury bond portfolios with differing times to maturity, from one year to more than ten years. We investigate whether bonds are risky assets; i.e., do they have low payoffs in bad times and therefore do investors need a premium to hold these assets? We use a consumption-based asset pricing model with Epstein-Zin-Weil recursive preferences to test the relation between the mean excess returns on bond portfolios and the riskiness of bonds. Our model has the following pricing factors: innovations to current consumption growth and innovations to expected future consumption growth. We use a VAR model to extract the two factors. A novelty of our application of VAR is that instead of specific predictive variables we use we use dynamic factors obtained from a large panel of macroeconomic and financial time series.

Our main results are as follows. Over our full sample period 1975–2006 the variation in the consumption betas related to expected future consumption growth shock of government bonds explains well the variation in the average excess bond returns. The market price of risk associated with this shock is positive and statistically significant. Further, portfolios of government bonds with longer maturity are more risky than the portfolios of bond with shorter maturity. The riskiness of bonds arises from the fact that bond returns covary positively with news in expected future consumption growth. Government bonds have high payoffs in good times and payoff badly in bad times thus requiring investors to demand a premium for holding such assets. In other words, government bonds are, on average, risky assets in the period 1975-2006. This conclusion is strongly supported by the results of the stochastic discount factor estimation, which suggest that expected future consumption growth innovation is relevant for pricing the set of tests assets.

Further we find that neither the news in current consumption growth nor the variance of the shocks to expected future consumption growth play any role in pricing the cross-section of excess nominal government bond returns. The associated factor risk premia are not statistically significant. This means that only the risk related to prospects in future consumption matters in pricing government bonds. This differs for example from the equity market where the volatility of future expected consumption growth is reported to be a priced factor (see for example Tedongap, 2007).

Finally, we also find support for our model in realistic estimates for the coefficient of relative risk aversion which further strengthen the results from the statistical tests. Overall the results imply that investors must be rewarded to hold government bond portfolios which are risky rather than safe assets and which may not be useful in hedging against risk related to long run prospects in consumption growth.

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Table 1 Summary Statistics of Excess Returns: Fama Maturity Portfolios

Sample Period: 1975-2006										
Summary statistics	BP1	BP2	BP3	BP4	BP5	BP6	BP7	BP8	BP9	BP10
Mean	0.338	0.361	0.422	0.468	0.521	0.533	0.532	0.550	0.625	0.886
St dev	1.318	1.683	2.080	2.354	2.560	2.825	2.986	3.300	3.843	5.638
Sharpe R	0.256	0.215	0.203	0.199	0.204	0.189	0.178	0.167	0.163	0.157
Minimum	-4.555	-5.516	-6.962	-7.852	-7.544	-8.088	-8.354	-9.458	-11.144	-16.082
Maximum	7.788	9.073	11.050	11.685	11.112	12.193	12.170	15.146	14.976	19.575
Autocorrelations										
ρ_1	-0.134	-0.122	-0.122	-0.119	-0.098	-0.084	-0.086	-0.114	-0.052	-0.059
ρ_2	0.043	0.017	0.030	0.035	0.052	0.023	0.043	0.035	0.042	0.016
ρ_3	0.144	0.165	0.139	0.140	0.156	0.159	0.149	0.110	0.121	0.117
ρ_4	0.006	0.024	0.024	0.012	0.024	0.012	0.022	0.008	0.003	-0.034

Notes: Table 1 presents summary statistics for quarterly excess returns on the 10 Fama Maturity Portfolios over the quarterly 30-day Treasury bill rates (reported here multiplied by 100). The quarterly holding period returns on the Fama Maturity Portfolios are computed using monthly returns obtained from the CRSP Monthly US Treasury database. Quarterly T-bill rates are obtained from the CRSP US Treasury and Inflation series. The Fama Maturity Portfolios, from the CRSP US Government Bond Files, consist of non-callable, non-flower notes and bonds and are defined by six-month or maturity intervals. In this table, BP1 represents excess returns on the portfolio holding securities that mature from 13 to 18 months, BP2 from 19 to 24, BP3 from 25 to 30, BP4 from 31 to 36, BP5 from 37 to 42, BP6 from 43 to 48, BP7 from 49 to 54, BP8 from 55 to 60, BP9 from 61 to 120, and BP10 greater than 120 months from the quote date. Our data span the sample period from 1975Q1 to 2006Q4. Also reported in this table are autocorrelations up to 4 quarters: those marked with asterisk are significant, exceeding the band $\pm 1.96/\sqrt{T} \approx \pm 0.173$.

Table 2 Summary Statistics for Consumption Growth

Sample Period: 1975-2006				
Summary Statictics	Mean	St dev	Minimum	Maximum
g_c	0.530	0.410	-1.090	1.459
$g_{c(P\&S)}$	0.792	0.401	-0.810	1.699
Autocorrelations	ρ_1	ρ_2	ρ_3	ρ_4
g_c	0.337*	0.194*	0.402*	0.150
$g_{c(P\&S)}$	0.300*	0.149*	0.383*	0.130

Notes: Table 2 presents summary statistics for quarterly consumption growth, calculated by taking log difference of real per capita consumption and multiplying by 100. Per capita consumption of nondurables and services is computed as in Hansen, Heaton and Li (2005). We also compute, following Piazzesi and Schneider, 2006, (P&S hereafter), an alternate measure of consumption. The source of data used to construct the consumption growth series is the U.S. Department of Commerce, Bureau of Economic Analysis. Both per capita consumption growth g_c, and P&S consumption growth $g_{c(P\&S)}$, span the sample period from 1975Q1 to 2006Q4. Also reported in this table are autocorrelations up to 4 quarters: those marked with asterisk are significant, exceeding the band $\pm 1.96/\sqrt{T} \approx \pm 0.173$.

Table 3 Selection of Dynamic Factors with Predictive Power for Consumption Growth

Regressors	LL-Ratio	$\bar{R}^2$	BIC	AIC
$\bar{F}_1$	-	0.229	1.060	1.043
$\bar{F}_2$	20.201*	0.304	0.980	0.946
$\bar{F}_3$	6.241*	0.323	0.975	0.923
$\bar{F}_4$	4.372*	0.335	0.980	0.910
$\bar{F}_5$	3.074	0.343	0.991	0.905
$\bar{F}_6$	2.414	0.348	1.006	0.903
$\bar{F}_7$	1.292	0.348	1.027	0.906
$\bar{F}_8$	0.049	0.345	1.055	0.917

Notes: Table 3 presents the procedure to select the subset of factors with predictive power for one-period-ahead per capita consumption growth. We start with a subsets consisting of a single factor ($\bar{F}_1$) and identify the subset, out of 8 candidates, that minimizes the information criteria (BIC and AIC) from the following regression: $g_{c,t+1} = \gamma' \bar{F}_{1t} + e_{t+1}$. Once the best predictor out of $\bar{F}_1$ is identified, we include an additional factor and expand it to $\bar{F}_2$, subsets comprising of two factors. We then run a regression of $g_{c,t+1}$ on and $\bar{F}_{2t}$ identify, out of 7 two-factor subset, the subset $\bar{F}_2$ that consists of factors that minimizes the BIC and AIC. We repeat this procedure until we reach $\bar{F}_8$.

The following subsets of factors are identified via the above procedure:
$\bar{F}_1 = \begin{bmatrix} \hat{f}_2 \end{bmatrix}$, $\bar{F}_2 = \begin{bmatrix} \hat{f}_2 & \hat{f}_8 \end{bmatrix}$, $\bar{F}_3 = \begin{bmatrix} \hat{f}_1 & \hat{f}_2 & \hat{f}_8 \end{bmatrix}$, $\bar{F}_4 = \begin{bmatrix} \hat{f}_1 & \hat{f}_2 & \hat{f}_5 & \hat{f}_8 \end{bmatrix}$,
$\bar{F}_5 = \begin{bmatrix} \hat{f}_1 & \hat{f}_2 & \hat{f}_4 & \hat{f}_5 & \hat{f}_8 \end{bmatrix}$, $\bar{F}_6 = \begin{bmatrix} \hat{f}_1 & \hat{f}_2 & \hat{f}_3 & \hat{f}_4 & \hat{f}_5 & \hat{f}_8 \end{bmatrix}$,
$\bar{F}_7 = \begin{bmatrix} \hat{f}_1 & \hat{f}_2 & \hat{f}_3 & \hat{f}_4 & \hat{f}_5 & \hat{f}_7 & \hat{f}_8 \end{bmatrix}$, $\bar{F}_8 = \begin{bmatrix} \hat{f}_1 & \hat{f}_2 & \hat{f}_3 & \hat{f}_4 & \hat{f}_5 & \hat{f}_6 & \hat{f}_7 & \hat{f}_8 \end{bmatrix}$

We also calculate the log-likelihood ratio (LL-ratio) between regressions of $g_{c,t+1}$ on (i) $\bar{F}_{kt}$ and (ii) $\bar{F}_{(k+1)t}$, where $k = 1, ..., 7$, to check whether the additional factor included in $\bar{F}_{(k+1)t}$ matters significantly in improving the fit of the model. Asterisks in the column indicate significance: the calculated statistics exceed a 5% critical value of χ^2_1 (3.84).

Table 4 Estimation of Factor-Augmented VAR

	g_c	$\hat{f}_1$	$\hat{f}_2$	$\hat{f}_5$	$\hat{f}_8$
g_c(-1)	0.123	-0.026	-0.074	-0.062	0.024
	(0.075)	(0.061)	(0.054)	(0.034)	(0.030)
	[0.079]	[0.062]	[0.054]	[0.033]	[0.029]
$\hat{f}_1$(-1)	-0.084	0.612	-0.149	0.032	0.044
	(0.064)	(0.052)**	(0.046)**	(0.029)	(0.026)*
	[0.067]	[0.051]**	[0.046]**	[0.028]	[0.025]*
$\hat{f}_2$(-1)	-0.669	0.767	0.403	-0.097	0.054
	(0.096)**	(0.078)**	(0.069)**	(0.043)**	(0.038)
	[0.094]**	[0.076]**	[0.071]**	[0.043]**	[0.037]
$\hat{f}_5$(-1)	-0.274	-0.141	0.130	0.521	-0.119
	(0.132)**	(0.106)	(0.094)	(0.059)**	(0.052)**
	[0.132]**	[0.104]	[0.096]	[0.057]**	[0.052]**
$\hat{f}_8$(-1)	0.701	-0.483	0.168	-0.118	0.389
	(0.174)**	(0.140)**	(0.124)	(0.078)	(0.069)**
	[0.166]**	[0.138]**	[0.125]	[0.077]	[0.068]**
R^2	0.356	0.654	0.246	0.342	0.207
$\bar{R}^2$	0.342	0.637	0.230	0.328	0.189
BIC	0.993	0.562	0.321	-0.610	-0.856
AIC	0.907	0.475	0.234	-0.697	-0.943

Notes: Table 4 presents the estimation results of a factor-augmented VAR (1) for per capita consumption growth g_c, and 4 dynamic factors as endogenous variables: i.e., $Z_{t+1} = AZ_t + w_{t+1}$, where $Z_t = \left[\begin{array}{cc} g_{c,t} & \bar{F}_{4t} \end{array}\right]$, $\bar{F}_{4t} = \left[\begin{array}{cccc} \hat{f}_{1t} & \hat{f}_{2t} & \hat{f}_{5t} & \hat{f}_{8t} \end{array}\right]$ and $w_{t+1} \sim iid$.

Each column in the table corresponds to an equation in the VAR. Two types of standard errors are reported for the coefficient estimates: (i) OLS standard errors in parentheses and (ii) bootstrapped standard errors in square brackets. Asterisks indicate statistical significance of the coefficient estimates at *10% and ** 5% levels.

Table 5 Summary Statistics and Autocorrelations: Pricing Factors

Sample Period: 1975-2006				
Summary Statictics	Mean	St dev	Minimum	Maximum
$\eta_{c,t}$	0.010	0.332	-0.919	0.728
$\varepsilon_{c,t}$	-0.041	0.454	-1.674	0.953
Autocorrelations	ρ_1	ρ_2	ρ_3	ρ_4
$\eta_{c,t}$	0.000	-0.048	0.333*	0.076
$\varepsilon_{c,t}$	-0.294*	-0.101	0.438*	-0.123

Notes: Table 5 presents summary statistics for the two pricing factors: current consumption growth shock, $\eta_{c,t}$ and expected future consumption growth shock, $\varepsilon_{c,t}$, both extracted from a VAR with real per capita consumption growth and the 4 dynamic factors. Consumption growth and pricing factors span the sample period from 1975Q1 to 2006Q4. Also reported in this table are autocorrelations up to 4 quarters: those marked with asterisk are significant, exceeding the band $\pm 1.96/\sqrt{T} \approx \pm 0.173$.

Table 6 Factor Betas for Fama Maturity Portfolios

	BP1	BP2	BP3	BP4	BP5	BP6	BP7	BP8	BP9	BP10
β_η	-0.396	-0.542	-0.739	-0.845	-0.997	-1.098	-1.240	-1.408	-1.343	-1.563
pval, S0	[0.207]	[0.180]	[0.157]	[0.150]	[0.117]	[0.119]	[0.097]	[0.098]	[0.133]	[0.176]
pval, S6	[0.204]	[0.137]	[0.159]	[0.155]	[0.128]	[0.134]	[0.109]	[0.104]	[0.138]	[0.170]
β_ε	1.284	1.606	1.901	2.093	2.131	2.318	2.301	2.508	2.980	4.196
pval, S0	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.001]	[0.001]	[0.001]	[0.001]
pval, S6	[0.005]	[0.004]	[0.004]	[0.004]	[0.004]	[0.004]	[0.005]	[0.007]	[0.004]	[0.003]

Notes: Table 6 presents the OLS estimates of β^i_η, β^i_ε from the following time-series regressions of excess returns on bond portfolio i on each of the pricing factor separately: $R^{ei}_t = \beta^i_{0\eta} + \beta^i_\eta \eta_{c,t} + e^i_{\eta t}$ and $R^{ei}_t = \beta^i_{0\varepsilon} + \beta^i_\varepsilon \varepsilon_{c,t} + e^i_{\varepsilon t}$. The bond portfolios are the 10 Fama Maturity Portfolios. The two pricing factors are respectively the current consumption growth shock, $\eta_{c,t}$ and the expected future consumption growth shock $\varepsilon_{c,t}$. Excess returns on bond portfolios and pricing factors span the period from 1975Q1 to 2006Q4. The estimates of β measure the riskiness of bond portfolios and are reported in this table along with their p-values for the t-statistics in square brackets. The p-values are based GMM-corrected standard errors with 0 and 6 NW lags ("pval, S0" and "pval, S6" respectively) that allow for heteroskedasticity and autocorrelation in the errors.

Table 7 FMB Cross-Sectional Regression: Fama Maturity Portfolios

	λ_η	λ_ε		χ^2	
estimates	0.032	0.193			
se, FMB iid	(0.139)	(0.134)		45.181	RMSE = 0.030
	[0.412]	[0.093]		[0.000]	MAE = 0.025
se, FMB iid-Sh	(0.151)	(0.144)		38.043	
	[0.419]	[0.109]		[0.000]	R^2= 0.96
se, GMM S0	(0.150)	(0.147)		33.695	$\bar{R}^2$= 0.95
	[0.418]	[0.113]		[0.000]	
se, GMM S6	(0.140)	(0.135)		47.194	
	[0.412]	[0.095]		[0.000]	
uncentered R^2	0.997			H_0:R^2= 1	H_0:R^2= 0
se(R^2) S0	(0.009)		pval S0	[0.606]	[0.047]
se(R^2) S6	(0.008)		pval S6	[0.539]	[0.106]
uncentered $\bar{R}^2$	0.996				

Notes: Table 7 presents the estimation results of the cross-sectional regrression using the second stage of Fama-MacBeth procedure (FMB) for the two-factor model. The test assets are the 10 Fama Maturity Portfolios and the pricing factors are the current consumption growth shock, $\eta_{c,t}$ and the expected future consumption growth shock $\varepsilon_{c,t}$. Excess returns on bond portfolios and pricing factors span the period from 1975Q1 to 2006Q4. The estimates of λ (factor risk prices) are reported in this table along with their standard errors in parentheses and the p-values for the t-statistics in square brackets. "se, FMB iid" are simple standard errors under the *iid* assumption. "se, FMB iid-Sh" are standard errors following Shanken (1992). The Shanken correction factor is 1.188. "se, GMM S0" and "se, GMM S6" are GMM-corrected standard errors (with 0 and 6 NW lags respectively) that also account for the problem associated with generated regressors and also allow for heteroskedasticity and autocorrelation in the errors.

We also report in the Table the χ^2 statistics, calculated as $\hat{\alpha}' Var(\hat{\alpha})^{-1}\hat{\alpha}$, that are used for a test of joint zero pricing errors $H_0 : \alpha = 0$. The test statistics asymptotically follow a χ^2_{N-L} distribution under the null hypothesis, where degrees of freedom are the number of the test assets (N) less the number of the parameters to estimate (L). The p-values for the calculated statistics are reported in square brackets.

Further, the Table contains the p-values (inside the square brackets) for testing the hypotheses on the true uncentered R^2, specifically $H_0 : R^2 = 1$ and $H_0 : R^2 = 0$, following Kan, Robotti and Shanken (2009). Uncentered R^2, its standard errors (in paretheses) and its adjusted version are reported as well.

RMSE represents root mean square pricing errors and MAE is mean absolute error. R^2 and $\bar{R}^2$ are cross-sectional R-square and adjusted R-square respectively.

Table 8 Stochastic Discount Factor Estimation using GMM: Fama Maturity Portfolios

Panel A: Weighting Matrix $W = I$

	b_η	b_ε	J_T		
estimates	-0.263	1.015			
se, S0	(1.274)	(0.537)	33.695	RMSE = 0.030	$\hat{\gamma}$ = 2.015
	[0.421]	[0.048]	[0.000]	MAE = 0.025	
se, S6	(1.329)	(0.713)	47.194		
	[0.424]	[0.096]	[0.000]		

Panel B: Weighting Matrix $W = Cov(R^e, R^{e\prime})^{-1}$

	b_η	b_ε	J_T		
estimates	-1.895	0.821			
se, S0	(0.905)	(0.451)	29.283	RMSE = 0.103	$\hat{\gamma}$ = 1.821
	[0.035]	[0.053]	[0.000]	MAE = 0.090	
se, S6	(0.924)	(0.766)	53.273		
	[0.037]	[0.158]	[0.000]		

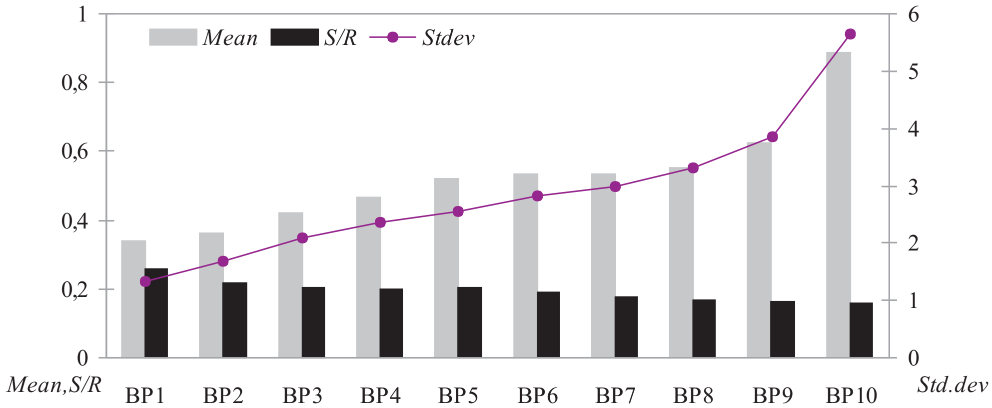
Notes: Table 8 presents the results of estimation of the stochastic discount factor (SDF) representation of the two-factor model using GMM. The test assets are the 10 Fama Maturity Portfolios and the pricing factors are the current consumption growth shock, $\eta_{c,t}$ and the expected future consumption growth shock $\varepsilon_{c,t}$. Excess returns on bond portfolios and pricing factors span the period from 1975Q1 to 2006Q4. We use two weighting matrices: the identity matrix $W = I$ and the Hansen-Jagannathan (HJ)-type weighting matrix $W = [Cov(R^e, R^{e\prime})]^{-1}$. The estimation results using each weighting matrix are presented in Panels A and B respectively. The coefficient b_j captures whether a pricing factor f_j is marginally useful in pricing assets given the presence of other factors.

We compute standard errors of the parameter estimates with 0 and 6 NW lags ("se, S0" and "se, S6" respectively). We report them in parenthesis. We include as well the the p-values for the calculated t-statistics in square brackets.

We also report in the Table are the J_T statistics, calculated following Eq.(25), that are used to test whether the pricing errors are jointly zero $H_0 : \alpha = 0$. The statistics assyptotically follows a χ^2_{N-L} distribution under the null hypothesis, where degrees of freedom are equal to the number of moment conditions $(N + L)$ less the number of parameters that need to be estimated $(2L)$. The p-values for the calculated statistics are provided in square brackets.

RMSE represents root mean square pricing errors and MAE is mean absolute error. The coefficient of relative risk aversion γ is calculated from the relation: $\hat{b}_\varepsilon = \hat{\gamma} - 1$. We do not report the estimation results for μ_f in the interest of brevity.

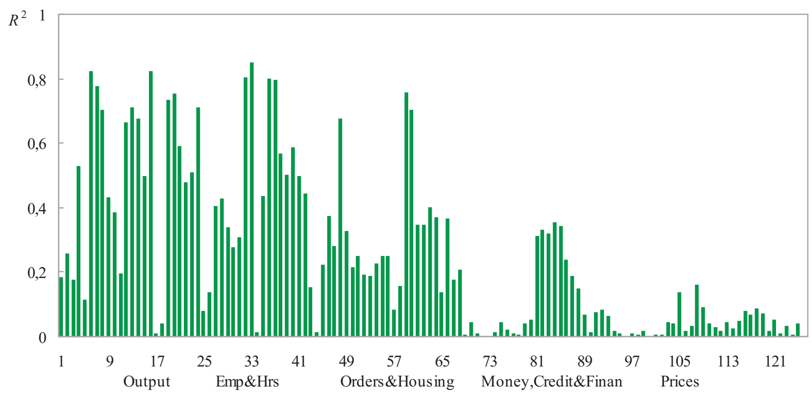
Figure 1 Characteristics of Excess Returns: Fama Maturity Portfolios



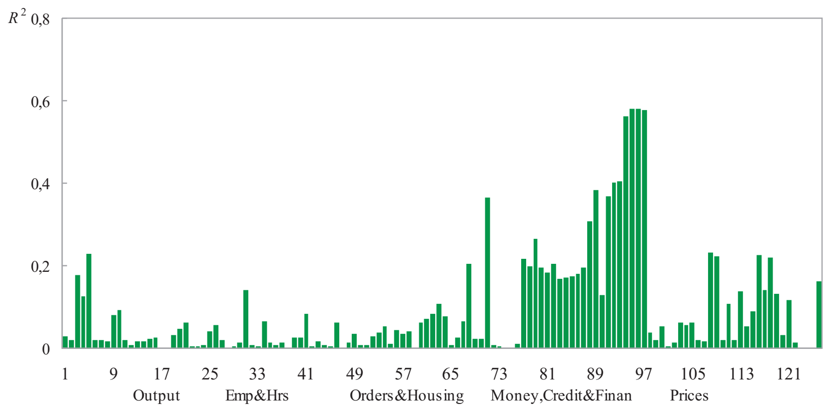
Notes: Figures 1 plots means, Sharpe ratios ("S/R") and standard deviations ("Std.dev") of quarterly returns on 10 Fama Maturity Portfolios in excess of quarterly 30-day US Treasury bill rates. The sample spans the period from 1975Q1 to 2006Q4.

Figure 2 Marginal R-squares for Selected Dynamic Factors

A. Marginal R^2 for $\hat{f}_1$



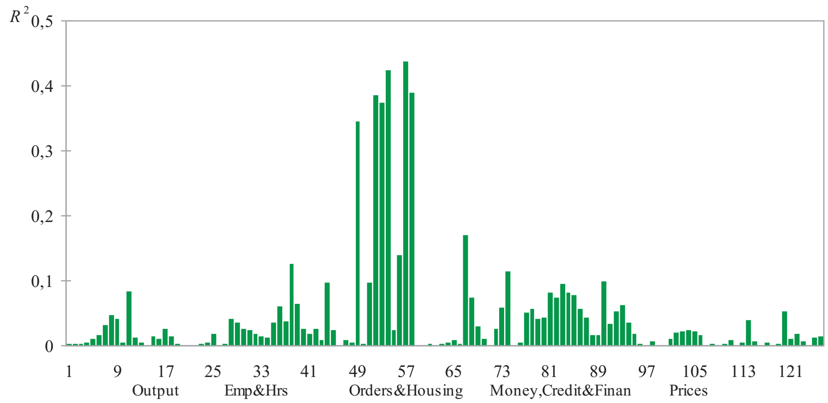
B. Marginal R^2 for $\hat{f}_2$



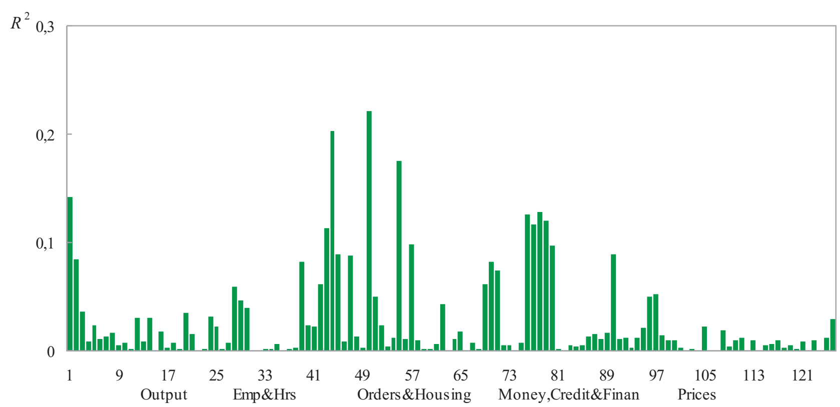
Notes: Figure 2 plots marginal R-squares (on the y-axis) from regressing the numbered macroeconomic and financial series (on the x-axis) onto dynamic factors $\left[\hat{f}_1 \quad \hat{f}_2 \quad \hat{f}_5 \quad \hat{f}_8 \right]$ that are included in the factor-augmented VAR(1). The factors are estimated via principal component analysis using data spanning the period 1960Q1 to 2006Q4. See the Appendix for a description of the series.

Figure 2 Marginal R-squares for Dynamic Factors (continued)

C. Marginal R^2 for $\hat{f}_5$

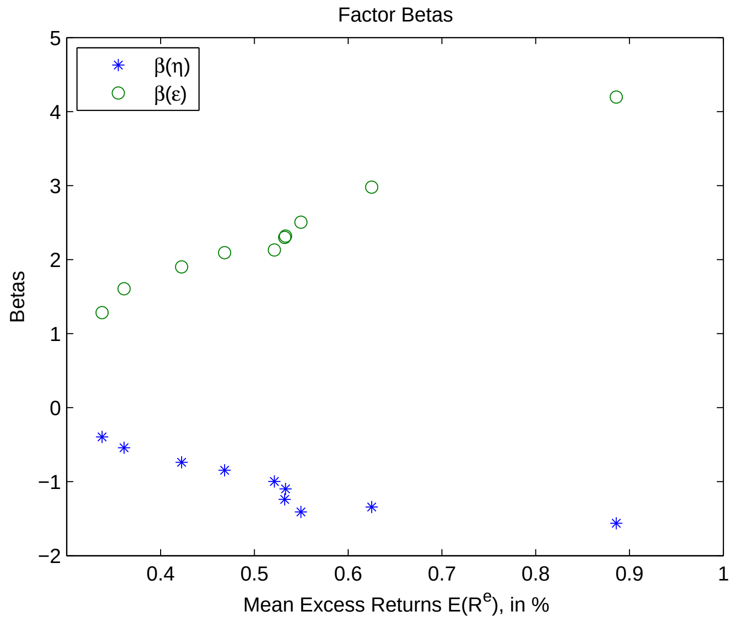


D. Marginal R^2 for $\hat{f}_8$



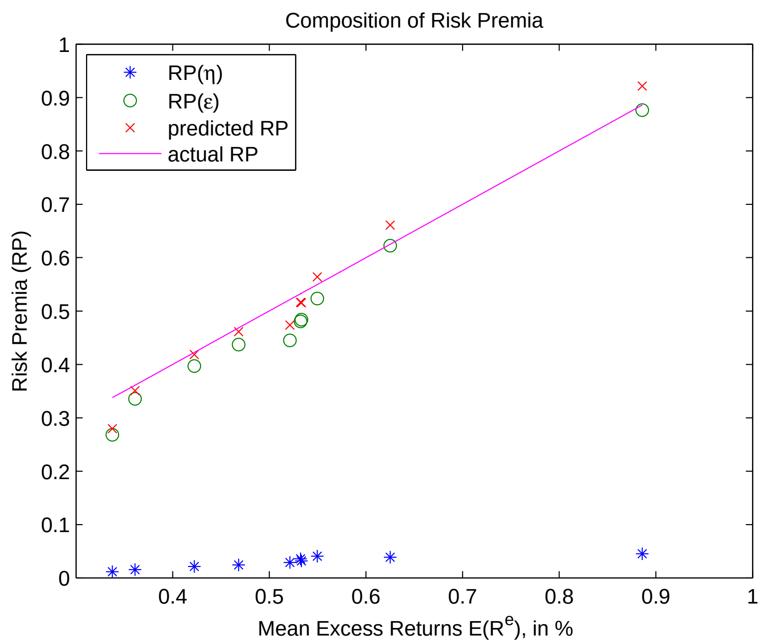
Notes: See previous page.

Figure 3 Cross-Sectional Variation in Betas



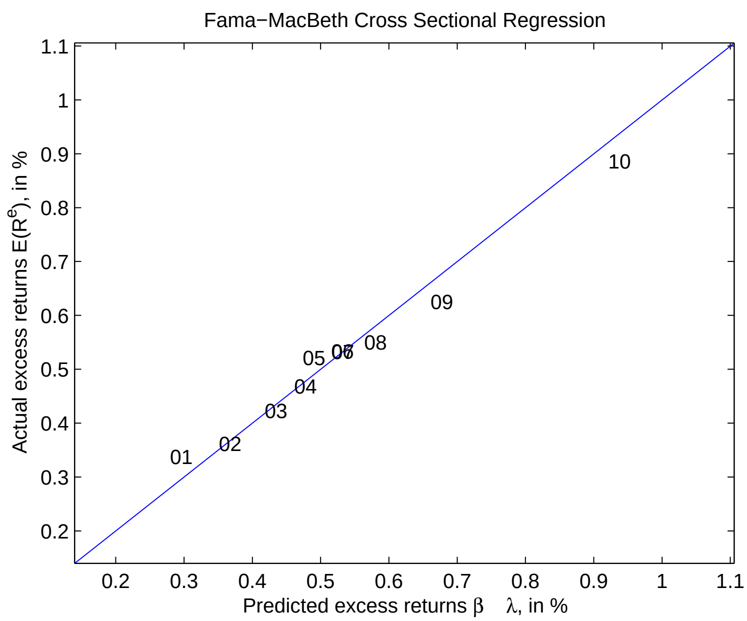
Notes: Figure 3 plots the OLS estimates of β_η^i and β_ε^i in the following time-series regressions of excess returns on each bond portfolio i on the pricing factor $j \in \{\eta, \varepsilon\}$: $R_t^{ei} = \beta_0^i + \beta_j^i f_{j,t} + e_t^i$ against the observed mean excess returns of test assets. The test assets are the 10 Fama Maturity Portfolios. The two pricing factors are respectively the current consumption growth shock $\eta_{c,t}$ and the expected future consumption growth shock $\varepsilon_{c,t}$. Excess returns on bond portfolios and pricing factors span the period from 1975Q1 to 2006Q4.

Figure 4 Composition of Risk Premia related to two pricing factors



Notes: Figure 4 plots risk premia related to two pricing factors against the observed mean excess returns of test assets. The risk premium for each pricing factor $j \in \{\eta, \varepsilon\}$ and each test asset $i = 1, 2, \ldots, 10$ is calculates as $\lambda_j \beta_j^i$. The two pricing factors are respectively the current consumption growth shock $\eta_{c,t}$ and the expected future consumption growth shock. The test assets are the 10 Fama Maturity Portfolios. Excess returns on bond portfolios and pricing factors span the period from 1975Q1 to 2006Q4. "predicted RP" is the sum of the risk premia related to the two pricing factors and this is the total risk premium for given test asset predicted by the model. "actual RP" is the observed risk premium for this asset and it equals its mean excess return so the pink line on the figure is a 45 degree line from the origin.

Figure 5 Actual vs Predicted Mean Excess Returns: Cross-Sectional Regression



Notes: Figure 5 plots the mean excess returns predicted from the two-factor model estimated using FMB Cross-Sectional Regression (horizontal axis) against the actual mean excess returns on the 10 Fama Maturity Portfolios over 30-day US Treasury bill rates (vertical axis). The data are quarterly, spanning the period from 1975Q1 to 2006Q4. The predicted and actual mean excess returns line up along a 45 degree line from the origin, suggesting that the observed returns are consistent with risks measured from the two-factor model.