Competitive Behavior in Market Games:

Evidence and Theory*

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September 2008

Abstract

We explore whether competitive outcomes arise in an experimental implementation of a *market game*, introduced by Shubik (1972). Market games obtain Pareto inferior (strict) Nash equilibria, in which some, and possibly all, markets are closed. We find that subjects do not coordinate on autarkic Nash equilibria, but favor more efficient "full" Nash equilibria in which all markets are open and there is a large volume of trade. We further find that as the number of agents participating in the market game increases, the full Nash equilibria they achieve come closer to approximating the associated Walrasian equilibrium of the economy. Motivated by these findings, we investigate theoretically whether evolutionary forces lead to Walrasian outcomes in such games. We introduce a strong version of evolutionary stable strategies (*SESS*) for finite populations. Our concept requires stability against deviations by coalitions of agents. A small coalition of trading agents is sufficient for Pareto improving trade to be generated. In addition, provided that agents lack market power, Nash equilibria corresponding to approximate competitive outcomes constitute the only approximate SESS.

Keywords: Market Games, Experiments, Full Nash Equilibrium, Walrasian Equilibrium, Evolutionary Stability.

JEL Codes: C72, C73, C92, D51.

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[*] We thank Martin Shubik, Steve Spear and seminar participants at several conferences and universities for useful comments and suggestions on an earlier draft. The usual disclaimer applies.

1 Introduction

Walrasian equilibrium is a cornerstone of modern economics. It is, therefore, not surprising that the question of price formation has received considerable attention in general equilibrium theory. The tâtonnement process has been used extensively in this context.[1] The study of tâtonnement, however, has produced largely negative results, and this has led some researchers to conclude that decentralized information about prices alone is not sufficient to bring the economy to a Walrasian equilibrium. In addition, and perhaps more importantly, the tâtonnement has been criticized for lacking micro-foundations since the price adjustment process is not the outcome of individual optimization.

Even setting the traditional stability question aside, Walrasian equilibrium may be challenged on the basis of complexity considerations. Can "unsophisticated" agents learn to behave in such a way that an outside observer of the economy will see a Walrasian equilibrium allocation? We begin this paper by exploring whether agents exhibit behavior consistent with Walrasian equilibrium in a laboratory implementation of a *market game*, introduced by Shubik (1972).[2] Market games are one of the structures that give rise to competitive outcomes when agents lack market power. Thus, it has served as a non-cooperative foundation for the Walrasian equilibrium. Even in large economies, however, in addition to approximately Walrasian outcomes, market games obtain Pareto inferior (strict) Nash equilibria, in which some, and possibly all, markets are closed due to a coordination failure. While market games offer an interesting benchmark, to our knowledge there are no attempts to experimentally implement a market game setup. Our work, thus, attempts to fill this gap and is in the spirit of exploring properties of different market structures in the laboratory.

Our experimental findings reveal that subjects placed in a two-good pure exchange environment that is subject to market game rules, do not coordinate on autarkic Nash equilibria, where markets are closed (even though such equilibria are strict) but instead favor more efficient "full" Nash equilibria in which all markets are open and there is a large volume of trade. We further find that as the number of agents participating in the market game increases, the full Nash equilibria they achieve come closer to approximating the associated Walrasian equilibrium of the economy. Indeed, our findings suggest that if the economy is sufficiently large, the market game mechanism will reliably lead agents to coordinate on Walrasian equilibrium allocations.

Motivated by these observations, and in order to understand our experimental findings, we built a theoretical model of a market game.[3] We consider a pure exchange economy with a large, finite number of agents

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[1] See Arrow and Hurwicz (1959) for a classic reference.

[2] There is extensive literature on market games. Standard references include Shapley (1977), Shapley and Shubik (1977), Dubey and Shubik (1977), and Mas-Colell (1982).

[3] We remark that our paper reverses the standard sequence in experimental economics research, in which an (often existing) theoretical model is implemented experimentally. In building a theoretical model in order to understand regularities observed in experimental data we follow a long tradition established in the natural sciences.

and a finite number of goods as in Postlewaite and Schmeidler (1978). Instead of imposing Nash-behavior, we study this game from an evolutionary point of view. After all, competitive outcomes are often justified by appealing to the natural selection of behavior that is more "fit."[4] Our story is not explicitly dynamic. Rather, we demonstrate that certain outcomes can be disturbed by the introduction of a small number of "noise-traders," some of whom can become better off in relative terms by choosing different trading patterns. Interestingly, the lack of market power is necessary for this to be true.[5]

We introduce a strong version of evolutionary stable strategies (*SESS*) for asymmetric, finite games and demonstrate that, in an approximate sense that we make precise, (partial) autarky outcomes are not SESS. Roughly speaking, *SESS* requires stability against all coalitions consisting of at most one agent per population. A suitable small-size coalition can generate trade and open a market. Thus, evolutionary forces provide an avenue through which the economy can avoid situations where some markets are closed due to a coordination failure. In addition, feasible outcomes in which all markets are open, and in which non-Walrasian prices prevail in some markets, can be disturbed by a small coalition. More precisely, we demonstrate that if the game is sufficiently large so that agents' market power is insignificant, Nash equilibria that support approximate Walrasian equilibria of the underlying economy are the only approximate *SESS*.

We can summarize the intuition behind our theoretical findings as follows. Since a single agent cannot create beneficial trade, Pareto inferior outcomes, in which some markets are closed, cannot be disturbed by one agent. On the other hand, the introduction of trading agents *from several sides of the market* is sufficient to rule out such outcomes. All other non-Nash states that involve trade, but not necessarily individual optimization, can be disturbed by a single agent who chooses the best basket at given prices. An important ingredient in our analysis is that the number of agents in the economy under study is much greater than the size of the deviating coalition. Consequently, while such coalitions can change certain agents' baskets, they have a small effect on prices. As a result, if the economy is close to a Walrasian equilibrium, no deviations by small-size coalitions can lead to improvements in an approximate sense. Thus, consistent with the traditionally held view, our findings provide support for the belief that evolutionary forces lead to competitive outcomes, but *only* when individual agents lack market power.[6]

The remainder of the paper is organized as follows. In the next Section we provide a motivating example of a simple, 2×2 market game that will be used in our experiment. Section 3 presents the experimental design and findings. Section 4 contains our main theoretical findings, beginning with the evolutionary solution

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[4] See Alchian (1950) for one of the first attempts to formalize this argument. See, for example, Weibull (1995), Vega-Redondo (1996), or Samuelson (1997) for a review of evolutionary models.

[5] Our results are related to Dubey and Shubik (1978), who introduce an outside agency that ensures that arbitrarily small amounts of bids and asks are present in all markets. Our argument, however, does not rely on the existence of such an agency. In addition, we impose minimal rationality requirements on our agents, and we explicitly consider non-Nash outcomes.

[6] More precisely, the Nash equilibrium in which all markets are open may fail to be evolutionary stable if it does not correspond to an approximate Walrasian equilibrium. This is in contrast with some recent papers in the literature, notably Vega-Redondo (1997).

concept used, followed by a description of a general market game environment and finally an application of evolutionary stability to this market game and statement of the main theoretical findings of the paper. Finally, Section 5 relates these findings to the existing literature. A technical Appendix follows.

2 2×2 Market Games

We begin by presenting a simple example of a market game that involves an Edgeworth box pure exchange economy with two types of agents and two consumption goods. This example serves two roles. First, it introduces the main setup in which our experiment we will based. Second, it motivates some of the issues that we will concentrate on in our theoretical study of more general market games.

There are two populations of equal size, each consisting of n agents. We will refer to agents in the first population as *type* I and those in the second population as *type* II. There are two consumption goods, x and y. Populations are distinguished by preferences and endowments. Agents of type i, where $i \in \{I, II\}$, have continuous, strictly increasing, strictly convex smooth preferences denoted by $\succeq^i$ and endowment $w^i = (w^i_x,\ w^i_y) \in \mathbb{R}^2_{++}$. We shall restrict ourselves to economies with a unique, interior Walrasian equilibrium.

Agents participate in a market game as in those studied by Shapley and Shubik (1977). Each agent can bid a positive amount of one good in exchange for an amount of the other good. The exchange rates, or prices, are determined as the ratios of the aggregate bids for the two commodities.

More precisely, let b^i_j be the amount of good j bid by agent i. Let $B_j = \sum_{i=1}^{2n} b^i_j$ be the total amount of good j bid by all agents. Bidding takes place according to the following rules. For all agents of type i, $i \in \{I, II\}$, we assume:

$$b^i_x = 0 \text{ or } b^i_y = 0, \text{ or both,} \tag{1}$$

$$0 \leq b^i_x \leq w_x \text{ and } 0 \leq b^i_y \leq w_y. \tag{2}$$

The first condition requires that agents may bid a positive amount of only *one* of the two goods. The second condition requires that these individual bids are feasible. The resulting allocation (also referred to as agent i's consumption basket) is determined by the following rules:

$$\begin{aligned} x^i &= w^i_x - b^i_x + b^i_y \frac{B_x}{B_y} \\ y^i &= w^i_y - b^i_y + b^i_x \frac{B_y}{B_x}. \end{aligned} \tag{3}$$

In order to "visualize" the mechanism, one could imagine the existence of a trading post where the total bids of the two goods are aggregated and then the total amount of good $x(y)$ bid is distributed across the good $y(x)$ bidders in proportions equal to the relative size of their individual bids.

A standard approach is to consider the Nash problem that each individual faces in this game. Assuming that agent i is a bidder of good x, this problem is as follows: given the total bids by all other agents, choose b_x^i so as to maximize $\succeq^i$ subject to constraints (1)- (2) and similarly for bidders of good y. Since agents of the same type are identical, it makes sense to consider outcomes where agents of the same type make the same bids and achieve the same allocation. It is straightforward to verify that $b_x^i = 0$, $b_y^i = 0$ is a strict, "autarkic" Nash equilibrium of this game. Unless the endowment point itself is the Walrasian equilibrium allocation, this equilibrium results in a Pareto inferior outcome. It is also straightforward to verify that there exists another strict Nash equilibrium in which trade occurs. This equilibrium is Pareto superior to autarky, yet, it is not efficient since agents in a finite game have some market power. This inefficiency, however, diminishes as the number of agents increases. In that case, the Nash equilibrium with trade approximates the Walrasian allocation of the underlying economy as each agent's bid becomes insignificant relative to the aggregate bid. We consider a numerical example next.

2.1 A Symmetric Example

Assume $n > 1$ of each of the two player types, I and II. The two types have the following endowments and preferences over amounts of the two goods, x and y:

$$w^I = (10, 200),\ u^I = x^2 y,\ \text{and}\ w^{II} = (200, 10),\ u^{II} = xy^2. \tag{4}$$

For the given preferences, the demand functions for the two player types are given by:

$$\begin{aligned} x^I &= \frac{2(p_x w_x^I + p_y w_y^I)}{3p_x}, \\ y^I &= \frac{(p_x w_x^I + p_y w_y^I)}{3p_y}, \\ x^{II} &= \frac{(p_x w_x^{II} + p_y w_y^{II})}{3p_x}, \\ y^{II} &= \frac{2(p_x w_x^{II} + p_y w_y^{II})}{3p_y}. \end{aligned}$$

Using market clearing and the endowments $w^i = (w_x, w_y)$ given in (4), the (unique) competitive equilibrium is given by:

$$\overline{p_x} = \overline{p_y} = 1;\ \overline{x}^I = 140;\ \overline{y}^I = 70;\ \overline{x}^{II} = 70;\ \overline{y}^{II} = 140. \tag{5}$$

The Nash problem for agent i of type I (and similarly for a type II agent) is:

$$\begin{aligned} & \max_{b_y^i \in [0, w_y^I]} \left(10 + \frac{b_y^i}{B_y} B_x\right)^2 (200 - b_y^i) \\ = & \max_{b_y^i \in [0, w_y^I]} \left(10 + \frac{b_y^i}{(b_y^i + \sum_{j \in I, j \neq i} b_y^j)} B_x\right)^2 (200 - b_y^i). \end{aligned}$$

The FOC for this problem gives

$$2\left[10+\frac{b_y^i}{\left(b_y^i+\sum_{j\in I,j\neq i}b_y^j\right)}B_x\right](200-b_y^i)B_x\left[\frac{\sum_{j\in I,j\neq i}b_y^j}{\left(b_y^i+\sum_{j\in I,j\neq i}b_y^j\right)^2}\right]$$
$$-\left[10+\frac{b_y^i}{\left(b_y^i+\sum_{j\in I,j\neq i}b_y^j\right)}B_x\right]^2=0.$$

Similarly for the type II agent. The two equations can be solved for the symmetric Nash bids:

$$2(200-b_y^i)B_x\left[\frac{\sum_{j\in I,j\neq i}b_y^j}{\left(b_y^i+\sum_{j\in I,j\neq i}b_y^j\right)^2}\right]=10+\frac{b_y^i}{\left(b_y^i+\sum_{j\in I,j\neq i}b_y^j\right)}B_x,$$

$$2(200-b_x^i)B_y\left[\frac{\sum_{j\in II,j\neq i}b_x^j}{\left(b_x^i+\sum_{j\in II,j\neq i}b_x^j\right)^2}\right]=10+\frac{b_x^i}{\left(b_x^i+\sum_{j\in II,j\neq i}b_x^j\right)}B_y.$$

In the symmetric Nash equilibrium,

$$b_x^i=b_x,\ b_y^i=b_y,\ \text{for } i=1,...,n,$$

and

$$B_x=nb_x,\ B_y=nb_y.$$

Thus, in the symmetric Nash equilibrium

$$2(200-b_y)nb_x\frac{(n-1)\,b_y}{(nb_y)^2}=(10+\frac{b_y}{nb_y}nb_x),$$
$$2(200-b_x)nb_y\frac{(n-1)\,b_x}{(nb_x)^2}=(10+\frac{b_x}{nb_x}nb_y),$$

or

$$2\,(n-1)\,(200-b_y)b_x = (10+b_x)nb_y, \tag{6}$$
$$2\,(n-1)\,(200-b_x)b_y = (10+b_y)nb_x. \tag{7}$$

Subtracting (7) from (6) we obtain

$$2\,(n-1)\,200(b_x-b_y)=10n(b_y-b_x). \tag{8}$$

Assuming $n>1$, equation (8) reveals that all Nash equilibrium solutions are such that $b_x=b_y$. Substitution of the latter restriction into (6) or (7) yields precisely two solutions. The first is the strict "autarkic" Nash equilibrium, where $b_x=b_y=0$, and allocations are equal to initial endowments. The second solution is the symmetric, full-trade Nash equilibrium, where

$$b_x=b_y=\frac{390n-400}{3n-2}>0,\ n>1.$$

No. of Each Player Type, n	Group Size $2n$	Full NE bids $b_x = b_y$	Full NE Allocation Type I; Type II	WE Allocation Type I; Type II
2	4	95	(105,105); (105,105)	(140, 70); (70, 140)
10	20	125	(135,75) ; (75, 135)	(140, 70); (70, 140)

Table 1: Equilibrium Predictions: NE=Nash Equilibrium, WE=Walrasian Equilibrium

The symmetric, full trade Nash equilibrium bids imply, via (1)-(3) and using (4) the following full Nash equilibrium allocation as a function of n:

$$\begin{aligned}
\widehat{x}^I &= 10 + \frac{390n - 400}{3n - 2}; \\
\widehat{y}^I &= 200 - \frac{390n - 400}{3n - 2}; \\
\widehat{x}^{II} &= 200 - \frac{390n - 400}{3n - 2}; \\
\widehat{y}^{II} &= 10 + \frac{390n - 400}{3n - 2}.
\end{aligned} \tag{9}$$

The parameterization used in this illustration was chosen because it yields integer values for the full Nash equilibrium allocation in the case where $n = 2$ and 10 (see Table 1 below) as well as for the competitive equilibrium allocation (5). These allocations will be used in the experiment of the following section.

3 The Experiment

This section derives the experimental findings that motivate our theoretical model. We begin by describing our experimental design. We then discuss our main experimental findings.

3.1 Experimental Design

The experiment uses a 2×2 design where the treatment variables are 1) the number of players in a group, 4 or 20, divided up equally between the two consumer types (I and II) so there are $n = 2$ or $n = 10$ of each player type, and 2) the initial endowment for each type, which was either given by $\{w^I = (10, 200)$, $w^{II} = (200, 10)\}$ or was set equal to the Pareto superior "full" trade Nash equilibrium levels, given in Table 1, and calculated according to (9). For comparison purposes, this table also reports the unique Walrasian competitive equilibrium allocation for all economies studied in this experiment using the calculation given in (5). Notice that as the number of subject increases from $n = 2$ of each type to $n = 10$ of each type that the full Nash equilibrium allocation comes closer to the Walrasian competitive equilibrium allocation, an observation we will revisit later.

The experimental design, summarized in Table 2, indicates that in two treatments, one with $n = 2$ and the other with $n = 10$, the initial endowments $\{w^I = (10, 200), w^{II} = (200, 10)\}$, were far away from the full

	Initial Endowment: $\{w^I = (10, 200), w^{II} = (200, 10)\}$	At Full NE Allocation
Group Size = 4	3 Sessions (16 Sbj/Sess., 4 Groups)	0 Sessions
Group Size = 20	3 Sessions (20 Sbj/Sess., 1 Group)	3 Sessions (20 Sbj/Sess., 1 Group)

Table 2: Experimental Design

Nash equilibrium predictions. In a third treatment, $n = 10$ subjects of each type began with endowments equal to the trade Nash equilibrium predictions $\{w^I = (135, 75),\ w^{II} = (75, 135)\}$. The case where $n = 2$ players of each type start out at the trade Nash equilibrium prediction was not studied experimentally. For each of the three cells in our experimental design, we have conducted three sessions for a total of nine sessions involving 168 subjects.

At the start of a session, subjects were randomly divided up into two equal-sized groups and assigned the roles of type I or type II players. Subjects were given instruction on the objectives of both player types but were informed that they would remain in the *same* role (type I or type II) in all 25 rounds of the experiment. Notice from Table 2 that in the $n = 2$ treatment, we used 16 subjects per session. In these sessions, the 16 subjects were randomly divided up into four groups of 4 subjects in each of the 25 rounds played, with each group consisting of two type I and two type II players. In the $n = 10$ treatment, we used 20 subjects per session, ten of type I and ten of type II players.

Subjects were given written instructions which were read out loud in an effort to make the instructions "common knowledge." A copy of the instructions used in one of the $n = 10$ treatments is given in the Appendix; instructions for the other treatments are similar. Subjects were informed that they would participate in 25 rounds of decision-making, with each round consisting of a repetition of the exact same decision problem. They were instructed that there were two goods, x and y, and they were informed of their initial endowments of these goods at the start of each round, and that these initial endowments would be the same at the start of every round. They were further instructed that their payoff for the round would depend on their *final* allocation of the two goods. They were also informed of their payoff function over final allocations: $\pi^I = x^2y$ or $\pi^{II} = xy^2$.[7]

The sequence of events in each round of the experiment was as follows. Subjects were reminded of their initial endowment of goods x and y and of their payoff (utility) function, all of which remained the same in every round. They were then asked to make one of three trading decisions: (1) trade good x for good y, (2) trade good y for good x, or, (3) no trade. Subjects who chose option (1) or (2) (trade) were then asked how

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[7] To aid subjects in calculating these payoffs, they were given tables displaying payoffs for type I and type II players as a function of the final allocations of x and y for a large (though incomplete) set of final allocations; this payoff table included the payoffs from not trading (final allocation = initial allocation) and from a wide variety of other final allocations including the trade Nash equilibrium allocation (see the instructions in the Appendix for the payoff tables shown to subjects).

many units of good x (b_x^i), or good y (b_y^i), they wanted to trade for the other good (y or x, respectively). Trade amounts were restricted to be integers between 1 unit and the subject's total endowment of the good offered in trade. Notice that while choices were limited to just one type of trade (or no trade), we did *not* restrict the choice of good that either player type was allowed to trade, and *no trade* of either good was always a choice option. Once all subjects had made their trading decisions, the computer program calculated $B_x = \sum_i b_x^i$ and $B_y = \sum_i b_y^i$ for each group and then determined each player's end-of-round allocation according the allocation rules (3). The manner in which these final allocations were made by the computer program was carefully explained to subjects.[8]

At the end of each round, subjects were informed of (i) the amount they chose to trade (bid) of one good for the other (if any), (ii) the total amounts bid of goods x and y, B_x and B_y, by all members of their group (size 4 or 20) including themselves, and even if they chose no trade, (iii) the fraction of B_x or B_y they individually acquired from their bid (if any), (iv) their individual final allocation of goods x and y, and finally, (v) their individual payoff in points for the round and the dollar value of that point total (the conversion rate of 100,000 points = \$1 was public information). To make this information as salient as possible, we not only showed it to subjects on their computer screens but we also asked them to record all these pieces of information on individual record sheets. Once this information was provided and recorded the round was over. If the 25th round had not yet been played, a new round would then begin.

Subjects were instructed that, at the end of the session, one round would be chosen randomly from all 25 rounds played and their dollar earnings for that round would comprise part of their earnings for the session. In addition, subjects were awarded a \$5 show-up payment. Total earnings averaged \$17.28 per subject in the three $n = 2$ sessions and \$18.42 per subject in the six $n = 10$ sessions. Each session lasted about 90 minutes.

Subjects were recruited from the undergraduate population at the University of Pittsburgh. No subject participated in more than one session.

3.2 Experimental Results

We begin by considering trading decisions, "end-of-round" or "final" allocations, and payoff efficiency across the three treatments of our experimental design. Specifically, we report (1) the average frequency with which each player type chose "No Trade," (2) the average amount bid of each good by each player type (among those players choosing to trade), (3) the average, final allocation for each player type, and finally, as a measure of efficiency, (4) the average payoff earned by each player type as a percentage of the full trade Nash equilibrium (Full NE) payoff.

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[8] Indeed, before playing, subjects had to answer a number of quiz questions that tested their understanding both of the experimental design, the trading rules, the final-allocation market mechanism and their understanding of the payoff table. The interested reader is referred to the experimental instructions, provided in the Appendix, for further details.

To avoid possible learning effects (for now), we calculate all of four statistics for each player type using data from the last 5 rounds of each session (we will consider behavior over all 25 rounds and individual behavior later in this section). These aggregate statistics are reported in Tables 3-4, where average amounts bid ("Avg. Bid"), and average final allocations ("Final Alloc."), are represented as *pairs* in the format (amount of good x, amount of good y).

Session	$n =$	**Type I**, $w^I = (10, 200)$				**Type II**, $w^{II} = (200, 10)$			
		% No Trade	Avg. Bid	Final Alloc.	Pay. Eff. % Full NE	% No Trade	Avg. Bid	Final Alloc.	Pay. Eff. % Full NE
1	2	0.00	(0,95)	(91, 105)	0.727	0.00	(81,0)	(119, 105)	1.152
2	2	0.00	(0,83)	(115, 117)	1.264	0.00	(105,0)	(95, 93)	0.737
3	2	0.00	(0,103)	(121, 97)	1.194	0.00	(111,0)	(89, 113)	0.924
Avg. 1-3	2	0.00	(0,94)	(109, 106)	1.062	0.00	(99,0)	(101, 104)	0.937
Full NE	2	0.00	(0,95)	(105, 105)	1.000	0.00	(95,0)	(105, 105)	1.000
WE	–	–	(0,130)	(140, 70)	1.185	–	(130,0)	(70, 140)	1.185
1	10	0.00	(0,130)	(129, 70)	0.833	0.00	(119,0)	(81, 140)	1.111
2	10	0.00	(0,113)	(124, 87)	0.933	0.00	(114,0)	(86, 123)	0.948
3	10	0.00	(0,126)	(140, 74)	1.013	0.00	(130,0)	(70, 136)	0.902
Avg.1-3	10	0.00	(0,123)	(131, 77)	0.926	0.00	(121,0)	(79, 133)	0.987
Full NE	10	0.00	(0,125)	(135, 75)	1.000	0.00	(125,0)	(75, 135)	1.000
WE	–	–	(0,130)	(140, 70)	1.004	–	(130,0)	(70, 140)	1.004

Table 3: Averages by Player Type I, II, from the Last Five rounds of Sessions where $w^I = (10, 200)$, $w^{II} = (200, 10)$

Consider first the results for the $n = 2$ and $n = 10$ treatments where subjects start out with $w^I = (10, 200)$ and $w^{II} = (200, 10)$ as reported in Table 3. The results in both treatments are striking: we see that all subjects are choosing to engage in trade (% No Trade = 0.0), so subjects have clearly chosen to move *away* from the strict, autarkic Nash equilibrium. Further, as a first test of the subjects' understanding of the game, we observe that type I subjects are choosing only to bid good y for good x, as evidenced by the 0 in the first element of their bid pairs, and likewise, type II subjects are choosing only to bid good x for good y, as evidenced by the 0 in the second element of their bid pairs (and not the opposite type of trades). Such bidding behavior was *not* imposed on subjects but rather follows from the objective of maximizing one's type-specific payoff function. That is, subjects nearly always offered to bid for the good with the higher marginal payoff for their type, even though they were free to bid for either type of good, or not to bid at all. Most importantly, subjects in these treatments bid so as to achieve final allocations that are very close to the full Nash equilibrium (Full NE) by the final rounds of all sessions. Both the average amounts bid and the final, end-of-round allocations are very close to these full NE predictions in both the $n = 2$ and $n = 10$ treatments. Indeed, we also see that subject are achieving approximately 100% of the payoffs they could have earned had all played according to the full Nash equilibrium; in some instances certain player types are doing slightly better, due to less than complete coordination on the NE and the inefficiency of that equilibrium relative to the Walrasian equilibrium. Notice further that most of these averages lie below the Walrasian equilibrium allocation (WE) predictions, which is consistent with theoretical predictions. Perhaps most importantly, there is strong evidence for the comparative static prediction of greater trade volume (bid amounts) as the number of each player type increases from 2 to 10, with no change in efficiency. Using the three session–level averages for the bids of type I players (of good y for good x) or the bids of type II players (of good x for good y), a non-parametric, Mann-Whitney test confirms that we can reject the null hypothesis of no difference in bid amounts between the $n = 2$ and $n = 10$ treatments in favor of the alternative hypothesis that bids by both player types are *greater* when $n = 10$ than when $n = 2$ ($p = 0.05$ for both tests).[9]

This is powerful evidence that, as the number of agents in the economy increases; i.e., as the economy becomes large, strategic considerations (i.e., imperfect competition) diminish in importance, as the full trade NE allocation gradually approximates that of the perfectly competitive Walrasian equilibrium (WE). This is in contrast to experimental findings using double auction trading rules. Under conditions of incomplete information, such experiments suggest that the number of agents on either side of the market– the extent of market power – matters little for whether convergence to Walrasian, competitive equilibrium obtains.[10] The fact that strategic considerations (group size) matter in a predictable and intuitive way under the market game trading rules would seem to make the market game mechanism a compelling alternative to the double auction for understanding the conditions under which a competitive equilibrium may be reached.

		Type I, $w^I = (135, 75)$				**Type II**, $w^{II} = (75, 135)$			
Session	$n =$	% No Trade	Avg. Bid	Final Alloc.	Pay. Eff. % Full NE	% No Trade	Avg. Bid	Final Alloc.	Pay. Eff. % Full NE
1	10	0.48	(0,6)	(141, 69)	1.004	0.30	(6,0)	(69, 141)	0.988
2	10	0.32	(0,11)	(147, 65)	1.007	0.26	(13,0)	(63, 145)	0.928
3	10	0.34	(1,11)	(147, 65)	0.996	0.18	(13,1)	(63, 145)	0.953
Avg. 1-3	10	0.38	(0,9)	(145, 66)	1.002	0.25	(11,0)	(65, 144)	0.956
Full NE	10	1.00	(0,0)	(135, 75)	1.000	1.00	(0,0)	(75, 135)	1.000
WE	–	–	(0,5)	(140, 70)	1.004	–	(5,0)	(70, 140)	1.004

Table 4: Averages by Player Type I, II, from the Last Five rounds of Sessions where $w^I = (135, 75)$, $w^{II} = (75, 135)$, i.e., the Full NE allocation.

Next, we consider the case where groups of 20 subjects ($n = 10$ of each type) start out with endowments

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[9] The same test applied to the session level data on *payoff efficiency* relative to the full NE confirms that we cannot reject the null hypothesis of no difference in that statistic as n increases from 2 to 10 ($p \geq .35$ for all pairwise comparisons); that is, subjects are playing just as efficiently in groups of size 4 as they are in groups of size 20.

[10] For instance, markets organized under double-auction rules generate near-competitive outcomes even in the case of a single, monopoly seller (e.g., Smith (1981)), where the seller's efforts to achieve and sustain monopoly prices are ultimately unsuccessful, or in cases where one side of the market has opportunities for pre-trade collusion (Isaac and Plott (1981)).

equal to the full NE allocation, as reported in Table 4. Here we see that, contrary to the strict, full NE prediction, a majority of subjects *do* choose to engage in some trade – the percentage clicking on the "no trade" button (% No Trade) over the last five rounds of each session averages just 38% for type I and 25% for type II players. While this represents a statistically significant *increase* in the frequency of subjects choosing no trade over the $n = 10$ treatment where subjects did not start out at the full trade NE allocation ($p = .05$, Mann-Whitney test), the frequency of subjects choosing no trade is still far less than the predicted 100%. We make a couple of observations about this result. First, notice that the amounts traded are rather *small.* Type I agents trade, on average, just 9 units of good y for good x while type II agents trade, on average just 11 units of good x for good y. The alternative type of trade available to each player type averages 0 as in the other treatments. A Mann-Whitney test using session level data confirms that trade volume (bids of good y for x by type I players and bids of good x for y by type II players) is significantly less in the $n = 10$ treatment where subjects start out at the full NE allocation than in the $n = 10$ treatment where they do not ($p = .05$). Second, somewhat surprisingly, these small amounts of trade move subjects very close to the Walrasian equilibrium (WE) allocation. Subjects, on average, overshoot the WE to a small degree, by acquiring a little too much of the good with the higher marginal payoff. Indeed, because of this overshooting payoff efficiency generally lies below both the full Nash and the WE predicted levels.[11] While these findings violate the no-trade prediction of this treatment, the violation seems rather small and could be partially rationalized as follows: the full NE endowment is not Pareto optimal while the WE allocation is, and subjects are trying to find a way to achieve the latter, despite the game theoretic prediction that the WE is not a Nash equilibrium in this environment. Yet, there is a continuum of other Pareto optimal allocations and the power of WE in this context remains somewhat puzzling.

[Insert Figures 1-3 here.]

Thus far we have considered only *aggregate, session-level* averages. Figures 1-3 show all *individual* final allocations (averaged over the final 5 rounds) using data from all sessions of each of the three treatments. These individual allocations are situated within the Edgeworth box representing the pure exchange economy that we implemented experimentally. In Figure 1, we see that for the $n = 2$ treatment, where initial endowments are $w^I = (10, 200)$ and $w^{II} = (200, 10)$, by the final 5 rounds there remains considerable dispersion in individual final allocations, though the averages for the two player types are very close to the full NE prediction. This dispersion is attributable to the relatively greater market power subjects have in this treatment where they interact in groups of 4. By contrast, as Figure 2 shows, when the group size increases from 4 to 20 ($n = 10$ of each type) so that market power is greatly reduced, the dispersion in individual,

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[11] However, we again find that we cannot reject the null hypothesis of no difference in payoff efficiency for this treatment relative to the other two treatments involving $n = 2$ or 10 subjects of each type where initial endowments were far away from the full NE ($p \geq .35$, for all pairwise comparisons using the Mann-Whitney test on session–level data).

average final allocations is also greatly reduced, with the averages for each player type remaining very close to the full NE prediction. Finally, as Figure 3 shows, when subjects interact in groups of 20 and start out at the full NE, $w^I = (135, 75), w^{II} = (75, 135)$, the dispersion in individual, average final allocations is even further reduced. In this case, as noted earlier, the average final allocations are closer to the WE prediction than to the full NE.

[Insert Figures 4-6 here.]

In addition to considering allocations over the last 5 rounds of a session it is instructive to consider subjects' *bidding behavior* over the *entire* 25 rounds of each session so as to assess whether there is any evidence of trends or learning behavior over time. Figures 4-6 show time series on the average bids of good x for good y or of good y for good x by both player types using pooled data from all three sessions of a treatment. Please note the use of a different vertical scale in all three figures: The smaller vertical scale on the LHS is for type $I(II)$ bids of good $x(y)$ for good $y(x)$ - which are predicted to be zero – while the 10 × larger scale on the RHS is for type $I(II)$ bids of good $y(x)$ for good $x(y)$, which, in treatments where subjects' initial endowments were *not equal* to the full NE (Figures 4-5 only), are predicted to be strictly positive. Notice that, consistent with theoretical predictions, type I players quickly learn only to bid good y for good x, and type II players quickly learn only to bid good x for good y; the alternative types of bids (dashed lines in Figures 4-6), while available to subjects, are generally small (use the LHS scale) and converge to zero over time in all treatments. Second, notice that the *amounts* types $I(II)$ bid of good $y(x)$ for good $x(y)$ are *very close* to the full NE predictions after only a few rounds. Specifically, in Figure 4, where subjects interact in groups of size 4, it is predicted that type $I(II)$ bids 95 units of good $y(x)$ for good $x(y)$, and subjects are on average close to this prediction after 5 rounds of play. Similarly, in Figure 5 where subjects interact in groups of size 20, the prediction is for type $I(II)$ to bid 125 units of good $y(x)$ for good $x(y)$, and this prediction is, on average, close to being met after around 10 rounds of play. Finally, in Figure 6, where subjects interact in groups of size 20 but have endowments equal to the full NE, the prediction is that all bid amounts should be zero. Here we see that type $I(II)$ bids of good $y(x)$ for good $x(y)$ start out averaging around 20 and decrease to an average of around 10 over the 25 rounds, so there is some evidence that subjects in this treatment are learning not to bid, though it appears that this learning process was not complete in the time-frame allowed by our design. Alternatively, as noted earlier, subjects in this treatment are very close to achieving the WE allocation, which would involve type $I(II)$ bidding 5 units of good $y(x)$ for good $x(y)$. As mentioned before, such non-Nash bidding remains puzzling. Yet, while it cannot be rationalized as a Nash equilibrium, the WE allocation is payoff superior to the full NE, and given the number of subjects involved in trading, such payoff considerations may play a role for the findings in this treatment.

Summarizing our experimental design and findings, we have implemented a simple, 2-good, 2-player type market game in the laboratory; to our knowledge, this is the first-ever experimental test of this important class of games which serve as a bridge between non-cooperative game theory and general equilibrium analysis. Importantly we have found that the autarkic (no trade) outcome, while a strict Nash equilibrium, is never observed in our sessions where subjects' initial endowments are far from the full NE allocation. Instead, subjects quickly learn to bid so as to implement allocations that closely approximate the full NE allocations, on average. A further, striking finding is that the *size* of the economy, in terms of the number of agents of each player type, matters greatly for the realized outcome. In particular, we find that when subjects interact in small groups of size 4 (2 of each type) and thus individually possess some market power, the full NE allocation to which such groups converge is far from the competitive, WE allocation. By contrast, when subjects interact in groups of 20 (10 of each type) and thus individually possess comparatively less market power, the full NE allocation to which groups converge is much closer to the competitive WE allocation. As we discuss in the next Section, this finding is consistent with theoretical predictions in the context of a market game. Yet, it stands in sharp contrast to a large body of experimental work using double oral auctions (mentioned above) which, somewhat counter to economic intuition, shows that the size of the market economy matters little for whether or not a competitive equilibrium is achieved; competitive equilibria in experimental implementations of these auctions are reliably achieved with very small numbers of agents on either side of the market (three or less). Finally, when subjects' initial endowments are equal to the full NE allocation, we demonstrated that there is significantly less trade. In addition, and somewhat surprisingly, the trade that does occur moves allocations in the direction of the WE allocation.

Having presented strong evidence that human subjects eschew the strict, autarkic NE in favor of the full NE in all cases, and that as predicted, the full NE allocation approximates the competitive WE allocation as the size of the economy (number of each player type) grows, we next turn toward providing a theoretical explanation of these findings.[12]

4 The Theoretical Model

In order to help us understand the experimental data, our theoretical model must have two important features. First, it must be able to assist in equilibrium selection by distinguishing among strict Nash equilibria. More precisely the model must generate a process through which autarky can be escaped and a NE with trade might be reached. Secondly, the model must be able to distinguish between NE and WE and generate the right comparative statics. In particular, Nash behavior must approximate Walrasian behavior as agents'

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[12] In a way, our approach reverses the typical order of inquiry in the social sciences and follows a path that is more common in the natural sciences: having designed a sequence of experiments and having observed certain regularities in experimental data, we build a theoretical model in an attempt to understand this data.

market power declines. In what follows, we build a model that has these features. In order to encompass the first feature, we extend existing concepts in evolutionary game theory in order to account for the possibility of “coalitional deviations,” and in order for our concepts to be applicable in the context of market game-like exchange. In order to encompass the second feature, we will build on a version of a market game introduced by Postlewaite and Schmeidler (1978). Their version allows us to study properties of the NE with trade as the number of agents in the underlying economy becomes large. While our theoretical analysis is motivated by our experimental findings, we believe that it can be useful in other context. To this end, we first develop an abstract model that we later interpret in our specific context.

4.1 The Solution Concept

We begin by stating two existing definitions of evolutionary stability in the context of an abstract normal form game. First, consider a single population consisting of a continuum of identical agents, and assume that N agents are selected to play a normal-form game $\Gamma = (N, S, U)$, where S is the set of available (pure) strategies, and U represents payoffs. The definition of an *Evolutionary Stable Strategy (ESS)* for ($N = 2$)-player symmetric games is as follows (see Weibull, 1995):

Definition 1 *A strategy $s \in \Delta$ is an* **ESS** *if, for every strategy $t \in \Delta$, $t \neq s$, there exists $\varepsilon_t > 0$ such that*

$$U\left(s, (1-\varepsilon)s + \varepsilon t\right) > U\left(t, (1-\varepsilon)s + \varepsilon t\right), \tag{10}$$

for all $\varepsilon \in (0, \varepsilon_t)$, where Δ is the set of all mixed strategies.

Next, consider any finite population of size N. The definition of *ESS* for N-player symmetric games is as follows (see Schaffer, 1988, 1989):

Definition 2 *A strategy $s \in S$ is an* **ESS** *if, for any strategy $t \in \Delta$, $t \neq s$,*

$$U\left(s, (t, \overline{s})\right) \geq U(t, (s, \overline{s})), \tag{11}$$

where $(t, \overline{s})$ and $(s, \overline{s})$ denote the strategies of the other $(n-1)$ players. In particular, $(s, \overline{s})$ indicates that all other players play strategy s, while $(t, \overline{s})$ indicates that one player plays strategy t, while all other players play s.

Note that, unlike Nash equilibrium, the *ESS* criterion refers to *relative* performance. We will amend Schaffer's (1988) definition in two ways. First, we extend the definition of an *ESS* from one to multiple, distinct, finite populations. Second, we will require a strong version of evolutionary stability: one that requires stability against simultaneous deviations by multiple agents from different populations.

We first present the concept in the context of an example. In the next section, we will apply it to a market game. Assume that there are $K > 1$ finite populations. Each population, i, contains $n_i \geq 2$ agents. We assume that agents play an N-player game, Γ, where $N = n_1 + ... + n_K$. The game is assumed to have the following symmetry property. All players from population i have the same set of strategies, X^i, and the same payoff function, U^i. In other words, if two players (from the same population) play the same strategy, they will obtain the same payoffs. Hence, we indicate the normal form game as

$$\Gamma = \left(\{n_1 + ... + n_K\} ; \underbrace{S^1 \times ... \times S^1}_{n_1 \text{ times}} \times ... \times \underbrace{S^K \times ... \times S^K}_{n_K \text{ times}} ; \left(U^1; ...; U^K\right) \right).$$

In what follows, we will need to consider the situation where one agent from population i plays strategy t^i, while every other agent from that population plays strategy s^i. More generally, in the case where at most one agent in each population plays a strategy, t, which is different from the one chosen by every other agent in his population, the payoff of the agent from population i who plays a different strategy than his peers can be written as:

$$U^i \left(t^i; (t^1, \overline{s^1}); ...; (t^i, \overline{s^i}); ...; (t^K, \overline{s^K}) \right), \tag{12}$$

where, as before, $\left(t^i, \overline{s^i}\right)$ denotes that one agent from population i plays t^i, while all other agents from population i play s^i.

We are now ready to define our main concept.

Definition 3 *A symmetric strategy profile* $\overline{s} = \left(\underbrace{s^1, ..., s^1}_{n_1}; ...; \underbrace{s^K, ..., s^K}_{n_K} \right) \in \underbrace{S^1 \times ... \times S^1}_{n_1} \times ... \times \underbrace{S^K \times ... \times S^K}_{n_K}$ *is a* **Strong ESS** *(***SESS***) if, for all i,*

$$U^i \left(s^i; \gamma^1, ..., \gamma^K\right) \geq U^i \left(t^i; \gamma^1, ..., \gamma^K\right), \tag{13}$$

for any strategy $t^i \in S^i$, $t^i \neq s^i$, *and for all* γ^j, *such that* $\gamma^j = \left(s^j, \overline{s^j}\right)$, *or* $\gamma^j = \left(t^j, \overline{s^j}\right)$.

In other words, a notable feature of the *SESS* is that it requires stability against up to K simultaneous deviations (one per population). Clearly, this is a stronger concept than Schaffer's *ESS*. Thus, like *ESS*, *SESS* will not exist in general. An important feature of our concept is that while it requires a symmetric outcome, it can be applied to asymmetric games. Below, we give an example of a four-player coordination-like game in which *SESS* uniquely selects the Pareto efficient Nash equilibrium even though there is another *ESS*.

Example: Suppose that there are two populations (I and II), each consisting of two players. Each player has two strategies (a and b). Let $\theta_{I(II)}$ stand for the number of a-players in population $I(II)$. Payoffs are defined as follows.

$U_I(a,\theta_I,\theta_{II})$	$U_I(b,\theta_I,\theta_{II})$	$U_{II}(a,\theta_I,\theta_{II})$	$U_{II}(b,\theta_I,\theta_{II})$
$U_I(a,1,0)=0$	$U_I(b,0,0)=2$	$U_{II}(a,0,1)=0$	$U_{II}(b,0,0)=2$
$U_I(a,2,0)=0$	$U_I(b,1,0)=2$	$U_{II}(a,0,2)=0$	$U_{II}(b,0,1)=2$
$U_I(a,1,1)=3$	$U_I(b,0,1)=1$	$U_{II}(a,1,1)=3$	$U_{II}(b,1,0)=1$
$U_I(a,2,1)=3$	$U_I(b,1,1)=1$	$U_{II}(a,1,2)=3$	$U_{II}(b,1,1)=1$
$U_I(a,1,2)=4$	$U_I(b,0,2)=0$	$U_{II}(a,2,1)=4$	$U_{II}(b,2,0)=0$
$U_I(a,2,2)=4$	$U_I(b,1,2)=0$	$U_{II}(a,2,2)=4$	$U_{II}(b,2,1)=0$

For example, $U_I(a,1,0)=0$ means that the payoff of an a-player from population I, when all other players (one player in population I and two players in population II) play action b, is zero. This game obtains two symmetric strict Nash equilibria in which all agents play a or all play b, respectively. The a-equilibrium is an *SESS*. Notice, however, that the b-equilibrium is not an *SESS* since a coalition consisting of one agent per population (type) deviating to playing a will result in a payoff of 3 for each of the two deviators (instead of 1 for the b-players).

Later, we shall make use of the following approximate notion of an *SESS*.

Definition 4 *A symmetric strategy profile* $\overline{s}=\left(\underbrace{s^1,...,s^1}_{n_1};...;\underbrace{s^K,...,s^K}_{n_K}\right)\in\underbrace{S^1\times...\times S^1}_{n_1}\times...\times\underbrace{S^K\times...\times S^K}_{n_K}$ *is an* ϵ-**SESS** *if, for all* i,

$$U^i\left(s^i;\gamma^1,...,\gamma^K\right)\geq U^i\left(t^i;\gamma^1,...,\gamma^K\right)-\epsilon, \tag{14}$$

for any $t^i\neq s^i$, *and for all* γ^j, *such that* $\gamma^j=\left(s^j,\overline{s^j}\right)$, *or* $\gamma^j=\left(t^j,\overline{s^j}\right)$.

Thus, an ϵ-SESS requires that no agent can be better off by more than a small amount, ϵ. In the next section we motivate and use ϵ-*SESS* in the context of our main topic of study, a strategic market game.

4.2 The General Market Game

For our theoretical analysis we consider a finite, convex pure exchange economy with L consumption goods.[13] The economy is described by $\mathcal{E}=\left\langle I,X^i,w^i,u^i\right\rangle_{i\in I}$, where $I=\{1,...,nK\}$ is a finite set of agents belonging to $K>1$ different populations (or types); $X^i=\mathbb{R}_+^L$ denotes the consumption possibility set for agent i; $w^i\in\mathbb{R}_+^L$ is the endowment vector of agent i; and $u^i:\mathbb{R}_+^L\to\mathbb{R}$ is the utility function of agent i. Agents belonging to the same type have identical preferences and endowments. We assume that u^i is continuous, strictly increasing in all its variables, and strictly quasi-concave.

Agents participate in a strategic market game related to the one in Shapley and Shubik (1977). We will follow Postlewaite and Schmeidler (1978), henceforth **PS**, in specifying the market game corresponding to

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[13] The reader should think of the 2x2 model studied in our experimental section as a special case of the general market game studied here.

$\mathcal{E}$. An nK-person market game in normal form is defined as follows. For each $i \in I$, let $S^i = \{s^i = (b^i, q^i) \in \mathbb{R}_+^L \times \mathbb{R}_+^L : q^i \leq w^i\}$ be the set of strategies available to agent i. Given any (symmetric) nK-list of strategies $(b^i, q^i)_{i \in I}$, the payoff to agent i is denoted by $U^i\left((b^1, q^1); ...; (b^i, q^i); ...(b^{nK}, q^{nK})\right)$. Here, b^i denotes the vector of bids or "goods requested" by agent i, measured in abstract units of account, while q^i denotes the vector of goods offered by agent i. $U^i : S^1 \times ... \times S^{nK} \to \mathbb{R}$ is the von Neumann and Morgenstern utility function of agent i.

Individual agents have to satisfy a *balance or bankruptcy condition,* which requires that the total value of an agent's bids be less than the total "receipts" from his sales of goods. More precisely, the individual balance condition is given by

$$\sum_{l \in L} b_l^i \leq \sum_{l \in L} \frac{q_l^i}{\sum_{j \in I} q_l^j} \sum_{j \in I} b_l^j. \tag{15}$$

One issue is what happens to agents who violate the balance condition. This is particularly important in our case for two reasons. First, unlike **PS**, we explicitly consider non-Nash states in which this constraint might be violated. Second, since agents in our model are concerned with *relative* performance, they might take an action that will make them worse off in absolute terms if this leads to other agents of their type becoming further worse off. This could occur, for example, if an action by a single agent led to other agents' becoming bankrupt. This possibility would arise under the **PS** specification since they assume that agents who violate the balance condition have all their resources confiscated. With these considerations in mind, we impose the milder assumption that an agent whose total value of goods requested exceeds that of his total receipts has his bid vector "shaved" by an amount that is proportional to his overbidding. More precisely, let

$$\alpha^i = \frac{\sum_{l \in L} \frac{q_l^i}{\sum_{j \in I} q_l^j} \sum_{j \in I} b_l^j}{\sum_{l \in L} b_l^i} \tag{16}$$

and let

$$\widehat{b}_l^i = \begin{cases} \alpha^i b_l^i, & \text{if } \sum_{l \in L} b_l^i > \sum_{l \in L} \frac{q_l^i}{\sum_{j \in I} q_l^j} \sum_{j \in I} b_l^j \\ b_l^i, & \text{otherwise.} \end{cases} \tag{17}$$

The resulting consumption baskets (previously referred to as allocations) are defined as follows. For all $i \in I$, and $l \in L$, let the consumption of good l by agent i be given by

$$c_l^i = w_l^i - q_l^i + \frac{\widehat{b}_l^i}{\sum_{j \in I} \widehat{b}_l^j} \sum_{j \in I} q_l^j. \tag{18}$$

Note that each strategy profile $\left((b^1, q^1); ...; (b^i, q^i); ...(b^{nK}, q^{nK})\right)$ uniquely determines agents' consumption $\left(\left(c_1^1, ..., c_L^1\right), ..., \left(c_1^{nK}, ..., c_L^{nK}\right)\right)$ using (18).

A (symmetric) strategy profile $(\widehat{s}^1, ..., \widehat{s}^{nK})$ is a *Nash equilibrium* if for all $i \in I$ and all $s^i \in S^i$,

$$U^i(\widehat{s}^1, ..., \widehat{s}^i, ...\widehat{s}^{nK}) \geq U^i(\widehat{s}^1, ..., s^i, ...\widehat{s}^{nK}).$$

A Nash equilibrium is *full* if all markets are open; i.e., for all $l \in L$,

$$\sum_{i\in I} \widehat{b}_l^i > 0 \text{ and } \sum_{i\in I} \widehat{q}_l^i > 0.$$

We shall only consider economies where a full Nash equilibrium exists, in which there is (sufficiently large) positive trade in all commodities.

Proceeding as in **PS**, for all l, and for a distinguished agent i, we can write $B_l = b_l^i + B_l^{-i} = b_l^i + \sum_{j\in I, j\neq i} b_l^j$, and $Q_l = q_l^i + Q_l^{-i} = q_l^i + \sum_{j\in I, j\neq i} q_l^j$. Let $p_l = B_l/Q_l$ denote the average price of commodity l (provided that the denominator of this expression is strictly positive). Define an allocation $\widehat{z}$ resulting from a full Nash equilibrium to be ϵ-Walrasian if all markets are open, and there exists $\widehat{p} = (\widehat{p}_1, ..., \widehat{p}_L)$ such that for all $i \in I$, $\widehat{p}\widehat{z}^i = \widehat{p}w^i$, and

$$\#\{i \in I : \forall z^i,\ u^i\left(z^i\right) > u^i\left(\widehat{z}^i\right) \Rightarrow \widehat{p}z^i > \widehat{p}(1-\epsilon)w^i\} > (1-\epsilon)\#I,$$

where, as stated above, prices correspond to ratios of aggregate bids. We first state the main result of **PS**. It establishes the connection between full Nash equilibria of the market game and approximate Walrasian equilibria of the underlying economy.

Proposition 1 (**PS**)**:** *For any positive numbers α, β, and ϵ, any allocation resulting from a full Nash equilibrium in an economy $\mathcal{E} = \left\langle I, X^i, w^i, u^i \right\rangle_{i\in I}$ with $w^i < \beta(1, ...1)$ for all $i \in I$, $\sum_{i\in I} w^i > N\alpha(1, ..., 1)$, and $N > 16L\beta/\alpha\epsilon^2$ is ϵ-Walrasian.*

This completes the description of the market game. For a more detailed discussion of these concepts, we refer the reader to **PS.** Henceforth, we will concentrate on the evolutionary stability properties of the full Nash equilibria.

4.3 Evolutionary Stability

Before we analyze the market game from an evolutionary point of view, we introduce the main argument in an informal way. This will also serve as a motivating discussion for the concept introduced in the previous section. First, notice that no Nash equilibrium in which some markets are closed can be disturbed by a single deviating agent. This is because at least one agent *on each side of the market* is necessary for any trade. While the existence of such (partial) autarky Nash outcomes is plausible, it is also insightful to study under what conditions evolutionary forces will result in the "opening of markets," leading to a Pareto superior outcome. The fact that this requires multiple simultaneous deviations is exactly what *SESS* is designed to capture.

A separate issue from whether all markets will be open is whether evolution will give rise to an efficient or, more restrictively, to a Walrasian outcome. Even if no state in which some or all markets are closed corresponds to an *SESS*, one might ask whether states that correspond to full Nash equilibria are *SESS*. Here, a difficulty arises. The fact that we deal with a finite game implies that each individual agent has some market power. Of course, market power vanishes as the number of agents increases. This suggests that in the case where the economy is large enough, we can expect the above question to be answered in the affirmative, but only in an approximate sense.

To see this, let us assume that the economy is at a full Nash equilibrium. Suppose that an agent switches to a different bid/offer. Clearly, since the previous situation was a Nash equilibrium, the deviating agent will be worse off. However, this does *not* imply the evolutionary stability of full Nash equilibria. The reason is as follows. Since there is a finite number of agents, the deviation will result in slightly different prices for at least some agents. While the deviator is worse off under the new prices, the other agents of his type may be even more worse off or, in other words, the deviator could be better off *in relative terms*. Thus, the evolutionary stability of full Nash equilibria is not automatic. A continuity argument, however, guarantees that if the economy is large enough, a deviation by a small-size coalition cannot make the deviators better off by more than an arbitrarily small amount. Thus, a full Nash equilibrium of a large enough economy, which **PS** have shown to be approximately Walrasian, will also be an approximate *SESS*, provided that agents lack significant market power. Formalizing the details of this argument is the main purpose of this section. We begin by presenting a definition of ϵ-*SESS* in the context of a market game. Define the price for good l faced by agent i by $p_l^i = B_l^{-i}/Q_l^{-i}$, with the convention that $p_l^i = 0$ if $B_l^{-i} = Q_l^{-i} = 0$. Then,

Definition 5 *A symmetric strategy profile* $\overline{s} = \left(\overline{s}^1, ..., \overline{s}^1; ...; \overline{s}^K, ..., \overline{s}^K\right) \in \underbrace{S^1 \times ... \times S^1}_{n} \times ... \times \underbrace{S^K \times ... \times S^K}_{n}$ *is an* ϵ-**SESS** *of the market game if, for all* $i \in I$

$$U^i\left(\overline{s^i}; \widetilde{\mathbf{p}}^i\right) \geq U^i\left(t^i; \widetilde{\widetilde{\mathbf{p}}}^i\right) - \epsilon, \tag{19}$$

for any $t^i \in S^i$*, and for all* $\widetilde{\mathbf{p}}^i$*, where* $\widetilde{\mathbf{p}}^i = \left(\widetilde{B}_1^{-i}/\widetilde{Q}_1^{-i}, ..., \widetilde{B}_L^{-i}/\widetilde{Q}_L^{-i}\right)$, $\widetilde{\widetilde{\mathbf{p}}}^i = \left(\widetilde{\widetilde{B}}_1^{-i}/\widetilde{\widetilde{Q}}_1^{-i}, ..., \widetilde{\widetilde{B}}_L^{-i}/\widetilde{\widetilde{Q}}_L^{-i}\right)$ *and*

$$\begin{aligned}
\widetilde{B}_l^{-i} &= (n-2)\,\overline{b}_l^i + \sum_{k=1,k\neq i}^{K} (n-1)\,\overline{b}_l^k + \sum_{k=1}^{K} \widetilde{b}_l^k, \\
\widetilde{Q}_l^{-i} &= (n-2)\,\overline{q}_l^i + \sum_{k=1,k\neq i}^{K} (n-1)\,\overline{q}_l^k + \sum_{k=1}^{K} \widetilde{q}_l^k, \\
\widetilde{\widetilde{B}}^{-i} &= \sum_{k=1}^{K} (n-1)\,\overline{b}_l^k + \sum_{k=1,k\neq i}^{K} \widetilde{b}_l^k, \\
\widetilde{\widetilde{Q}}^{-i} &= \sum_{k=1}^{K} (n-1)\,\overline{q}_l^k + \sum_{k=1,k\neq i}^{K} \widetilde{q}_l^k.
\end{aligned}$$

As before, the above conditions require that a distinguished deviating agent be at most better off by ϵ relative to the other agents of his type when at most one agent per population deviates. The variables

$(\widetilde{B}^{-i}, \widetilde{Q}^{-i})$ and $(\widetilde{\widetilde{B}}^{-i}, \widetilde{\widetilde{Q}}^{-i})$ give rise to the resulting prices before and after the deviation by the distinguished agent. Throughout the paper we assume that the economy in question has a symmetric full Nash equilibrium. Our first result below is stated analogously to the result in **PS**, but in the context of our evolutionary analysis.

Consider an economy $\mathcal{E}$ and suppose that $(\widehat{s}^1, ..., \widehat{s}^1; ...; \widehat{s}^K ..., \widehat{s}^K)$ is a symmetric full Nash equilibrium profile where $\widehat{s}^i = \left(\widehat{b}^i, \widehat{q}^i\right)$. Let

$$\max_i \widehat{q}_l^i = \lambda_l > 0, \text{ for all } l \in \{1, ..., L\}, \tag{20}$$

denote the largest offer (per population) in this equilibrium. We have the following result.

Theorem 1 *Consider an economy $\mathcal{E}$ that for any positive numbers ϵ, β, and λ satisfies the following: (1) $(\lambda_1, ..., \lambda_L) > \lambda (1, ..., 1)$, (2) $w^i < \beta (1, ..., 1)$, and (3) there exists $\delta (\epsilon, \beta, \lambda) > 0$ such that $[2(1+K)\beta + (K-1)] \frac{L^2(K-1)K^2\beta^4}{\lambda^4} \frac{n^2}{(n-1)^3} < \delta (\epsilon, \beta, \lambda)$. Then, the symmetric full Nash equilibrium profile $(\widehat{s}^1, ..., \widehat{s}^1; ...; \widehat{s}^K ..., \widehat{s}^K)$ of the market game associated with $\mathcal{E}$ is an ϵ-SESS.*

The proof appears in the Appendix. We briefly discuss the conditions needed for the above Theorem. The first condition requires that the full Nash equilibrium involves a strictly positive amount of trade in all markets. The second condition is also employed in **PS**. It assumes that individual endowments are "small." Finally, the third condition requires that the number of agents belonging to each type, n, be sufficiently large.

The proof proceeds by deriving explicit bounds for the effects of a deviating coalition on the terms of trade. Such effects are small provided that the economy is sufficiently large. Given this fact, the proof establishes that the resulting change in consumption baskets and, thus, in utility, for the non-deviating agents is also small. A couple of remarks are in order. First, notice that the above proof uses the "large economy" assumption. This turns out to be a necessary condition for the result. This feature is consistent with our experimental findings and we consider it to be a central feature of our model as it suggests that evolutionary arguments can be used as a foundation for Walrasian equilibria only when agents lack market power. We discuss this issue further below. We now proceed by demonstrating two additional facts. First, outcomes in which some markets are closed are not consistent with *SESS*. Second, if all markets are open, only full Nash equilibrium outcomes can be consistent with *SESS*. We begin by studying the first assertion.

Theorem 2 *Consider an economy $\mathcal{E}$ and suppose that a symmetric strategy profile $(s^1, ..., s^1; ...; s^K ..., s^K)$ is associated with an outcome where market l is closed; i.e., $b_l^i = q_l^i = 0$, for all $i \in I$. Then, there exists $\epsilon_0 > 0$ such that the profile $(s^1, ..., s^1; ...; s^K ..., s^K)$ is not an ϵ_0-SESS.*

The proof of this Theorem appears later in the Section. First, we briefly discuss the idea behind it. As mentioned earlier, we assume that a full Nash equilibrium exists. We will demonstrate that, in any state in which a market is closed, there exists a coalition of agents (one agent per type) such that if the coalition opens the market, at least one member of the coalition can always become better off than any non-deviant agent of his type after trading. In this sense the proof of the theorem is "destructive." We will describe a coalition deviation which guarantees a higher payoff for one (but possibly not all!) of the deviating agents. Indeed, that at least one member of the deviating coalition is better off is all that is required to violate the SESS criterion. A stronger form of Theorem 2 holds for the case of the autarky (no trade) equilibrium. In that case, there exists a coalition C such that, by introducing trade, coalition C opens all markets and makes *all* of its members better off. No approximation argument is needed in that case. We next demonstrate this last result.

Proposition 2 *Consider an economy $\mathcal{E}$ and the strategy profile associated with no trade, $b_l^i = q_l^i = 0$ for all $i \in I$ and all $l \in \{1, ..., L\}$. This profile is not an SESS. Moreover, there exists a deviating coalition of agents (one per population) such that each member of the coalition obtains a higher payoff than he had in the autarky.*

Proof: Denote the strategy profile associated with no trade as $\left(\overline{\mathbf{0}}, ..., \overline{\mathbf{0}}\right)$. By assumption, there exists a full Nash equilibrium, $(\widehat{s}^1, ..., \widehat{s}^1; ...; \widehat{s}^K ..., \widehat{s}^K)$. In the full Nash equilibrium each agent obtains a payoff strictly higher than the corresponding payoff associated with a no-trade strategy profile; i.e.,

$$U^i(\widehat{s}^i; \widehat{\mathbf{p}}^i) > U^i\left(\overline{\mathbf{0}}; \mathring{\mathbf{p}}^i\right), \tag{21}$$

where $\mathring{\mathbf{p}}^i = \left(\mathring{\mathbf{p}}_1^i, ..., \mathring{\mathbf{p}}_L^i\right) = (0/0, ..., 0/0)$. Note that, in the symmetric full Nash equilibrium, all agents from the same population play the same strategy. Thus, agent i's consumption of good l in the full Nash equilibrium can be written as

$$\begin{aligned}\widehat{c}_l^i &= w_l^i - \widehat{q}_l^i + \frac{\widehat{b}_l^i}{\sum_{j \in I} \widehat{b}_l^j} \sum_{j \in I} \widehat{q}_l^j \\ &= w_l^i - \widehat{q}_l^i + \frac{\widehat{b}_l^i}{n \sum_{k=1}^K \widehat{b}_l^k} n \sum_{k=1}^K \widehat{q}_l^k \\ &= w_l^i - \widehat{q}_l^i + \frac{\widehat{b}_l^i}{\sum_{k=1}^K \widehat{b}_l^k} \sum_{k=1}^K \widehat{q}_l^k.\end{aligned}$$

Now restrict attention to the no-trade strategy profile. Consider a coalition C, consisting of exactly one agent per type (K agents in total), and suppose that each agent in C deviates to the strategy prescribed by the full Nash equilibrium. In that case, the following strategy profile arises $(\widehat{s}^1, \overline{\mathbf{0}}, ..., \overline{\mathbf{0}}; \widehat{s}^2, \overline{\mathbf{0}}, ..., \overline{\mathbf{0}}; \widehat{s}^K, \overline{\mathbf{0}}, ..., \overline{\mathbf{0}})$. Note that each deviant agent i's individual consumption of good l is given by

$$c_l^i = w_l^i - \widehat{q}_l^i + \frac{\widehat{b}_l^i}{\sum_{k=1}^K \widehat{b}_l^k} \sum_{k=1}^K \widehat{q}_l^k = \widehat{c}_l^i. \tag{22}$$

Since $c_l^i = \widehat{c}_l^i$, for any $l \in \{1, ..., L\}$, for each deviant agent i, we have that $U^i(\widehat{s}^i; \mathbf{p}^i) = U^i(\widehat{s}^i; \widehat{\mathbf{p}}^i)$, where $\mathbf{p}^i = \left(p_1^i, ..., p_L^i\right)$ is the price vector faced by the deviant agent i. Therefore,

$$U^i(\widehat{s}^i; \mathbf{p}^i) = U^i(\widehat{s}^i; \widehat{\mathbf{p}}^i) > U^i\left(\overline{\mathbf{0}}; \mathring{\mathbf{p}}^i\right).$$

Hence the strategy profile associated with no trade $\left(\overline{\mathbf{0}}, ..., \overline{\mathbf{0}}\right)$ is not an *SESS*. ■

The above Proposition is consistent with our experimental findings. It offers a strong sense in which autarky, although a strong Nash equilibrium, is not likely to be observed. A general version of the above Proposition is contained in Theorem 2. In the proof below we assume that two agents form a coalition. One of these agents gives her endowment of the good from the market which was closed initially in exchange for an arbitrarily small amount, $\eta > 0$, of another good. This exchange does not affect prices on all previously opened markets, but makes one of the agents in the coalition "much better off" than any non-deviant agent of her type.

Proof of Theorem 2: Consider a symmetric strategy profile $(s^1, ..., s^1; ...; s^K ..., s^K)$ which is associated with an outcome where market l is closed. Note that all agents from the same population play the same strategy. Thus, agent i's consumption of good h is given by

$$\begin{aligned} c_h^i &= w_h^i - q_h^i + \frac{b_h^i}{\sum_{j \in I} b_h^j} \sum_{j \in I} q_h^j \\ &= w_h^i - q_h^i + \frac{b_h^i}{n \sum_{k=1}^K b_h^k} n \sum_{k=1}^K q_h^k \\ &= w_h^i - q_h^i + \frac{b_h^i}{\sum_{k=1}^K b_h^k} \sum_{k=1}^K q_h^k. \end{aligned}$$

Since each strategy profile uniquely determines players' consumption through (18), we have that

$$U^i\left(s^i; s^{-i}\right) = u^i\left(c_1^i, ..., c_L^i\right). \tag{23}$$

By assumption, there exists a market l which is not open, $b_l^i = q_l^i = 0$, for all $i \in I$. Choose an arbitrary agent, say of type i, with $w_l^i > 0$. If all other markets (in addition to that of good l) are closed, the result follows directly from Proposition 2. Thus, assume that there exists good h and type j such that $b_h^j > \eta > 0$. Now, consider a coalition $C = \{i, j\}$ and suppose that C deviates by opening the market for good l by adopting

$$\widetilde{q}_l^i = w_l^i, \ \widetilde{b}_l^j = \eta, \tag{24}$$

in exchange for

$$\widetilde{b}_h^i = b_h^i + \eta, \ \widetilde{b}_h^j = b_h^j - \eta. \tag{25}$$

Note that the prices for all goods except good l are not affected by the deviation of coalition C. Thus, each non-deviant agent of type j has the same payoff as before. However, the deviating agent, j, obtains a strictly higher payoff than any non-deviant agent of his type. More precisely, denote

$$\epsilon_0 = \frac{2}{3}\left[u^j\left(c_1^j,...,c_{l-1}^j,\widetilde{c}_l^j,c_{l+1}^j,...,c_L^i\right) - u^j\left(c_1^j,...,c_{l-1}^j,c_l^j,c_{l+1}^j,...,c_L^i\right)\right] > 0.$$

Let $\widetilde{c}_l^j$ denote the resulting consumption of good l by the deviant agent j. We then have

$$\begin{aligned}
&u^j\left(c_1^j,...,c_{l-1}^j,\widetilde{c}_l^j,c_{l+1}^j,...,c_{h-1}^j,\widetilde{c}_h^j,c_{h+1}^j,...,c_L^i\right) - u^j\left(c_1^j,...,c_{l-1}^j,c_l^j,c_{l+1}^j,...,c_L^i\right) \\
&= \left(u^j\left(c_1^j,...,c_{l-1}^j,\widetilde{c}_l^j,c_{l+1}^j,...,c_{h-1}^j,\widetilde{c}_h^j,c_{h+1}^j,...,c_L^i\right) - u^j\left(c_1^j,...,c_{l-1}^j,\widetilde{c}_l^j,c_{l+1}^j,...,c_L^i\right)\right) \\
&\quad + \left(u^j\left(c_1^j,...,c_{l-1}^j,\widetilde{c}_l^j,c_{l+1}^j,...,c_L^i\right) - u^j\left(c_1^j,...,c_{l-1}^j,c_l^j,c_{l+1}^j,...,c_L^i\right)\right) \\
&> \frac{3}{2}\epsilon_0 - \\
&\left|u^j\left(c_1^j,...,c_{l-1}^j,\widetilde{c}_l^j,c_{l+1}^j,...,c_{h-1}^j,\widetilde{c}_h^j,c_{h+1}^j,...,c_L^i\right) - u^j\left(c_1^j,...,c_{l-1}^j,\widetilde{c}_l^j,c_{l+1}^j,...,c_L^i\right)\right|.
\end{aligned}$$

The function $u^j\left(c_1^j,...,c_L^i\right)$ is continuous in all its arguments. Therefore, for all $\widetilde{c}_h^j$ such that $\left|\widetilde{c}_h^j - c_h^j\right| = \eta < \delta_1(\epsilon_0)$, we have

$$\left|u^j\left(c_1^j,...,c_{l-1}^j,\widetilde{c}_l^j,c_{l+1}^j,...,c_{h-1}^j,\widetilde{c}_h^j,c_{h+1}^j,...,c_L^i\right) - u^j\left(c_1^j,...,c_{l-1}^j,\widetilde{c}_l^j,c_{l+1}^j,...,c_L^i\right)\right| < \frac{\epsilon_0}{2}.$$

Thus, the strategy profile associated with an outcome where at least one of the markets is not open is not an ϵ_0-*SESS* for $\eta < \delta_1(\epsilon_0)$. ■

Next, we demonstrate another necessary condition for evolutionary stability. The resulting allocation must be a symmetric full Nash equilibrium. Like before, for any strategy profile $\left(t^1,...,t^K\right)$, and for all goods j for which the corresponding market is open, let $\max_i q_l^i = \lambda_l > 0$ denote the largest offer (per population) in the given state. Since the definition of ϵ-*SESS* involves symmetry, it is sufficient to consider only symmetric profiles. We have the following.

Theorem 3 *Let $\mathcal{E}$ be such that all markets are open and consider any symmetric profile $\left(t^1,...,t^1;...;t^K,...,t^K\right)$ that does not constitute a symmetric Nash equilibrium of the underlying market game. Let β and λ be positive numbers for which the following conditions hold: (1) $(\lambda_1,...,\lambda_L) > \lambda\,(1,...,1)$, and (2) $w^i < \beta\,(1,...,1)$. Then, there exists $\epsilon_0 > 0$ such that if (3) $0 < \frac{\beta^4}{\lambda^4}LK^2\left(\lambda + (LK+L)\,\beta\right)\frac{n^3}{(n-1)^4} < \delta\left(\epsilon_0,\beta,\lambda\right)$, then the profile $\left(t^1,...,t^K\right)$ is not an ϵ_0-SESS.*

The proof appears in the Appendix. Summarizing our findings, Theorems 1-3 demonstrate that for any $\epsilon > 0$, if the economy $\mathcal{E}$ is "large," any symmetric full Nash equilibrium profile of the market game associated with $\mathcal{E}$ is an ϵ-SESS. Proposition 1, from **PS**, demonstrates that any allocation resulting from a full Nash equilibrium in a large enough economy is approximately Walrasian. In this sense, SESS provides support for approximately Walrasian outcomes in large economies. We think that these results are consistent with our experimental findings of the previous section. In this sense, they provide plausible theoretical explanations for the observed behavior in our rudimentary 2x2 experiments.

As we mentioned before, the above results will *not* hold in general if the economy is populated by a small number of agents. In that case, by having a non-negligible effect on prices, an agent deviating from the full Nash equilibrium allocation may be able to make himself better off relative to the other agents of his type. Therefore, full Nash equilibria may not correspond to ϵ-*SESS* if agents have significant market power. While this observation is consistent with the traditionally held view that competitive outcomes arise when individual agents are of insignificant size, it is distinct from Vega-Redondo (1997), in which a competitive outcome is evolutionary stable in the context of a Cournot oligopoly model where agents have significant market power. This suggests that whether a partial or a general equilibrium framework is assumed matters when determining the evolutionary stability of Walrasian outcomes.

5 Discussion

Experimental results on double auctions (DA) give remarkably strong support for Walrasian equilibrium (WE) outcomes. This is true even when there are a small number of agents on both sides of the market. Vernon Smith (1982) referred to this property as a "scientific mystery."[14] This raises the important question of whether Walrasian price formation crucially depends on the choice of market institution. Is this support particular to the way a DA "aggregates" agents' actions and information, or is it a property of WE itself? In the second case, the apparent emergence of WE should be shared by a *variety of market mechanisms*, in addition to the DA. This question can be studied in the context of several models of price formation such as the partial equilibrium setups of Cournot/Bertrand-like models. However, little experimental work has been done to investigate the performance of different "general equilibrium" market setups.

Market games as in Shapley and Shubik (1977) offer such mechanisms in that they involve the general equilibrium properties of the Walrasian paradigm while, at the same time, they are fully specified non-cooperative games. These two features make them particularly suitable for the study whether Walrasian prices will emerge under different specifications of the underlying economic environment; i.e., the agents' preferences, endowments, etc.

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[14] Smith (1982, p. 945) recognized that "many economists express surprise, if not discomfort, with the evidence [that convergence to competitive equilibrium obtains with small numbers of competing agents]." He adds "the idea that a competitive equilibrium is an ideal frictionless state not likely to be approached in any observable market — and certainly not without a large number of agents with its assumed concomitant "price taking" behavior — is a deeply ingrained belief based on untested theory going back to Cournot. Since Cournot's theory does not specify an institution, it is unclear in what context the theory is supposed to have relevance."

The role of private information in Smith's scientific mystery is not fully understood. Since they completely abstract from private information frictions, market games offer a rudimentary setup that allows us to isolate issues related to the emergence of WE from the effects of private information. More precisely, rational behavior in a market game requires a standard Nash equilibrium play, as opposed to a Bayesian equilibrium in which subjects must form consistent beliefs about other subjects' values, etc. The formation of such belief/action assessments is a rather demanding task. In contrast, the only relevant variable in a market game is the price, as defined by the ratio of the aggregate bids. As the above discussion suggests, since subjects are playing a "simpler" game, their performance should be expected to be at least as good as that exhibited in the more complex DA setup.[15]

Another feature of the k-DA is that it is subject to severe multiplicity of equilibrium. Although less severe, multiplicity is also present in strategic market games. More precisely, among other partial autarky outcomes, such games always obtain a strict equilibrium that involves no trade. One can, thus, ask whether agents in a laboratory setup will "learn" to open markets when this would lead to a Pareto superior outcome. While obviously interesting, the issue of opening new markets has not been sufficiently explored in an experimental setup.

Even if one expects that subjects find it easier to play "rationally" in a market game than in a DA, an interesting issue arises as to which outcome will be eventually selected. In all finite games there is a discrepancy between the Nash equilibrium with trade and the Walrasian outcome. Which of the two will the subjects' actions converge to? On the one hand, Nash equilibrium is the outcome consistent with rationality. Alternatively, since the Nash equilibrium is socially inefficient, one might also expect that subjects will find their way to a socially efficient outcome, perhaps through bids that implement Walrasian prices. The question of whether a Nash or a Walrasian outcome will be selected becomes particularly interesting when it is studied as a function of the number of subjects in the game.

The above question is important since, unlike the Walrasian outcome, even the most efficient Nash outcome involves an inefficiency due to market power. Satterthwaite and Williams (2002) study this inefficiency in a wide class of mechanisms and conclude that the "worst-case" inefficiency converges to zero at the fastest possible rate in a k-DA. Postlewaite and Schmeidler (1978) established the connection between the most efficient Nash equilibrium of a "large enough" market game and approximate Walrasian equilibria of the underlying economy. Cason and Friedman (1997) report experimental findings that are closer to the Bayes-

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[15] Within the DA framework, the k-DA (see, e.g., Satterthwaite and Williams (1991)) is the closest to our setup. Saterthwhite and Williams find theoretical support for WE when the economy is large. In addition, they perform simulations that offer some support for WE even when the number of agents is small. Experiments on the k-DA by Kagel and Vogt (1993) found that as the number of traders increases efficiency also increases but at a very slow rate. In related experimental work, Cason and Friedman (1997) study a single call market in which buyers and sellers bids and asks are collected and then a uniform market clearing price prevails when the market closes. They find support for the predictions of Bayes-Nash equilibrium. However, they too report that observed efficiency is below the equilibrium prediction.

Nash equilibrium than the Walrasian equilibrium. The performance of market games in that regard has not been investigated experimentally and our paper offers a first attempt in this direction.

Throughout the paper we required stability against coalitions consisting of K agents (one per population). One could ask whether our results would be different if we required stability against *any* coalitions of size K (possibly several per population). Indeed, our results would hold under this more general specification (we adopted the more restrictive notion for notational convenience). That is because any outcome that does not satisfy our notion of stability would not satisfy the more general notion. In addition, full Nash equilibria will satisfy the more general notion since, provided that the economy is sufficiently large, there is no coalition consisting of K agents that will have an appreciable effect on the price vector. Thus, the approximate evolutionary stability of full Nash equilibria will remain intact under the more general specification.[16]

Peck and Shell (1990) and Ghosal and Morelli (2004) study variations of market games in which competitive outcomes prevail even when the number of traders is small. It would be interesting to study whether our evolutionary story can be embedded in their setups. An important extension of our analysis concerns the relation between our static *SESS* concept and the asymptotically stable points of a suitably defined dynamic system describing the learning process. This extension is left to future research. Finally, an advantage of the proposed setup is that it is simple enough to be implemented in an experimental environment. In future work, we plan to study under what specifications of preferences and endowments human subjects will exhibit behavior consistent with *SESS* in a laboratory environment.

Future work includes studying economies with many goods, more general preferences and multiple Walrasian equilibria. In addition, we could investigate the properties of different dynamic learning process that can give rise to *SESS*.

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[16] Clearly, the two notions will not be equivalent in all games. Our concept could be used in biological examples in which simultaneous mutations, say by a male and a female, might be needed in order to increase population fitness. See Noldeke and Samuelson (2003) and references therein for related examples in biology.

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Appendix

Proof of Theorem 1: Fix an economy $\mathcal{E}$ and let $\epsilon > 0$ be given. Suppose that $(\widehat{s}^1, ..., \widehat{s}^1; ...; \widehat{s}^K ..., \widehat{s}^K)$ is a symmetric full Nash equilibrium profile. Each strategy profile uniquely determines players' consumption through (18). Thus, in a symmetric full Nash equilibrium, agent i's consumption of good l is given by

$$\begin{aligned}\widehat{c}_l^i &= w_l^i - \widehat{q}_l^i + \frac{\widehat{b}_l^i}{\sum_{j \in I} \widehat{b}_l^j} \sum_{j \in I} \widehat{q}_l^j \\ &= w_l^i - \widehat{q}_l^i + \frac{\widehat{b}_l^i}{n \sum_{k=1}^K \widehat{b}_l^k} n \sum_{k=1}^K \widehat{q}_l^k \\ &= w_l^i - \widehat{q}_l^i + \frac{\widehat{b}_l^i}{\sum_{k=1}^K \widehat{b}_l^k} \sum_{k=1}^K \widehat{q}_l^k.\end{aligned}$$

Since all markets are open, $\frac{\sum_{k=1}^K \widehat{q}_l^k}{\sum_{k=1}^K \widehat{b}_l^k} > 0$. Note that *any unilateral deviation*, $t^i \neq \widehat{s}^i$, from the full Nash equilibrium by an agent of type i does not improve his payoff; i.e.,

$$U^i\left(\widehat{s}^i; \gamma^i\right) - U^i\left(t^i; \gamma^i\right) = U^i\left(\widehat{s}^i; \widehat{s}^{-i}\right) - U^i\left(t^i; \widehat{s}^{-i}\right) \geq 0,$$

where $\gamma^j = \left(\widehat{s}^j, \widehat{s}^j\right)$ for all $j \neq i$.

Consider a coalition of agents, $C \neq \emptyset$, consisting of at most one agent per type, who deviates from the full Nash equilibrium. Denote by t^i a deviation by an agent of type i.

Let $\widetilde{\widehat{c}}_l^i$ $\left(\widetilde{g}_l^i\right)$ denote the resulting individual consumption of good l by a non-deviant (deviant) agent of type i. Then, for any $t^i \neq \widehat{s}^i$, and for all γ^j such that $\gamma^j = \left(\widehat{s}^j, \widehat{s}^j\right)$ or $\gamma^j = \left(t^j, \widehat{s}^j\right)$, we have

$$\begin{gathered}U^i\left(\widehat{s}^i; \gamma^1, ..., \gamma^K\right) - U^i\left(t^i; \gamma^1, ..., \gamma^K\right) \\ = U^i\left(\widehat{s}^i; \gamma^1, ..., \gamma^K\right) - U^i\left(\widehat{s}^i; \gamma^i\right) \\ + U^i\left(\widehat{s}^i; \gamma^i\right) - U^i\left(t^i; \gamma^i\right) \\ + U^i\left(t^i; \gamma^i\right) - U^i\left(t^i; \gamma^1, ..., \gamma^K\right) \geq \\ - \left|U^i\left(\widehat{s}^i; \gamma^1, ..., \gamma^K\right) - U^i\left(\widehat{s}^i; \gamma^i\right)\right| - \left|U^i\left(t^i; \gamma^i\right) - U^i\left(t^i; \gamma^1, ..., \gamma^K\right)\right|.\end{gathered}$$

Next, we consider the differences $\left|U^i\left(\widehat{s}^i; \gamma^1, ..., \gamma^K\right) - U^i\left(\widehat{s}^i; \gamma^i\right)\right|$ and

$\left|U^i\left(t^i; \gamma^i\right) - U^i\left(t^i; \gamma^1, ..., \gamma^K\right)\right|$. From (18) note that

$$\left|U^i\left(\widehat{s}^i; \gamma^1, ..., \gamma^K\right) - U^i\left(\widehat{s}^i; \gamma^i\right)\right| = \left|u^i\left(\widetilde{\widehat{c}}_1^i, ..., \widetilde{\widehat{c}}_L^i\right) - u^i\left(d_1^i, ..., d_L^i\right)\right|, \tag{26}$$

where d_l^i denotes the individual consumption of good l by a non-deviant agent of type i when only one agent of type i deviates from the symmetric Nash equilibrium. Likewise,

$$\left|U^i\left(t^i; \gamma^i\right) - U^i\left(t^i; \gamma^1, ..., \gamma^K\right)\right| = \left|u^i\left(\widetilde{\widetilde{g}}_1^i, ..., \widetilde{\widetilde{g}}_L^i\right) - u^i\left(\widetilde{g}_1^i, ..., \widetilde{g}_L^i\right)\right|, \tag{27}$$

where $\widetilde{\widetilde{g}}_l^i$ denotes the individual consumption of good l by a deviant agent of type i when only one agent of type i (the deviator himself) deviates from the symmetric Nash equilibrium.

The function $u^i\left(c_1^i, ..., c_L^i\right)$ is continuous in all arguments. Therefore, for all $\left(c_1^i, ..., c_L^i\right)$ such that $\max_l \left|c_l^i - \widetilde{\widehat{c}}_l^i\right| < \delta_1(\epsilon)$, we have

$$\left|u^i\left(\widetilde{\widehat{c}}_1^i, ..., \widetilde{\widehat{c}}_L^i\right) - u^i\left(c_1^i, ..., c_L^i\right)\right| < \frac{\epsilon}{2}. \tag{28}$$

Similarly, for all $\left(c_1^i, ..., c_L^i\right)$ such that $\max_l \left|c_l^i - \widetilde{g}_l^i\right| < \delta_2(\epsilon)$, we have

$$\left|u^i\left(\widetilde{g}_1^i, ..., \widetilde{g}_L^i\right) - u^i\left(c_1^i, ..., c_L^i\right)\right| < \frac{\epsilon}{2}. \tag{29}$$

Define $\delta(\epsilon) = \min\left\{\delta_1(\epsilon), \delta_2(\epsilon)\right\}$. Next, we need to estimate the value of $\max_l \left|\widetilde{\widehat{c}}_l^i - d_l^i\right|$. We have:

$$\begin{aligned}
\max_l \left|\widetilde{\widehat{c}}_l^i - d_l^i\right| = \Bigg| w_l^i - \widehat{q}_l^i + \frac{\widehat{b}_l^i}{\sum_{k=1}^K \widetilde{b}_l^k + (n-1)\sum_{k=1}^K \widehat{b}_l^k}\left[\sum_{k=1}^K \widetilde{q}_l^k + (n-1)\sum_{k=1}^K \widehat{q}_l^k\right] - \left(w_l^i - \widehat{q}_l^i\right) \\
- \left(\frac{\widehat{b}_l^i}{\widetilde{b}_l^i + \sum_{k\neq i} \widehat{b}_l^k + (n-1)\sum_{k=1}^K \widehat{b}_l^k}\left[\widetilde{q}_l^i + \sum_{k\neq i, k=1}^K \widehat{q}_l^k + (n-1)\sum_{k=1}^K \widehat{q}_l^k\right]\right)\Bigg|
\end{aligned}$$

$$= \widehat{b}_l^i \left|\frac{\sum_{k\neq i,k=1}^K \widetilde{q}_l^k + \left(\widetilde{q}_l^i + (n-1)\sum_{k=1}^K \widehat{q}_l^k\right)}{\sum_{k\neq i,k=1}^K \widetilde{b}_l^k + \left(\widetilde{b}_l^i + (n-1)\sum_{k=1}^K \widehat{b}_l^k\right)} - \frac{\sum_{k\neq i,k=1}^K \widehat{q}_l^k + \left(\widetilde{q}_l^i + (n-1)\sum_{k=1}^K \widehat{q}_l^k\right)}{\sum_{k\neq i,k=1}^K \widehat{b}_l^k + \left(\widetilde{b}_l^i + (n-1)\sum_{k=1}^K \widehat{b}_l^k\right)}\right|. \tag{30}$$

Denote

$$Q = \left(\widetilde{q}_l^i + (n-1)\sum_{k=1}^K \widehat{q}_l^k\right),$$

and

$$B = \left(\widetilde{b}_l^i + (n-1)\sum_{k=1}^K \widehat{b}_l^k\right).$$

Then,

$$\begin{aligned}
&\left|\frac{\sum_{k\neq i,k=1}^K \widetilde{q}_l^k + \left(\widetilde{q}_l^i + (n-1)\sum_{k=1}^K \widehat{q}_l^k\right)}{\sum_{k\neq i,k=1}^K \widetilde{b}_l^k + \left(\widetilde{b}_l^i + (n-1)\sum_{k=1}^K \widehat{b}_l^k\right)} - \frac{\sum_{k\neq i,k=1}^K \widehat{q}_l^k + \left(\widetilde{q}_l^i + (n-1)\sum_{k=1}^K \widehat{q}_l^k\right)}{\sum_{k\neq i,k=1}^K \widehat{b}_l^k + \left(\widetilde{b}_l^i + (n-1)\sum_{k=1}^K \widehat{b}_l^k\right)}\right| \\
&= \left|\frac{\sum_{k\neq i,k=1}^K \widetilde{q}_l^k + Q}{\sum_{k\neq i,k=1}^K \widetilde{b}_l^k + B} - \frac{\sum_{k\neq i,k=1}^K \widehat{q}_l^k + Q}{\sum_{k\neq i,k=1}^K \widehat{b}_l^k + B}\right| \\
&= \left|\frac{\left(\sum_{k\neq i,k=1}^K \widetilde{q}_l^k + Q\right)\left(\sum_{k\neq i,k=1}^K \widehat{b}_l^k + B\right) - \left(\sum_{k\neq i,k=1}^K \widehat{q}_l^k + Q\right)\left(\sum_{k\neq i,k=1}^K \widetilde{b}_l^k + B\right)}{\left(\sum_{k\neq i,k=1}^K \widetilde{b}_l^k + B\right)\left(\sum_{k\neq i,k=1}^K \widehat{b}_l^k + B\right)}\right| \\
&= \left|\frac{B\sum_{k\neq i,k=1}^K \widetilde{q}_l^k + Q\sum_{k\neq i,k=1}^K \widehat{b}_l^k + \left(\sum_{k\neq i,k=1}^K \widetilde{q}_l^k\right)\left(\sum_{k\neq i,k=1}^K \widehat{b}_l^k\right)}{\left(\sum_{k\neq i,k=1}^K \widetilde{b}_l^k + B\right)\left(\sum_{k\neq i,k=1}^K \widehat{b}_l^k + B\right)}\right.
\end{aligned}$$

$$
-\frac{B\sum_{k\neq i,k=1}^{K}\widehat{q}_l^k - Q\sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k - \left(\sum_{k\neq i,k=1}^{K}\widehat{q}_l^k\right)\left(\sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k\right)}{\left(\sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k + B\right)\left(\sum_{k\neq i,k=1}^{K}\widehat{b}_l^k + B\right)}\Bigg|
$$

$$
\leq \frac{B\left|\sum_{k\neq i,k=1}^{K}\widetilde{q}_l^k - \sum_{k\neq i,k=1}^{K}\widehat{q}_l^k\right|}{\left(\sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k + B\right)\left(\sum_{k\neq i,k=1}^{K}\widehat{b}_l^k + B\right)} + \frac{Q\left|\sum_{k\neq i,k=1}^{K}\widehat{b}_l^k - \sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k\right|}{\left(\sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k + B\right)\left(\sum_{k\neq i,k=1}^{K}\widehat{b}_l^k + B\right)}
$$

$$
+\frac{\left|\left(\sum_{k\neq i,k=1}^{K}\widetilde{q}_l^k\right)\left(\sum_{k\neq i,k=1}^{K}\widehat{b}_l^k\right) - \left(\sum_{k\neq i,k=1}^{K}\widehat{q}_l^k\right)\left(\sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k\right)\right|}{\left(\sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k + B\right)\left(\sum_{k\neq i,k=1}^{K}\widehat{b}_l^k + B\right)}.
$$

We now proceed by estimating the above expression. Using (5), we have

$$
b_l^i \leq \sum_{l\in L}\frac{q_l^i}{\sum_{j\in I} q_l^j}\sum_{j\in I} b_l^j \leq L\frac{\beta}{(n-1)\,\lambda}nK\beta, \tag{31}
$$

or, summing up,

$$
B = \left(\widetilde{b}_l^i + (n-1)\sum_{k=1}^{K}\widehat{b}_l^k\right) \leq \left(1+(n-1)\,K\right)LK\frac{\beta^2}{\lambda}\frac{n}{(n-1)}.
$$

Using assumption (2) of the Theorem, we also have

$$
Q = \left(\widetilde{q}_l^i + (n-1)\sum_{k=1}^{K}\widehat{q}_l^k\right) \leq \left(1+(n-1)\,K\right)\beta
$$

and

$$
\left|\sum_{k\neq i,k=1}^{K}\widetilde{q}_l^k - \sum_{k\neq i,k=1}^{K}\widehat{q}_l^k\right| \leq (K-1)\,\beta. \tag{32}
$$

Inequality (31) leads to

$$
\left|\sum_{k\neq i,k=1}^{K}\widehat{b}_l^k - \sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k\right| \leq (K-1)\,KL\frac{\beta^2}{\lambda}\frac{n}{(n-1)}.
$$

Finally, assumption (2) and inequality (31) give

$$
\left|\left(\sum_{k\neq i,k=1}^{K}\widetilde{q}_l^k\right)\left(\sum_{k\neq i,k=1}^{K}\widehat{b}_l^k\right) - \left(\sum_{k\neq i,k=1}^{K}\widehat{q}_l^k\right)\left(\sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k\right)\right|
$$

$$
\leq LK\,(K-1)^2\,\frac{\beta^3}{\lambda}\frac{n}{(n-1)}.
$$

Note that assumption (1) implies the following inequalities:

$$
B = \left(\widetilde{b}_l^i + (n-1)\sum_{k=1}^{K}\widehat{b}_l^k\right) \geq (n-1)\,\lambda,
$$

and

$$
\left(\sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k + B\right)\left(\sum_{k\neq i,k=1}^{K}\widehat{b}_l^k + B\right) \geq (n-1)^2\,\lambda^2.
$$

Thus, we obtain

$$\frac{B\left|\sum_{k\neq i,k=1}^{K}\widetilde{q}_l^k-\sum_{k\neq i,k=1}^{K}\widehat{q}_l^k\right|}{\left(\sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k+B\right)\left(\sum_{k\neq i,k=1}^{K}\widehat{b}_l^k+B\right)}+\frac{Q\left|\sum_{k\neq i,k=1}^{K}\widehat{b}_l^k-\sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k\right|}{\left(\sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k+B\right)\left(\sum_{k\neq i,k=1}^{K}\widehat{b}_l^k+B\right)}$$

$$+\frac{\left|\left(\sum_{k\neq i,k=1}^{K}\widetilde{q}_l^k\right)\left(\sum_{k\neq i,k=1}^{K}\widehat{b}_l^k\right)-\left(\sum_{k\neq i,k=1}^{K}\widehat{q}_l^k\right)\left(\sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k\right)\right|}{\left(\sum_{k\neq i,k=1}^{K}\widetilde{b}_l^k+B\right)\left(\sum_{k\neq i,k=1}^{K}\widehat{b}_l^k+B\right)}\leq$$

$$\frac{\left(1+\left(n-1\right)K\right)LK\frac{\beta^2}{\lambda}\frac{n}{(n-1)}\left(K-1\right)\beta+\left(1+\left(n-1\right)K\right)\beta\left(K-1\right)KL\frac{\beta^2}{\lambda}\frac{n}{(n-1)}}{\left(n-1\right)^2\lambda^2}$$

$$+\frac{LK\left(K-1\right)^2\frac{\beta^3}{\lambda}\frac{n}{(n-1)}}{\left(n-1\right)^2\lambda^2}=\frac{L\left(K-1\right)K\beta^2}{\lambda^3}\left|\frac{\left[2\left(1+\left(n-1\right)K\right)\beta+\left(K-1\right)\right]n}{\left(n-1\right)^3}\right|.$$

Now, we finally have

$$\max_l\left|\widetilde{\widehat{c}}_l^{\,i}-d_l^i\right|<$$

$$\frac{L^2\left(K-1\right)K^2\beta^4}{\lambda^4}\left|\frac{\left[2\left(1+\left(n-1\right)K\right)\beta+\left(K-1\right)\right]n^2}{\left(n-1\right)^4}\right|<$$

$$\frac{L^2\left(K-1\right)K^2\beta^4}{\lambda^4}\left|\frac{\left[2\left(1+K\right)\beta+\left(K-1\right)\right]n^2}{\left(n-1\right)^3}\right|<\delta(\epsilon). \tag{33}$$

Similarly, we obtain

$$\max_l\left|\widetilde{\widetilde{g}}_l^{\,i}-\widetilde{g}_l^i\right|<\delta(\epsilon). \tag{34}$$

Hence, from (33) – (34) and (28) – (29) we get

$$U^i\left(\widehat{s}^i;\gamma^1,...,\gamma^K\right)-U^i\left(t^i;\gamma^1,...,\gamma^K\right)\geq$$

$$-\left|U^i\left(\widehat{s}^i;\gamma^1,...,\gamma^K\right)-U^i\left(\widehat{s}^i;\gamma^i\right)\right|$$

$$-\left|U^i\left(t^i;\gamma^i\right)-U^i\left(t^i;\gamma^1,...,\gamma^K\right)\right|>-\frac{\epsilon}{2}-\frac{\epsilon}{2}=-\epsilon.$$

In other words, a full Nash equilibrium profile is an ϵ-*SESS*. ■

Proof of Theorem 3: Consider any symmetric Non-Nash strategy profile $\left(t^1,...,t^1;...;t^K,...,t^K\right)$ such that all markets are open. Then, there exists an agent, say from population i, such that by deviating to a different strategy, say $\widehat{s}^i$, he obtains a strictly higher absolute payoff; i.e.,

$$U^i\left(\widehat{s}^i;t^{-i}\right)>U^i\left(t^i;t^{-i}\right). \tag{35}$$

To complete the proof, we need to demonstrate that this also results in the payoffs to non-deviant agents changing only by a small amount. To this end, let

$$\epsilon_0 = \frac{2}{3}\left[U^i\left(\widetilde{s}^i; t^{-i}\right) - U^i\left(t^i; t^{-i}\right)\right] > 0. \tag{36}$$

Let $\widetilde{g}^i_l\left(c^i_l\right)$ denote the resulting individual consumption of good l by a non-deviant (deviant) agent of type i. Then

$$\begin{aligned} &U^i\left(\widetilde{s}^i; \gamma^i\right) - U^i\left(t^i; \gamma^i\right) \\ &= \left(U^i\left(\widetilde{s}^i; t^{-i}\right) - U^i\left(t^i; t^{-i}\right)\right) + \left(U^i\left(t^i; t^{-i}\right) - U^i\left(t^i; \gamma^i\right)\right) \\ &> \frac{3}{2}\epsilon_0 - \left|U^i\left(t^i; t^{-i}\right) - U^i\left(t^i; \gamma^i\right)\right|, \end{aligned} \tag{37}$$

where $\gamma^j = \left(t^j, \overline{t^j}\right)$ for all $j \neq i$.

We now consider the difference $\left|U^i\left(t^i; t^{-i}\right) - U^i\left(t^i; \gamma^i\right)\right|$. Note that, from (18),

$$\left|U^i\left(t^i; t^{-i}\right) - U^i\left(t^i; \gamma^i\right)\right| = \left|u^i\left(\widetilde{g}^i_1, ..., \widetilde{g}^i_L\right) - u^i\left(\widetilde{\widetilde{g}}^i_1, ..., \widetilde{\widetilde{g}}^i_L\right)\right|, \tag{38}$$

where $\widetilde{\widetilde{g}}^i_l$ denotes the individual consumption of good l by a non-deviant agent of type i when only one agent of type i deviates from the symmetric strategy profile $\left(t^1, ..., t^1; ...; t^K, ..., t^K\right)$.

The function $u^i\left(\widetilde{g}^i_1, ..., \widetilde{g}^i_L\right)$ is continuous in all its arguments. Therefore, for all $\left(\widetilde{\widetilde{g}}^i_1, ..., \widetilde{\widetilde{g}}^i_L\right)$ such that $\max_l \left|\widetilde{g}^i_l - \widetilde{\widetilde{g}}^i_l\right| < \delta_1(\epsilon_0)$, we have

$$\left|u^i\left(\widetilde{g}^i_1, ..., \widetilde{g}^i_L\right) - u^i\left(\widetilde{\widetilde{g}}^i_1, ..., \widetilde{\widetilde{g}}^i_L\right)\right| < \frac{\epsilon_0}{2}. \tag{39}$$

Like in the proof of Theorem 1, we now need to estimate the value of $\max_l \left|\widetilde{g}^i_l - \widetilde{\widetilde{g}}^i_l\right|$. We have:

$$\begin{aligned} &\max_l \left|\widetilde{g}^i_l - \widetilde{\widetilde{g}}^i_l\right| = \\ &\left| w^i_l - q^i_l + \frac{b^i_l}{n\sum_{k=1}^K b^k_l}\left[n\sum_{k=1}^K q^k_l\right]\right. \\ &\left. - \left(w^i_l - q^i_l + \frac{b^i_l}{\widetilde{b}^i_l + \sum_{k\neq i,k=1}^K b^k_l + (n-1)\sum_{k=1}^K b^k_l}\left[\widetilde{q}^i_l + \sum_{k\neq i,k=1}^K q^k_l + (n-1)\sum_{k=1}^K q^k_l\right]\right)\right| \\ &= b^i_l\left|\frac{q^i_l + \left(\sum_{k\neq i,k=1}^K q^k_l + (n-1)\sum_{k=1}^K q^k_l\right)}{b^i_l + \left(\sum_{k\neq i,k=1}^K b^k_l + (n-1)\sum_{k=1}^K b^k_l\right)} - \frac{\widetilde{q}^i_l + \left(\sum_{k\neq i,k=1}^K q^k_l + (n-1)\sum_{k=1}^K q^k_l\right)}{\widetilde{b}^i_l + \left(\sum_{k\neq i,k=1}^K b^k_l + (n-1)\sum_{k=1}^K b^k_l\right)}\right|. \end{aligned}$$

Denote

$$Q = \left(\sum_{k\neq i,k=1}^K q^k_l + (n-1)\sum_{k=1}^K q^k_l\right)$$

and

$$B=\left(\sum_{k\neq i,k=1}^{K}b_l^k+(n-1)\sum_{k=1}^{K}b_l^k\right).$$

Then,

$$\left|\frac{q_l^i+\left(\sum_{k\neq i,k=1}^{K}q_l^k+(n-1)\sum_{k=1}^{K}q_l^k\right)}{b_l^i+\left(\sum_{k\neq i,k=1}^{K}b_l^k+(n-1)\sum_{k=1}^{K}b_l^k\right)}-\frac{\widetilde{q}_l^i+\left(\sum_{k\neq i,k=1}^{K}q_l^k+(n-1)\sum_{k=1}^{K}q_l^k\right)}{\widetilde{b}_l^i+\left(\sum_{k\neq i,k=1}^{K}b_l^k+(n-1)\sum_{k=1}^{K}b_l^k\right)}\right|$$

$$=\left|\frac{q_l^i+Q}{b_l^i+B}-\frac{\widetilde{q}_l^i+Q}{\widetilde{b}_l^i+B}\right|=\left|\frac{\left(q_l^i+Q\right)\left(\widetilde{b}_l^i+B\right)-\left(\widetilde{q}_l^i+Q\right)\left(b_l^i+B\right)}{\left(b_l^i+B\right)\left(\widetilde{b}_l^i+B\right)}\right|$$

$$=\left|\frac{q_l^iB+\widetilde{b}_l^iQ+q_l^i\widetilde{b}_l^i-\widetilde{q}_l^iB-b_l^iQ-\widetilde{q}_l^ib_l^i}{\left(b_l^i+B\right)\left(\widetilde{b}_l^i+B\right)}\right|=\left|\frac{\left(q_l^i-\widetilde{q}_l^i\right)B+\left(\widetilde{b}_l^i-b_l^i\right)Q+\left(q_l^i\widetilde{b}_l^i-\widetilde{q}_l^ib_l^i\right)}{\left(b_l^i+B\right)\left(\widetilde{b}_l^i+B\right)}\right|$$

$$\leq\frac{\left|q_l^i-\widetilde{q}_l^i\right|B+\left|\widetilde{b}_l^i-b_l^i\right|Q+\left|q_l^i\widetilde{b}_l^i-\widetilde{q}_l^ib_l^i\right|}{\left(b_l^i+B\right)\left(\widetilde{b}_l^i+B\right)}. \tag{40}$$

Note that from assumption (2) we have:

$$B=\left(\sum_{k\neq i,k=1}^{K}b_l^k+(n-1)\sum_{k=1}^{K}b_l^k\right)\leq(nK-1)\,\beta, \tag{41}$$

$$Q=\left(\sum_{k\neq i,k=1}^{K}q_l^k+(n-1)\sum_{k=1}^{K}q_l^k\right)\leq(nK-1)\,\beta, \tag{42}$$

$$\left|q_l^i-\widetilde{q}_l^i\right|\leq\beta, \tag{43}$$

$$\left|\widetilde{b}_l^i-b_l^i\right|\leq L\frac{\beta}{(n-1)\,\lambda}nK\beta, \tag{44}$$

and

$$\left|q_l^i\widetilde{b}_l^i-\widetilde{q}_l^ib_l^i\right|\leq L\frac{\beta}{(n-1)\,\lambda}nK\beta^2. \tag{45}$$

In addition, from assumption (1),

$$B=\left(\sum_{k\neq i,k=1}^{K}b_l^k+(n-1)\sum_{k=1}^{K}b_l^k\right)\geq(n-1)\,\lambda, \tag{46}$$

and

$$\left(b_l^i+B\right)\left(\widetilde{b}_l^i+B\right)\geq(n-1)^2\,\lambda^2. \tag{47}$$

Inequalities (41) − (47) imply that

$$\frac{\left|q_l^i-\widetilde{q}_l^i\right|B+\left|\widetilde{b}_l^i-b_l^i\right|Q+\left|q_l^i\widetilde{b}_l^i-\widetilde{q}_l^ib_l^i\right|}{\left(b_l^i+B\right)\left(\widetilde{b}_l^i+B\right)}\leq$$

$$\left|\frac{\beta\left(nK-1\right)\beta+L\frac{\beta}{(n-1)\lambda}nK\left(nK-1\right)\beta^2+L\frac{\beta}{(n-1)\lambda}nK\beta^2}{\left(n-1\right)^2\lambda^2}\right|\leq$$

$$\frac{\beta^2}{\lambda^3}\left|\frac{\lambda\left(nK-1\right)\left(n-1\right)+LKn\left(nK-1\right)\beta+L\beta nK}{\left(n-1\right)^3}\right|.$$

Since, from assumption (2),

$$b_l^i\leq L\frac{\beta}{\left(n-1\right)\lambda}nK\beta,$$

we obtain

$$\max_l\left|\widetilde{g}_l^i-\widetilde{\widetilde{g}}_l^i\right|\leq$$

$$\frac{\beta^2}{\lambda^3}\left|\frac{\lambda\left(nK-1\right)\left(n-1\right)+LKn\left(nK-1\right)\beta+L\beta nK}{\left(n-1\right)^3}\right|L\frac{\beta}{\left(n-1\right)\lambda}nK\beta$$

$$<\frac{\beta^4}{\lambda^4}LK\left|\frac{\lambda Kn^2+LK^2n^2\beta+LK\beta n}{\left(n-1\right)^3}\right|\frac{n}{\left(n-1\right)}\leq$$

$$\frac{\beta^4}{\lambda^4}LK^2\left(\lambda+\left(LK+L\right)\beta\right)\frac{n^3}{\left(n-1\right)^4}\leq\delta_0(\epsilon_0).$$

Finally,

$$U^i\left(\widetilde{s}^i;\gamma^i\right)-U^i\left(t^i;\gamma^i\right)>$$

$$\frac{3}{2}\epsilon_0-\left|U^i\left(t^i;t^{-i}\right)-U^i\left(t^i;\gamma^i\right)\right|\geq\frac{3}{2}\epsilon_0-\frac{1}{2}\epsilon_0\geq\epsilon_0.$$

Therefore, the profile $\left(t^1,...,t^K\right)$ is not an ϵ_0-*SESS*. ■

Appendix: Instructions Used in the Experiment

Here we provide the instructions used in treatment (1) involving groups of 4 players, and treatment (2) involving groups of 20 players. In both of these treatments, Type I (1) players begin with an (X,Y) endowment of (10,200) while Type II (2) players begin with an (X,Y) endowment of (200,10). Instructions for treatment (3), where groups of 20 players start at the full Nash equilibrium are omitted as they are identical to the instructions for treatment (2) except that the initial endowments are changed to (135,75) for Type 1 and (75,135) for Type 2 (the full NE values). Note further that all three treatments make use of the same payoff tables for Types 1 and 2, so these payoff tables only appear once in this appendix.

[Treatment (1) involving groups of 4 players].

Instructions

Welcome to this experiment in economic decision-making. Funds for this experimental study have been provided by the University of Pittsburgh. Please read these instructions carefully as they explain how you earn money from the decisions that you make. There is NO TALKING for the duration of this experiment. If you have a question, please raise your hand.

Today's session involves 16 participants. At the start of the session, all participants will be divided up equally into one of two types, Type 1 or Type 2. The types differ only in their preferences for the two types of goods, Good X and Good Y, as will be explained below. Your type will be shown on your computer screen and will not change for the duration of the experiment.

Sequence of Play in Each Period

In today's experiment you will participate in 25 periods of decision-making. At the start of each period, you will be randomly and anonymously matched with three other participants: one of the same type as you and two of the opposite type. All possible matchings involving two Type 1 and two Type 2 players are equally likely. You will never know the identity of any member of your group nor will they know your identity even after the session is over.

You will start the period with an initial "endowment" of Good X and of Good Y that will be shown to you on your computer screen. This initial endowment depends on your type and since your type never changes, your initial endowment will be the same at the start of each period. After you are randomly matched with three other participants, all four of you must simultaneously make a choice. Your choices are: 1) Trade Good X for Good Y, 2) Trade Good Y for Good X or 3) No Trade. Click on the radio button next to your choice and then click the Submit button. You can change your choice any time prior to clicking the Submit button.

If you choose options 1 or 2: Trade Good X for Good Y or Trade Good Y for Good X, then an input box will appear. In this box, you must type in the quantity of the good you wish to trade for the other good. Quantity amounts are restricted to be integers (no decimals, please). The minimum quantity you can offer to trade is 1 unit and the maximum quantity you can offer to trade is your entire endowment of that good (X or Y).

If you choose option 3: No Trade, you will not participate in any trade of Good X or Y for the other good and will end the period with your initial endowment of these two goods.

Your End-Of-Period Allocation

Your end-of-period-allocation is the amount of good X and good Y you have at the end of the period. This end-of-period allocation determines your payoff in points for the round as explained below.

If you chose **No Trade**, your end-of-period allocation is the SAME as your initial endowment of Good X and Good Y.

If you chose to **Trade Good X for Good Y**, or to **Trade Good Y for Good X**, then your end-of-period allocation depends on the amount you offered to trade and the amounts and goods the other three members of your group chose to trade.

Let Σx be the total amount of good x offered for trade by all members of your group including you and let Σy be the total amount of good Y offered for trade by all members of your group including you.

If you chose to Trade Good X for Good Y and you offered x units of Good X, then your end of period allocation is determined as follows:

End of period allocation of good X = Initial amount of Good X – x (amount offered in trade).
End of period allocation of good Y = Initial amount of Good Y + $(x / \Sigma x)\times \Sigma y$.

If you chose to Trade Good Y for Good X and offered y units of Good Y, then your end of period allocation is determined as follows:

End of period allocation of good X = Initial amount of Good X + $(y / \Sigma y)\times \Sigma x$
End of period allocation of good Y = Initial amount of Good Y – y (amount offered in trade).

Notice several things.

1) If you offer to trade Good X for Y or Good Y for X, your end of period allocation of the good you are trading is always reduced by the amount you offer in trade relative to your endowment level of that good.

2) If you are the only one offering to trade a good (X or Y), then $x / \Sigma x$ or $y / \Sigma y$ will be 1 – that is, you will get 100 percent of the other good offered in trade (Y or X), if any of that good *is* offered. Otherwise, your fraction or share of the other good (if offered) is proportional to you relative contribution to the total supply of the good you are offering in trade, $x / \Sigma x$ or $y / \Sigma y$.

3) If you offer to trade Good X for Y (Good Y for X) and $\Sigma y=0$ ($\Sigma x=0$), you do not acquire any additional units of Y (X). That is, in order to increase your end of period allocation of one good relative to your initial endowment level, there must be “supply” and “demand” on both sides of the market, that is, both Σy and Σx must be positive.

Your Payoff Each Period

Your payoff in points each period depends on your type and your end-of-period allocation.

Let x denote your end-of-period allocation of Good X and let y denote your end-of-period allocation of Good Y.

If you are Type 1, then your payoff in points for the period = $x^2 \times y$.

If you are Type 2, then your payoff in points for the period = $x \times y^2$.

The computer program will calculate your end-of-period points for you and this number will be reported to you on your screen.

Information Feedback and Record Keeping

At the end of each period, you will be reminded of your trading decision (if any) and informed of the total amount of good X and good Y offered by all members of your group including yourself (Σx and Σy). If you offered to trade a good, you will also learn the fraction of the total amount of that good that was provided by you and the amount of the other good you acquired as the result of that trade (if any). Finally, you will be told your final allocation of good X and good Y for the period, your total points calculated using the payoff function for your type and your cash earnings for the round in the event that round is chosen for payment (as discussed below).

Following each round of play, please record this information on your record sheet under the appropriate headings.

Payments

At the end of the 25 periods played in today's session, one of the 25 periods will be chosen at random. The points you earned in that round will be converted into dollars at the rate of $1=100,000 points. For your convenience, we report your dollar payoff for each period, but remember that only one period will be randomly chosen for actual payment in cash at the end of the session.

For your convenience, we attach two Tables, one for Type 1 and one for Type 2. These tables show the dollar payments you can earn from various end-of-period allocations using the conversion factor of 100,000 points = $1. Notice that these tables do not include all possible or feasible payoffs, but they are comprehensive enough to give you some sense of what your payoffs would be for various end-of-period allocations you might achieve. In reading these tables notice that as you increase your end of period allocation of both goods (move down and to the right), your payoffs increase. The shaded bands in these Tables are there to help you assess payoff increases. Roughly speaking, your payoff doubles as you move down and to the right from the midpoint of one shaded band to the midpoint of the next lower band.

The computer program will convert your end of period point total into dollars and this will be reported to you on your decision screen.

Initial Endowments

If you are Type 1 you will start each period with 10 units of Good X and 200 units of Good Y.

If you are Type 2 you will start each period with 200 units of Good X and 10 units of Good Y.

Things to Consider

1. If you choose No Trade, then your end-of-period allocation equals your initial endowment. Your payoff in points if you are Type 1 is: $10\times10\times200=20{,}000$ points, and your payoff if you are Type 2 is $200\times10\times10=20{,}000$ points. In either case, your dollar earnings would be $(20,0000/100,0000)=$0.20. If you choose not to trade your payoff is certain – your end-of period allocation is unaffected by the decisions of other members of your four-player group.

2. If you are Type 1, then your payoff increases over your endowment payoff if you successfully acquire more of either Good X or Good Y. As the payoff table for Type 1 players indicates, your payoff increases more rapidly if you succeed in obtaining Good X in trade (trade Y for X) than if you succeed in obtaining Good Y in trade (trade Good X for Y). However any increase in your payoff over your endowment payoff will depend on other players in your group offering to trade some of the good opposite to the good you are offering to trade.

3. If you are Type 2, then your payoff increases as you successfully acquire more of either Good X or Good Y. As the payoff table for Type 2 players indicates, your payoff increases more rapidly if you succeed in obtaining Good Y in trade (trade X for Y) than if you succeed in obtaining Good X in trade (trade Good Y for X). However again, any increase in your payoff over your endowment payoff will depend on other players in your group offering to trade some of the good opposite to the good you are offering to trade.

Questions

Now is the time for questions. I am happy to answer any questions about the rules of play and payoff determination as described in these instructions.

Quiz

Before proceeding, we ask that you answer the attached quiz questions. You do not need to write your name on this quiz and it does not affect your payoff in any way. We just want to verify that you have comprehended these instructions. If any quiz questions are answered incorrectly, we will go over the relevant part of the instructions again.

Pre-Experiment Quiz: Please answer the questions in the space provided. You do not need to write your name on this quiz. When you are done pass your quiz answers to the experimenter. We will review any incorrect answers before proceeding.

The numbers in these quiz questions are examples only. Actual numbers in the experiment may be quite different.

1. True or false: Your Type, 1 or 2, will be the same in all periods.

2. True or false: You *must* offer to trade either good X for good Y or good Y for good X.

3. Suppose you initially have 10 units of good X and 200 units of good Y. *If you choose to trade*, what is the minimum and maximum units of good X can you offer in trade?
   What is the minimum and maximum units of good Y can you offer in trade?

4. Suppose you are Type 2: You initially have 200 units of Good X and 10 units of Good Y.
   a. Suppose you offer 50 units of good X for good Y. What is your end of period allocation of good X?

   b. Suppose the total amount offered of good X (Σx) including your own contribution is 100, what is your share of the total amount of good Y (Σy)?

   c. If the total amount offered of good Y (Σy) is 210, what is your end of period allocation of good Y?

   d. Using your end of period allocation of goods X and Y (your answers to parts a and c), what would be your payoff in dollars for this period if it were the one chosen at random for payment?

5. Suppose you are Type 1: You initially have 10 units of Good X and 200 units of Good Y.
   a. Suppose you offer 150 units of good Y for good X. What is your end of period allocation of good Y?

   b. Suppose the total amount offered of good Y including your own contribution (Σy) is 150, what is your share of the total amount of good X (Σx)?

   c. If the total amount offered of good X (Σx) is 0, what is your end of period allocation of good X?

   d. Using your end of period allocation of goods X and Y (your answers to parts a and c), what would be your (approximate) payoff in dollars for this period if it were the one chosen at random for payment?

[Treatment (2) involving groups of 20 players].

Instructions

Welcome to this experiment in economic decision-making. Funds for this experimental study have been provided by the University of Pittsburgh. Please read these instructions carefully as they explain how you earn money from the decisions that you make. There is NO TALKING for the duration of this experiment. If you have a question, please raise your hand.

Today's session involves 20 participants. At the start of the session, all participants will be divided up equally into one of two types, Type 1 and Type 2, so that there are precisely 10 of each type. The types differ only in their preferences for the two types of goods, Good X and Good Y, as will be explained below. Your type will be shown on your computer screen and will not change for the duration of the experiment.

Sequence of Play in Each Period

In today's experiment you will participate in 25 periods of decision-making. In each period you will make decisions that will affect outcomes for all other participants and the decisions of all other participants will affect your outcome as well. You will never know the identity of any other individual nor will they know your identity even after the session is over.

You will start the period with an initial "endowment" of Good X and of Good Y that will be shown to you on your computer screen. This initial endowment depends on your type and since your type never changes, your initial endowment will be the same at the start of each period. All participants must then simultaneously make a choice. Your choices are: 1) Trade Good X for Good Y, 2) Trade Good Y for Good X or 3) No Trade. Click on the radio button next to your choice and then click the Submit button. You can change your choice any time prior to clicking the Submit button.

If you choose options 1 or 2: Trade Good X for Good Y or Trade Good Y for Good X, then an input box will appear. In this box, you must type in the quantity of the good you wish to trade for the other good. Quantity amounts are restricted to be integers (no decimals, please). The minimum quantity you can offer to trade is 1 unit and the maximum quantity you can offer to trade is your entire endowment of that good (X or Y).

If you choose option 3: No Trade, you will not participate in any trade of Good X or Y for the other good and will end the period with your initial endowment of these two goods.

Your End-Of-Period Allocation

Your end-of-period-allocation is the amount of good X and good Y you have at the end of the period. This end-of-period allocation determines your payoff in points for the round as explained below.

If you chose **No Trade**, your end-of-period allocation is the SAME as your initial endowment of Good X and Good Y.

If you chose to **Trade Good X for Good Y**, or to **Trade Good Y for Good X**, then your end-of-period allocation depends on the amount you offered to trade and the amounts and goods the other participants chose to trade.

Let Σx be the total amount of good x offered for trade by participants including you and let Σy be the total amount of good Y offered for trade by all participants including you.

If you chose to Trade Good X for Good Y and you offered x units of Good X, then your end of period allocation is determined as follows:

End of period allocation of good X = Initial amount of Good X – x (amount offered in trade).
End of period allocation of good Y = Initial amount of Good Y + $(x / \Sigma x) \times \Sigma y$.

If you chose to Trade Good Y for Good X and offered y units of Good Y, then your end of period allocation is determined as follows:

End of period allocation of good X = Initial amount of Good X + $(y / \Sigma y) \times \Sigma x$
End of period allocation of good Y = Initial amount of Good Y – y (amount offered in trade).

Notice several things.

1) If you offer to trade Good X for Y or Good Y for X, your end of period allocation of the good you are trading is always reduced by the amount you offer in trade relative to your endowment level of that good.

2) If you are the only one offering to trade a good (X or Y), then $x / \Sigma x$ or $y / \Sigma y$ will be 1 – that is, you will get 100 percent of the other good offered in trade (Y or X), if any of that good *is* offered. Otherwise, your fraction or share of the other good (if offered) is proportional to you relative contribution to the total supply of the good you are offering in trade, $x / \Sigma x$ or $y / \Sigma y$.

3) If you offer to trade Good X for Y (Good Y for X) and $\Sigma y=0$ ($\Sigma x=0$), you do not acquire any additional units of Y (X). That is, in order to increase your end of period allocation of one good relative to your initial endowment level, there must be “supply” and “demand” on both sides of the market, that is, both Σy and Σx must be positive.

Your Payoff Each Period

Your payoff in points each period depends on your type and your end-of-period allocation.

Let *x* denote your end-of-period allocation of Good X and let *y* denote your end-of-period allocation of Good Y.

If you are Type 1, then your payoff in points for the period = $x^2 \times y$.

If you are Type 2, then your payoff in points for the period = $x \times y^2$.

The computer program will calculate your end-of-period points for you and this number will be reported to you on your screen.

Information Feedback and Record Keeping

At the end of each period, you will be reminded of your trading decision (if any) and informed of the total amount of good X and good Y offered by all participants including yourself (Σx and Σy). If you offered to trade a good, you will also learn the fraction of the total amount of that good that was provided by you and the amount of the other good you acquired as the result of that trade (if any). Finally, you will be told your final allocation of good X and good Y for the period, your total points calculated using the payoff function for your type and your cash earnings for the round in the event that round is chosen for payment (as discussed below).

Following each round of play, please record this information on your record sheet under the appropriate headings.

Payments

At the end of the 25 periods played in today's session, one of the 25 periods will be chosen at random. The points you earned in that round will be converted into dollars at the rate of $1=100,000 points. For your convenience, we report your dollar payoff for each period, but remember that only one period will be randomly chosen for actual payment in cash at the end of the session.

For your convenience, we attach two Tables, one for Type 1 and one for Type 2. These tables show the dollar payments you can earn from various end-of-period allocations using the conversion factor of 100,000 points = $1. Notice that these tables do not include all possible or feasible payoffs, but they are comprehensive enough to give you some sense of what your payoffs would be for various end-of-period allocations you might achieve. In reading these tables notice that as you increase your end of period allocation of both goods (move down and to the right), your payoffs increase. The shaded bands in these Tables are there to help you assess payoff increases. Roughly speaking, your payoff doubles as you move down and to the right from the midpoint of one shaded band to the midpoint of the next lower band.

The computer program will convert your end of period point total into dollars and this will be reported to you on your decision screen.

Initial Endowments

If you are Type 1 you will start each period with 10 units of Good X and 200 units of Good Y.

If you are Type 2 you will start each period with 200 units of Good X and 10 units of Good Y.

Things to Consider

4. If you choose No Trade, then your end-of-period allocation equals your initial endowment. Your payoff in points if you are Type 1 is: $10 \times 10 \times 200 = 20{,}000$ points, and your payoff if you are Type 2 is $200 \times 10 \times 10 = 20{,}000$ points. In either case, your dollar earnings would be \$(20,0000/100,0000)=\$0.20. If you choose not to trade your payoff is certain – your end-of period allocation is unaffected by the decisions of other participants.

5. If you are Type 1, then your payoff increases over your endowment payoff if you successfully acquire more of either Good X or Good Y. As the payoff table for Type 1 players indicates, your payoff increases more rapidly if you succeed in obtaining Good X in trade (trade Y for X) than if you succeed in obtaining Good Y in trade (trade Good X for Y). However any increase in your payoff over your endowment payoff will depend on other participants offering to trade some of the good opposite to the good you are offering to trade.

6. If you are Type 2, then your payoff increases as you successfully acquire more of either Good X or Good Y. As the payoff table for Type 2 players indicates, your payoff increases more rapidly if you succeed in obtaining Good Y in trade (trade X for Y) than if you succeed in obtaining Good X in trade (trade Good Y for X). However again, any increase in your payoff over your endowment payoff will depend on other participants offering to trade some of the good opposite to the good you are offering to trade.

Questions

Now is the time for questions. I am happy to answer any questions about the rules of play and payoff determination as described in these instructions.

Quiz

Before proceeding, we ask that you answer the attached quiz questions. You do not need to write your name on this quiz and it does not affect your payoff in any way. We just want to verify that you have comprehended these instructions. If any quiz questions are answered incorrectly, we will go over the relevant part of the instructions again.

Pre-Experiment Quiz: Please answer the questions in the space provided. You do not need to write your name on this quiz. When you are done pass your quiz answers to the experimenter. We will review any incorrect answers before proceeding.

The numbers in these quiz questions are examples only. Actual numbers in the experiment may be quite different.

6. True or false: Your Type, 1 or 2, will be the same in all periods.

7. True or false: You *must* offer to trade either good X for good Y or good Y for good X.

8. Suppose you initially have 10 units of good X and 200 units of good Y. *If you choose to trade*, what is the minimum and maximum units of good X can you offer in trade?
   What is the minimum and maximum units of good Y can you offer in trade?

9. Suppose you are Type 2: You initially have 200 units of Good X and 10 units of Good Y.
   a. Suppose you offer 50 units of good X for good Y. What is your end of period allocation of good X?

   b. Suppose the total amount offered of good X (Σx) including your own contribution is 100, what is your share of the total amount of good Y (Σy)?

   c. If the total amount offered of good Y (Σy) is 210, what is your end of period allocation of good Y?

   d. Using your end of period allocation of goods X and Y (your answers to parts a and c), what would be your payoff in dollars for this period if it were the one chosen at random for payment?

10. Suppose you are Type 1: You initially have 10 units of Good X and 200 units of Good Y.
    a. Suppose you offer 150 units of good Y for good X. What is your end of period allocation of good Y?

    b. Suppose the total amount offered of good Y including your own contribution (Σy) is 150, what is your share of the total amount of good X (Σx)?

    c. If the total amount offered of good X (Σx) is 0, what is your end of period allocation of good X?

    d. Using your end of period allocation of goods X and Y (your answers to parts a and c), what would be your payoff in dollars for this period if it were the one chosen at random for payment?

Type 1 Player Payoffs in Dollars

End of Period Y

End of Period X

X/Y	10	15	25	35	45	55	65	75	85	95	105	115	125	135	140	150	200	250	300
10	0.010	0.015	0.025	0.035	0.045	0.055	0.065	0.075	0.085	0.095	0.105	0.115	0.125	0.135	0.140	0.150	0.200	0.250	0.300
15	0.023	0.034	0.056	0.079	0.101	0.124	0.146	0.169	0.191	0.214	0.236	0.259	0.281	0.304	0.315	0.338	0.450	0.563	0.675
25	0.063	0.094	0.156	0.219	0.281	0.344	0.406	0.469	0.531	0.594	0.656	0.719	0.781	0.844	0.875	0.938	1.250	1.563	1.875
35	0.123	0.184	0.306	0.429	0.551	0.674	0.796	0.919	1.041	1.164	1.286	1.409	1.531	1.654	1.715	1.838	2.450	3.063	3.675
45	0.203	0.304	0.506	0.709	0.911	1.114	1.316	1.519	1.721	1.924	2.126	2.329	2.531	2.734	2.835	3.038	4.050	5.063	6.075
55	0.303	0.454	0.756	1.059	1.361	1.664	1.966	2.269	2.571	2.874	3.176	3.479	3.781	4.084	4.235	4.538	6.050	7.563	9.075
65	0.423	0.634	1.056	1.479	1.901	2.324	2.746	3.169	3.591	4.014	4.436	4.859	5.281	5.704	5.915	6.338	8.450	10.563	12.675
75	0.563	0.844	1.406	1.969	2.531	3.094	3.656	4.219	4.781	5.344	5.906	6.469	7.031	7.594	7.875	8.438	11.250	14.063	16.875
85	0.723	1.084	1.806	2.529	3.251	3.974	4.696	5.419	6.141	6.864	7.586	8.309	9.031	9.754	10.115	10.838	14.450	18.063	21.675
95	0.903	1.354	2.256	3.159	4.061	4.964	5.866	6.769	7.671	8.574	9.476	10.379	11.281	12.184	12.635	13.538	18.050	22.563	27.075
105	1.103	1.654	2.756	3.859	4.961	6.064	7.166	8.269	9.371	10.474	11.576	12.679	13.781	14.884	15.435	16.538	22.050	27.563	33.075
115	1.323	1.984	3.306	4.629	5.951	7.274	8.596	9.919	11.241	12.564	13.886	15.209	16.531	17.854	18.515	19.838	26.450	33.063	39.675
125	1.563	2.344	3.906	5.469	7.031	8.594	10.156	11.719	13.281	14.844	16.406	17.969	19.531	21.094	21.875	23.438	31.250	39.063	46.875
135	1.823	2.734	4.556	6.379	8.201	10.024	11.846	13.669	15.491	17.314	19.136	20.959	22.781	24.604	25.515	27.338	36.450	45.563	54.675
140	1.960	2.940	4.900	6.860	8.820	10.780	12.740	14.700	16.660	18.620	20.580	22.540	24.500	26.460	27.440	29.400	39.200	49.000	58.800
150	2.250	3.375	5.625	7.875	10.125	12.375	14.625	16.875	19.125	21.375	23.625	25.875	28.125	30.375	31.500	33.750	45.000	56.250	67.500
200	4.000	6.000	10.000	14.000	18.000	22.000	26.000	30.000	34.000	38.000	42.000	46.000	50.000	54.000	56.000	60.000	80.000	100.000	120.000
250	6.250	9.375	15.625	21.875	28.125	34.375	40.625	46.875	53.125	59.375	65.625	71.875	78.125	84.375	87.500	93.750	125.000	156.250	187.500
300	9.000	13.500	22.500	31.500	40.500	49.500	58.500	67.500	76.500	85.500	94.500	103.500	112.500	121.500	126.000	135.000	180.000	225.000	270.000

Type 2 Player Payoffs in Dollars

End of Period Y

End of Period X

X/Y	10	15	25	35	45	55	65	75	85	95	105	115	125	135	140	150	200	250	300
10	0.010	0.023	0.063	0.123	0.203	0.303	0.423	0.563	0.723	0.903	1.103	1.323	1.563	1.823	1.960	2.250	4.000	6.250	9.000
15	0.015	0.034	0.094	0.184	0.304	0.454	0.634	0.844	1.084	1.354	1.654	1.984	2.344	2.734	2.940	3.375	6.000	9.375	13.500
25	0.025	0.056	0.156	0.306	0.506	0.756	1.056	1.406	1.806	2.256	2.756	3.306	3.906	4.556	4.900	5.625	10.000	15.625	22.500
35	0.035	0.079	0.219	0.429	0.709	1.059	1.479	1.969	2.529	3.159	3.859	4.629	5.469	6.379	6.860	7.875	14.000	21.875	31.500
45	0.045	0.101	0.281	0.551	0.911	1.361	1.901	2.531	3.251	4.061	4.961	5.951	7.031	8.201	8.820	10.125	18.000	28.125	40.500
55	0.055	0.124	0.344	0.674	1.114	1.664	2.324	3.094	3.974	4.964	6.064	7.274	8.594	10.024	10.780	12.375	22.000	34.375	49.500
65	0.065	0.146	0.406	0.796	1.316	1.966	2.746	3.656	4.696	5.866	7.166	8.596	10.156	11.846	12.740	14.625	26.000	40.625	58.500
75	0.075	0.169	0.469	0.919	1.519	2.269	3.169	4.219	5.419	6.769	8.269	9.919	11.719	13.669	14.700	16.875	30.000	46.875	67.500
85	0.085	0.191	0.531	1.041	1.721	2.571	3.591	4.781	6.141	7.671	9.371	11.241	13.281	15.491	16.660	19.125	34.000	53.125	76.500
95	0.095	0.214	0.594	1.164	1.924	2.874	4.014	5.344	6.864	8.574	10.474	12.564	14.844	17.314	18.620	21.375	38.000	59.375	85.500
105	0.105	0.236	0.656	1.286	2.126	3.176	4.436	5.906	7.586	9.476	11.576	13.886	16.406	19.136	20.580	23.625	42.000	65.625	94.500
115	0.115	0.259	0.719	1.409	2.329	3.479	4.859	6.469	8.309	10.379	12.679	15.209	17.969	20.959	22.540	25.875	46.000	71.875	103.500
125	0.125	0.281	0.781	1.531	2.531	3.781	5.281	7.031	9.031	11.281	13.781	16.531	19.531	22.781	24.500	28.125	50.000	78.125	112.500
135	0.135	0.304	0.844	1.654	2.734	4.084	5.704	7.594	9.754	12.184	14.884	17.854	21.094	24.604	26.460	30.375	54.000	84.375	121.500
140	0.140	0.315	0.875	1.715	2.835	4.235	5.915	7.875	10.115	12.635	15.435	18.515	21.875	25.515	27.440	31.500	56.000	87.500	126.000
150	0.150	0.338	0.938	1.838	3.038	4.538	6.338	8.438	10.838	13.538	16.538	19.838	23.438	27.338	29.400	33.750	60.000	93.750	135.000
200	0.200	0.450	1.250	2.450	4.050	6.050	8.450	11.250	14.450	18.050	22.050	26.450	31.250	36.450	39.200	45.000	80.000	125.000	180.000
250	0.250	0.563	1.563	3.063	5.063	7.563	10.563	14.063	18.063	22.563	27.563	33.063	39.063	45.563	49.000	56.250	100.000	156.250	225.000
300	0.300	0.675	1.875	3.675	6.075	9.075	12.675	16.875	21.675	27.075	33.075	39.675	46.875	54.675	58.800	67.500	120.000	187.500	270.000

Figure 1: Average Final Allocations, Last 5 Rounds,
Treatment Where n=2 and Initial Endowments for Type 1, 2 are: (10, 200), (200,10)

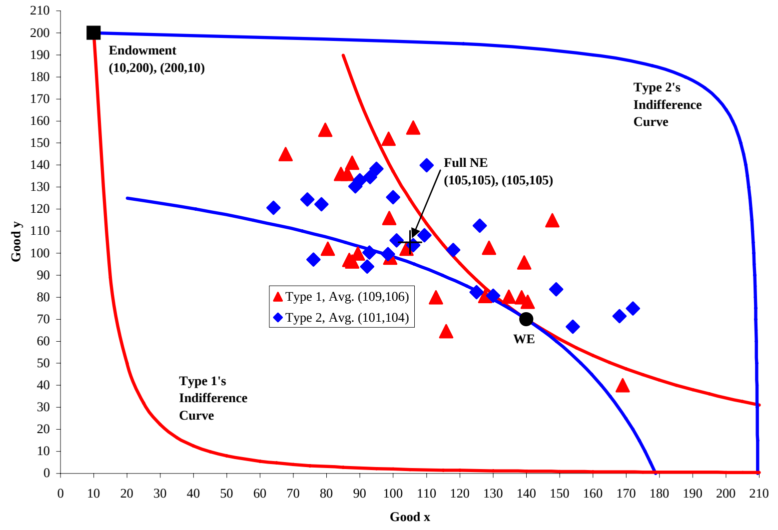

Figure 2: Average Final Allocations, Last 5 Rounds,
Treatment Where n=10 and Initial Endowments for Type 1, 2 are: (10, 200), (200,10)

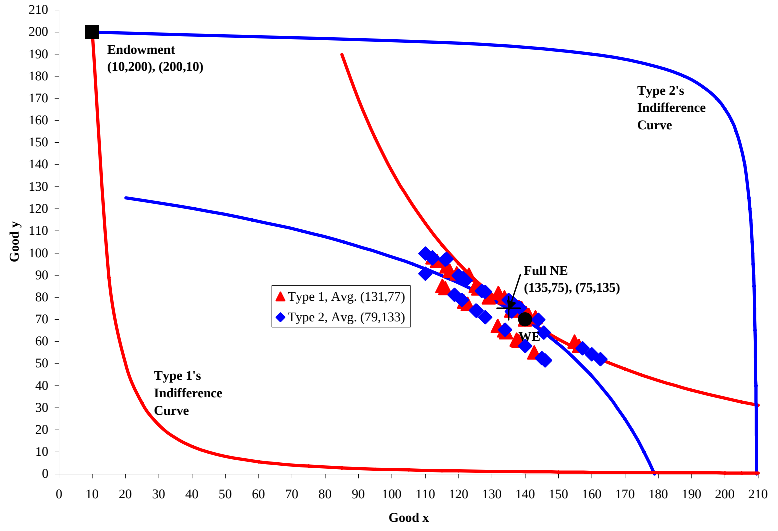

Figure 3: Average Final Allocations, Last 5 Rounds,
Treatment Where n=10 and Initial Endowments for Type 1, 2 are: (135, 75), (75,135)

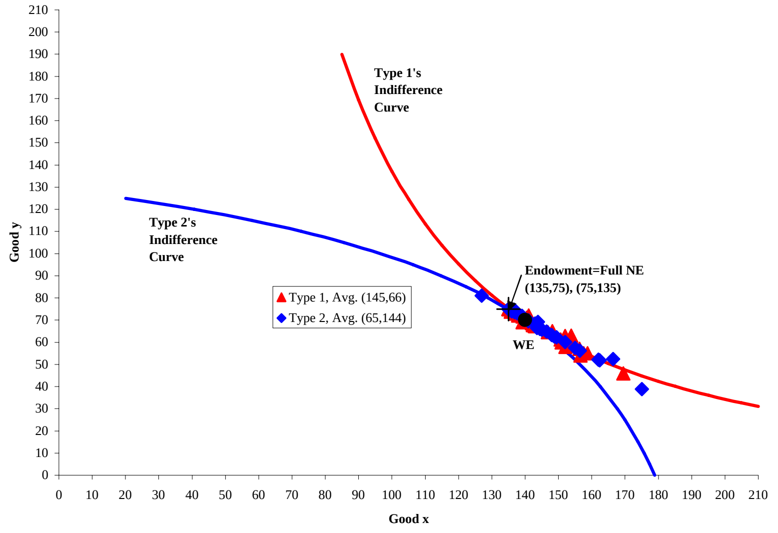

Figure 4: Average Bids Over All Rounds of a Session
Treatment Where n=2 and Initial Endowments for Type 1, 2 are: (10, 200), (200,10)

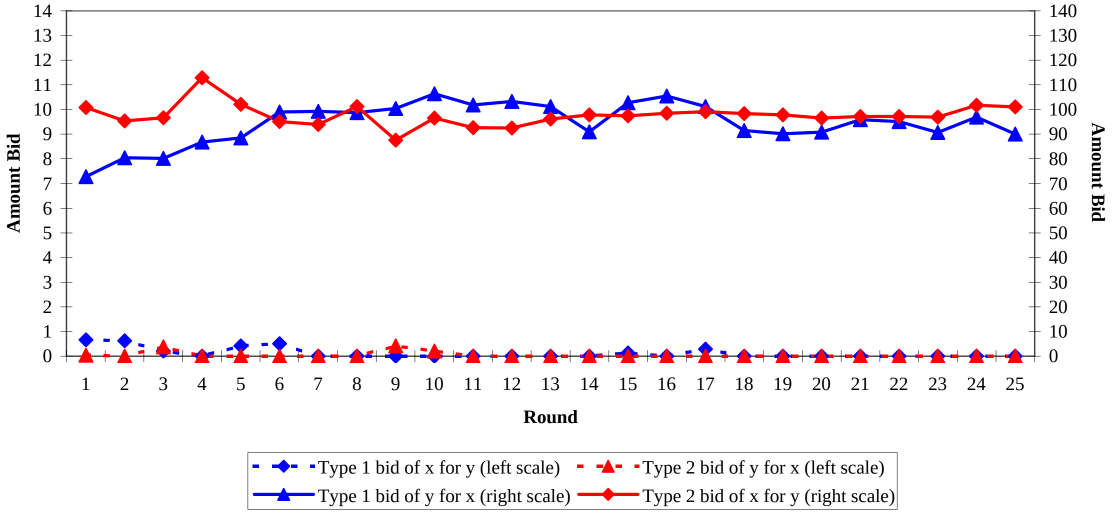

Figure 5: Average Bids Over All Rounds of a Session
Treatment Where n=10 and Initial Endowments for Type 1, 2 are: (10, 200), (200,10)

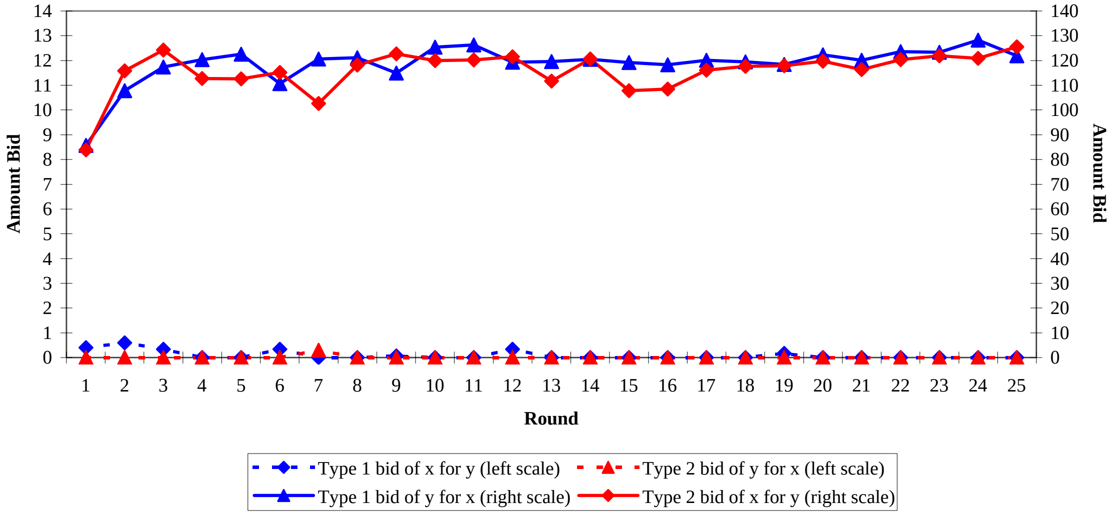

Figure 6: Average Bids Over All Rounds of a Session
Treatment Where n=10 and Initial Endowments for Type 1, 2 are: (135, 75), (75,135)